Resolving Fermat's Last Theorem by Prime Factor Method and Proof in 5 steps

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Abstract

In number theory Fermat's last theorem states that no three positive integers a,b,c satisfy the equation $a^n + b^n = c^n$ for all exponents including $n \geq 3$. This can be proved simply by considering the prime factors associated with one of the members a or b in the equation. It is found that when the exponent is greater than 2 , one or two numbers are invariably either complex or irrational. The non-existence of the relation with all are whole integers proves FLT for all exponents $n \geq 3$ .However if the exponents are not same then the three member multi-power relation becomes possible. An unique property associated with these relations is described by Beal conjecture. The Pythagoras theorem, FLT and Beal conjecture are different forms of one and the same multi-power relation under different condition. The inherent linkage between them helps to prove both the Fermat's last theorem and Beal conjecture in five successive steps.

Key words

Fermat's Last Theorem, equal power relations, multi-power relation, prime factors, Beal conjecture, number theory.

I.Introduction

For all exponents greater than 2 , the numerical relation in the form $a^n + b^n = c^n$ cannot exist when a,b and c are restricted to have whole integers which is called Fermat's Last Theorem (FLT) , but is true when irrational and complex numbers are allowed for at least one or two members. After the Fermat's assertion, many people [1-3] attempted to prove FLT for different values of exponent and verified with the help of number theory. After the introduction of computer technology people attempted to support the proof of FLT for a wide range of exponents 3 - 4,000,000 [4]. A proof of FLT was given by Taylor and Wills [5,6] with the help of modern concepts of mathematics – elliptic curve and modular theory. In general there must be two or more ways, one simple and another complicated, to get the same result for a given mathematical problem. Nature loves symmetry and likes always to be very simple. In fact it is our description that shows the nature to be illusory, which implies that there must be more natural and naive proof for FLT. This article describes the simple and natural proof of FLT. Using the same method used to prove Fermat's last theorem one can demonstrate the Beal conjecture which is close analogous to FLT[7-10]. The Pythagorean relation with all whole integers can be used to prove both FLT and Beal conjecture within five steps.

II. Prime factor method – A simple proof of FLT

Without any complexity in mathematical description, FLT can be proved with the knowledge of the prime factors associated with one of the members of the triple (a,b,c). Any number can be represented in terms of its prime factors. An odd number will have only odd prime factors denoted by P_o where as all even numbers will invariably have 2 as one of the prime factors and besides that it may have one or more odd prime factors. Accordingly an od and even number can be represented in terms of its prime factors as,

$$N(odd) = P_{01}^{\alpha} P_{02}^{\beta} P_{03}^{\gamma} \dots\dots\dots\dots\dots\dots\dots\dots P_{om}^{\omega}$$

$$N(even) = 2^m P_{01}^{\alpha} P_{02}^{\beta} \dots\dots\dots\dots\dots\dots\dots\dots P_{0m}^{\omega}$$

Where $\alpha,\beta,\gamma,\dots\dots\dots\omega$ may have any value from 0 depending upon the value of the base numbers a,b,c. m has a minimum value of 1 in the case of singly even and its value increases depending upon the degree of evenness. If all the powers are zero, the base number becomes 1 and for a number greater than unity, at least one or more exponents will be non-zero.

In $a^n + b^n = c^n$, a^n is equal to a product of two factors $(c^{n/2} - b^{n/2})(c^{n/2} + b^{n/2})$ and if the prime factors associated with a (or b) is known , then they can be distributed for these two factors. By solving the two equations, one can determine the unknown base numbers in terms of the known. For simplicity let us assume $a = P_{01}^{\alpha} P_{02}^{\beta} P_{03}^{\gamma}$

$$a^n = P_{01}^{n\alpha} P_{02}^{n\beta} P_{03}^{n\gamma} = (c^{n/2} - b^{n/2})(c^{n/2} + b^{n/2})$$

In any three member equal power relation $a^n + b^n = c^n$, c is always greater than both a and b, but c^m is less than $a^m + b^m$ when $m < n$ and greater when $m > n$. This is true even when m is a fraction. Since $(c^{n/2} - b^{n/2}) < a^{n/2}$,

$$c^{n/2} + b^{n/2} = k[P_{01}^{n\alpha/2} P_{02}^{n\beta/2} P_{03}^{n\gamma/2}]$$

$$c^{n/2} - b^{n/2} = [P_{01}^{n\alpha/2} P_{02}^{n\beta/2} P_{03}^{n\gamma/2}]/k$$

where $k > 1$, need not be an integer and may have all possible values fraction, complex, and irrational.

Solving these two equations, we get,

$$c^n = P_{01}^{n\alpha} P_{02}^{n\beta} P_{03}^{n\gamma} [(k^2 + 1)/2k]^2$$

$$b^n = P_{01}^{n\alpha} P_{02}^{n\beta} P_{03}^{n\gamma} [(k^2 - 1)/2k]^2$$

and substituting these values, we get

$$a^n + a^n [(k^2 - 1)/2k]^2 = a^n [(k^2 + 1)/2k]^2$$

If one is able to convert it into $a^n + b^n = c^n$ with a,b,c are all positive integers,, it is equal to say that FLT is disproved, otherwise it confirms. The above equation can be rearranged as $[(2k)^{2/n}a]^n + [a(k^2 - 1)^{2/n}]^n = [a(k^2 + 1)]^{2/n}]^n$

The terms inside each bracket will be whole number only when $n = 2$. For $n \geq 3$ one or more numbers will be either irrational or complex.

When $2k = \alpha^{n/2}$ then $(2k)^{2/n} = \alpha$

$(k^2 - 1) = \beta^{n/2}$ then $(k^2 - 1)^{2/n} = \beta$, and

$(k^2 + 1) = \gamma^{n/2}$ then $(k^2 + 1)^{2/n} = \gamma$

Subtracting one from the other,

$(\gamma^{n/2} - \beta^{n/2}\} = 1$, No whole integral solution is obtained for all possible values of ,β and γ . It substantiates Fermat's last theorem. However there are many solutions when $n = 2$, where the condition reduces to $\gamma - \beta = 1$

III.FLT and Beal Conjecture

The Pythagoras theorem, FLT and Beal conjecture are different form of one and the same multi-power relation under different condition. The most general form of three member multi-power relation is $a^{\alpha} + b^{\beta} = c^{\gamma}$, where both the base numbers (a,b,c) and the exponents (α,β,γ) are all supposed to be whole integers. Under different conditions, it gives Pythagorean relation $(\alpha = \beta = \gamma = 2)$, Fermat's last theorem $(\alpha = \beta = \gamma = n$ and $n > 2)$ and Beal conjecture (α, β, γ are not equal, at least one is different and $\alpha = \beta = \gamma \neq 2$).

When a,b,c,α,β and γ are all positive integers with α,β,γ are greater than 2, then a,b,c will invariably have a common prime factor (Beal conjecture) ,when the exponent of one or two base numbers is 2, the common prime factor is not to be a necessary condition and when the exponents of all the base numbers are equal, the common prime factor present can be cancelled out and reduces ultimately to an irreducible form of the relation

(Fermat's last theorem). In this paper a simple proof for FLT and Beal conjecture is derived from the realistic Pythagoras theorem.

IV.FLT in Five Steps

Step.1,Consider any known Pythagorean triples (a,b,c) satisfying $a^2 + b^2 = c^2$, where a,b and c are all whole integers. Dividing by a^2 throughout gives reduced Pythagorean relation as $1 + (b/a)^2 = (c/a)^2$

Step,2 Let us suppose an unknown three member like power relation $x^n + y^n = z^n$, where all possible values of x,y and z are allowed., which may be whole integers or fractions or irrational or even complex.

Step 3. Multiplying throughout the reduced Pythagorean relation by x^p gives $x^p + x^p(b/a)^2 = x^p(c/a)^2$

Step 4. Comparing term by term with the unknown relation $y^p = x^p(b/a)^2 + k = (x\alpha)^p$ and $z^p = x^p(c/a)^2 + k = (x\beta)^p$ where α and β are assumed to be whole integers and k will be zero when $(b/a)^2 = \alpha^p$ and $(c/a)^2 = \beta^p$ if not k may have some positive or negative value. Since it is added in both sides, the equation will not be affected.

Step 5. Substituting the derived values for $(b/a)^2$ and $(c/a)^2$ in the reduced Pythagorean relation $1 + \alpha^p = \beta^p$. No integer or fractional values of α and β will satisfy this relation for all values of p except p = 2.

V.Beal conjecture in five steps

Step.1 Consider any known Pythagorean triples (a,b,c) satisfying $a^2 + b^2 = c^2$, where a,b and c are all whole integers. Dividing by a^2 throughout gives reduced Pythagorean relation as $1 + (b/a)^2 = (c/a)^2$

Step 2. Let us suppose an unknown three member unlike power relation $x^p + y^q = z^r$ where x,y,z,,p,q and r are considered to be whole integers.

Step 3. Multiplying throughout the reduced Pythagorean relation by x^p gives $x^p + x^\alpha(b/a)^2 = x^p(c/a)^2$

Step 4.Comparing term by term with the unknown relation, $y^q = x^p(b/a)^2 = (x\alpha)^p$ and $z^r = x^p(c/a)^2 = x^{p+k}$ where α and k are all assumed to be whole integers.

Step 5. Substituting the derived values for $(b/a)^2$ and $(c/a)^2$ in the reduced Pythagorean relation we arrive at a mathematically possible $1 + \alpha^p = x^k$. Few typical solutions and the corresponding multi-power relations are $1 + 2^3 = 3^2$ gives $3^3 + 6^3 = 3^5$, $1+ 7 = 2^3$ gives $7^3 + 7^4 = 14^3$ and few general solutions are $1 + (2^n - 1\} = 2^n$ gives $(2^n - 1)^n + (2^n - 1)^{n+1} = [2(2^n - 1)]^n$ and $1 + n^m = (n^m + 1)$ gives $(n^m + 1)^m + [n(n^m + 1)]^m = (n^m + 1)^{m+1}$.

VI.Conclusion

The proof of FLT and Beal conjecture in five steps shows that three member equal power relation is not possible with all the base numbers are integers for all exponents greater than 2 since $1 + \alpha^n \neq \beta^n$ .However such relations are possible when irrational and complex numbers are allowed or when the exponents of the base numbers are allowed to be different since $1 + \alpha^n = \beta^m$ ..When it is multiplied with a common multiplier (β^n) which becomes the common prime factor in Beal conjecture we get a multi-power relation $\beta^n + (\alpha\beta)^n = \beta^{m+n}$, while the common factor α^m gives $\alpha^m + \alpha^{m+n} = (\alpha\beta)^m$.

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