SHORTEST PATH PROBLEM BY MINIMAL SPANNING TREE ALGORITHM USING BIPOLAR NEUTROSOPHIC NUMBERS

M. Mullai[a]*, S. Broumi[b], A. Stephen[c]
[a] *Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.*
[b] Laboratory of Information processing, B.P 7955, Sidi Othman, Casablanca, Morocco.
[c] Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

Abstract

Normally, Minimal Spanning Tree algorithm is used to find the shortest route in a network. Neutrosophic set theory is used when incomplete, inconsistancy and indeterminacy occurs. In this paper, Bipolar Neutrosophic Numbers are used in Minimal Spanning Tree algorithm for finding the shortest path on a network when the distances are inconsistant and indeterminate and it is illustrated by a numerical example.
Keywords: Neutrosophic set, Neutrosophic number, Bipolar Neutrosophic Number, Minimal spanning tree algorithm, Shortest path..

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1 Introduction

In 1965, Zadeh[23] introduced the concept of fuzzy set in order to deal with uncertainity. After that, Atanassov[1]incorporated the degree of non-membership in the concept of fuzzy set as an independent component and defined the concept of intuitionistic fuzzy set. Smarandache [18] coined the term degree of indeterminacy as an independent component and defined the concept of neutrosophic set to deal with incomplete, indeterminate and inconsistent information instead in reality. Neutrosophic set is a generalization of the theory of fuzzy set, intuitionistic fuzzy set. Neutrosophic set theory is used in many fields when incomplete, inconsistant or indeterminacy occurs. In a Neutrosophic logic, a proposition has a degree of truth (T), a degree of indeterminancy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$. Neutrosophic theory is very useful to the researchers for finding better solutions for real world problems when incomplete and inconsistant datas are given.

2 Preliminaries

DEFINITION 2.1 *[17-18] Let X be a space of points (objects) with generic elements in X denoted by x. Then the Neutrosophic Set A is an object having the form* $A=\{< x : T_A(x),\ I_A(x),\ F_A(x) >,\ x\epsilon X\}$*, where the function* $T,\ I,\ F : X \to]^-0, 1^+[$ *define respectively the truth-membership function, an indeterminacy membership-function, and a falsity-membership function of the element* $x\epsilon X$ *to the set A with the condition:*
$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. *The function* $T_A(x),\ I_A(x)$ *and* $F_A(x)$ *are real standard or nonstandard subsets of* $]^-0, 1^+[$.

DEFINITION 2.2 *[23] Let X be a space of points (objects) with generic elements in X denoted by x. A Single Valued Neutrosophic Set A is characterized by truth-membership function* $T_A(x)$*, an indeterminacy-membership function* $I_A(x)$ *and a falsity-membership function* $F_A(x)$*. For each point x in X* $T_A(x),\ I_A(x),\ F_A(x)\ \epsilon[0, 1]$*. A Single Valued Neureosophic Set A can be written as* $A = \{< x : T_A(x),\ I_A(x),\ F_A(x) >,\ x \in X\}$.

DEFINITION 2.3 *[1] A Bipolar Neutrosophic Set A in X is defined as an object of the form* $A = \{< x : T^p(x),\ I^p(x),\ F^p(x),\ T^n(x),\ I^n(x),\ F^n(x) >:\ x\epsilon X\}$*, where* $T^p,\ I^p,\ F^p : X \to [0, 1]$ *and* $T^n,\ I^n,\ F^n :\ X \to [-1, 0]$*. The positive membership degree* $T^p(x),\ I^p(x),\ F^p(x)$ *denotes the truth membership, indeterminate membership and false membership of an element belongs to X corresponding to a bipolar neutrosophic set A and the negative membership degree* $T^n(x),\ I^n(x),\ F^n(x)$ *denotes the truth membership, indeterminate membership and false membership of an element belongs to X to some implicit counter-property corresponding to a Bipolar Neutrosophic Set A.*

DEFINITION 2.4 *[8] Let*
$\widetilde{A_1} =< T_1^p,\ I_1^p,\ F_1^p,\ T_1^n,\ I_1^n,\ F_1^n >$ *and*
$\widetilde{A_2} =< T_2^p,\ I_2^p,\ F_2^p,\ T_2^n,\ I_2^n,\ F_2^n >$ *be two Bipolar Neutrosophic Numbers and* $\lambda > 0$*. Then, the operations of these numbers are defined as follows:*

$$\begin{aligned}
(i)\widetilde{A}_1 \bigoplus \widetilde{A}_2 &= < T_1^p + T_2^p - T_1^pT_2^p,\ I_1^pI_2^p,\\
&\quad F_1^pF_2^p,\ -(T_1^nT_2^n),\\
&\quad -(-I_1^n - I_2^n - I_1^nI_2^n),\\
&\quad -(-F_1^n - F_2^n - F_1^nF_2^n) > .\\
(ii)\widetilde{A}_1 \bigotimes \widetilde{A}_2 &= < T_1^pT_2^p,\ I_1^p + I_2^p - I_1^pI_2^p,\\
&\quad F_1^p + F_2^p - F_1^pF_2^p,\\
&\quad -(-T_1^n - T_2^n - T_1^nT_2^n),\\
&\quad -(I_1^nI_2^n),\ -(F_1^nF_2^n) > .
\end{aligned}$$

$$\begin{aligned}
(iii)\lambda\widetilde{A}_1 &= < 1 - (1 - T_1^p)^\lambda,\ (I_1^p)^\lambda,\ (F_1^p)^\lambda,\\
&\quad -(-T_1^n)^\lambda,\ -(-I_1^n)^\lambda,\\
&\quad -(1 - (1 - (-F_1^n))^\lambda) > .\\
(iv)\widetilde{A}_1^\lambda &= < (T_1^p)^\lambda,\ 1 - (1 - I_1^p)^\lambda,\ 1 - (1 - F_1^p)^\lambda,\\
&\quad -(1 - (1 - (-T_1^n))^\lambda),\ -(-I_1^n)^\lambda,\\
&\quad -(-F_1^n)^\lambda >,\ where\ \lambda > 0.
\end{aligned}$$

Definition 2.5 *[8]. In order to make a comparisons between two BNNs, the score function is applied to compare the grades of BNNs. This function shows that greater is the value, the greater is the Bipolar Neutrosophic Sets and by using this concept paths can be ranked. Let* $\widetilde{A} = < T^p,\ I^p,\ F^p,\ T^n,\ I^n,\ F^n >$ *be a bipolar neutrosophic number. Then, the score function* $s(\widetilde{A})$*, accuracy function* $a(\widetilde{A})$ *and certainty function* $c(\widetilde{A})$ *of a BNN are defined as follows:*

$$\begin{aligned} (i)s(\widetilde{A}) &= (\frac{1}{6})[T^p + 1 - I^p + 1 - F^p + 1 + \\ &\quad T^n - I^n - F^n] \\ (ii)a(\widetilde{A}) &= T^p - F^p + T^n - F^n \\ (iii)c(\widetilde{A}) &= T^p - F^n \end{aligned}$$

3 Bipolar Neutrosophic Minimal Spanning Tree Algorithm:

The Minimal spanning tree algorithm in [10] deals with linking the nodes of a network, directly or indirectly, using the shortest length of connecting branches. A typical application occurs in the construction of paved roads that link several rural towns. The road between two towns may pass through one or more towns. The most economical design of the road system calls for minimizing the total miles of paved roads, a result that is achieved by implementing the minimal spanning tree algorithm. When it is not possible to identify the distances between two places accurately, neutrosophic theory is used in that situation. Here, we introduce the concept of Bipolar Neutrosophic numbers as distances(lengths) in Minimal Spanning Tree algorithm to find the shortest route in a network.
The procedure is given as follows.
Let $N = \{1, 2, ..., n\}$ be the set of nodes of the network and define

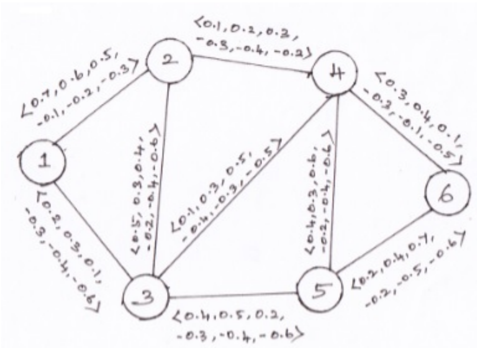
$\tilde{C}_k$ = Set of nodes that have been permanently connected at iteration k
$\overline{\tilde{C}_k}$ = Set of nodes as yet to be connected permanently after iteration k
Step 0:
Set $\tilde{C}_0 = \phi$ and $\overline{\tilde{C}_0} = N$
Step 1:
Start with any node i in the unconnected set $\overline{\tilde{C}_0}$ and set $\tilde{C}_0 = \{i\}$ which renders $\overline{\tilde{C}_0} = N - \{i\}$.
Set $k = 2$.
General Step k:
Select a node j^*, in the unconnected set $\overline{\tilde{C}}_{k-1}$ by using score function $s(\widetilde{A})$ in definition 2.5.
Link j^* permanently to $\tilde{C}_{k-1}$ and remove it from $\overline{\tilde{C}}_{k-1}$;

$$\begin{aligned} \tilde{C}_k &= vC_{k-1} + \{j^*\},\ \overline{vC_k} \\ &= \overline{\tilde{C}}_{k-1} - \{j^*\} \end{aligned}$$

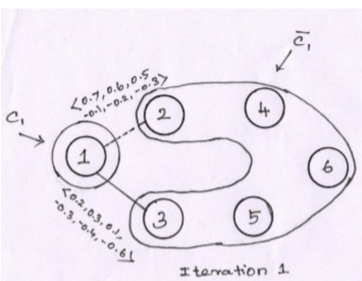
If the set of unconnected nodes, $\overline{\tilde{C}}_k$, is empty, Stop.
Otherwise, set k=k+1 and repeat the step.

This algorithm is illustrated by the following numerical example:
Numerical Example:

Consider a small network shown in the following figure in which each arc length is represented by a Bipolar Neutrosophic Number. This problem is to find the shortest path between source node and destination node on the given network.



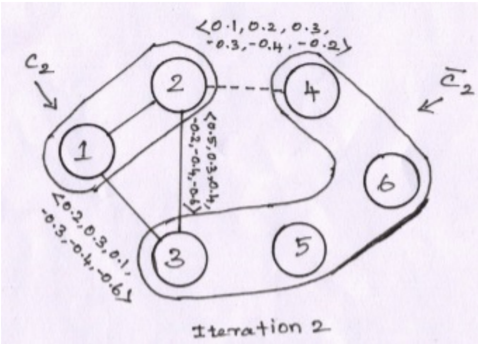
Solution:
Iteration 0:
Set $\tilde{C}_0 = \phi$ and $\overline{\tilde{C}_0} = N$.
Iteration 1:
Let $\tilde{C}_1 = \{1\}$ and $\overline{\tilde{C}_1} = \{2,\ 3,\ 4,\ 5,\ 6\}$
$s(< 0.7,\ 0.6,\ 0.5,\ -0.1,\ -0.2,\ -0.3 >)$



$= (\frac{1}{6}) \times (4.2 - 1.2)$
$= 0.5$
$s(< 0.2,\ 0.3,\ 0.1,\ -0.3,\ -0.4,\ -0.6 >)$
$= (\frac{1}{6}) \times (4.2 - 0.7)$
$= 0.583$
minimum$\{< 0.7,\ 0.6,\ 0.5,\ -0.1,\ -0.2,\ -0.3 >,$
$< 0.2,\ 0.3,\ 0.1,\ -0.3,\ -0.4,\ -0.6 >\}$
$=< 0.7,\ 0.6,\ 0.5,\ -0.1,\ -0.2,\ -0.3 >$.

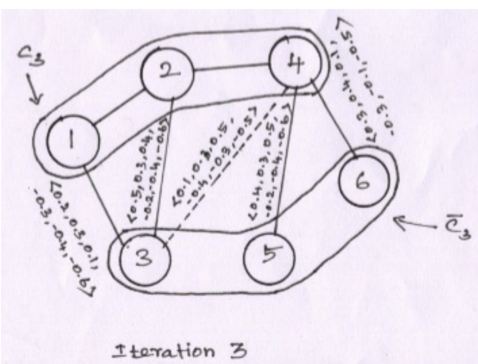
Iteration 2:

Let $\tilde{C}_2 = \{1,\ 2\}$ and $\overline{\tilde{C}_2} = \{3,\ 4,\ 5,\ 6\}$
$s(< 0.2,\ 0.3,\ 0.1,\ -0.3,\ -0.4,\ -0.6 >)$
$= (\frac{1}{6}) \times (4.2 - 0.7)$
$= 0.583$



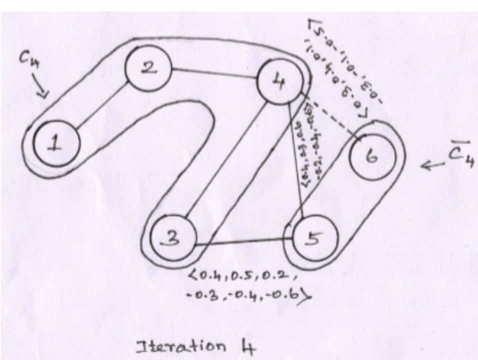
$s(< 0.1,\ 0.2,\ 0.3,\ -0.3,\ -0.4,\ -0.2 >)$
$= (\frac{1}{6}) \times (3.7 - 0.8)$
$= 0.48$
$s(< 0.5,\ 0.3,\ 0.4,\ -0.2,\ -0.4,\ -0.6 >)$
$= (\frac{1}{6}) \times (4.5 - 0.9)$
$= 0.6$
$\text{minimum}\{< 0.2,\ 0.3,\ 0.1,\ -0.3,\ -0.4,\ -0.6 >,$
$< 0.1,\ 0.2,\ 0.3,\ -0.3,\ -0.4,\ -0.2 >,$
$< 0.5,\ 0.3,\ 0.4,\ -0.2,\ -0.4,\ -0.6 >\}$
$=< 0.1,\ 0.2,\ 0.3,\ -0.3,\ -0.4,\ -0.2 >$.

Iteration 3:
Let $\tilde{C}_3 = \{1,\ 2,\ 3\}$ and $\overline{\tilde{C}_3} = \{4,\ 5,\ 6\}$



$s(<0.2,\ 0.3,\ 0.1,\ -0.3,\ -0.4,\ -0.6>)$
$= (\frac{1}{6}) \times (4.2 - 0.7)$
$= 0.583$
$s(<0.5,\ 0.3,\ 0.4,\ -0.2,\ -0.4,\ -0.6>)$
$= (\frac{1}{6}) \times (4.5 - 0.9)$
$= 0.6$
$s(<0.3,\ 0.4,\ 0.1,\ -0.3,\ -0.5,\ -0.5>)$
$= (\frac{1}{6}) \times (3.9 - 0.8)$
$= 0.51$
$s(<0.4,\ 0.3,\ 0.6,\ -0.2,\ -0.4,\ -0.6>)$
$= (\frac{1}{6}) \times (4.4 - 1.1)$
$= 0.55$
$s(<0.1,\ 0.3,\ 0.5,\ -0.4,\ -0.3,\ -0.5>)$
$= (\frac{1}{6}) \times (3.9 - 1.2)$
$= 0.45$
minimum$\{<0.2,\ 0.3,\ 0.1,\ -0.3,\ -0.4,\ -0.6>,$
$<0.5,\ 0.3,\ 0.4,\ -0.2,\ -0.4,\ -0.6>,$
$<0.3,\ 0.4,\ 0.1,\ -0.3,\ -0.5,\ -0.5>,$
$<0.4,\ 0.3,\ 0.6,\ -0.2,\ -0.4,\ -0.6>,$
$<0.1,\ 0.3,\ 0.5,\ -0.4,\ -0.3,\ -0.5>\}$
$=<0.1,\ 0.3,\ 0.5,\ -0.4,\ -0.3,\ -0.5>.$

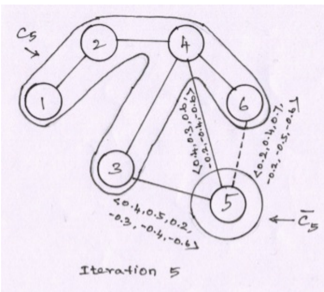
Iteration 4:
Let $\tilde{C}_4 = \{1,\ 2,\ 3,\ 4\}$ and $\overline{\tilde{C}_4} = \{5,\ 6\}$



$s(<0.3,\ 0.4,\ 0.1,\ -0.3,\ -0.1,\ -0.5>)$
$= (\frac{1}{6}) \times (3.9 - 0.8)$
$= 0.51$
$s(<0.4,\ 0.3,\ 0.6,\ -0.2,\ -0.4,\ -0.6>)$
$= (\frac{1}{6}) \times (4.4 - 1.1)$
$= 0.55$
$s(<0.4,\ 0.5,\ 0.2,\ -0.3,\ -0.4,\ -0.6>)$
$= (\frac{1}{6}) \times (4.4 - 1)$
$= 0.566$
minimum$\{<0.3,\ 0.4,\ 0.1,\ -0.3,\ -0.1,\ -0.5>,$
$<0.4,\ 0.3,\ 0.6,\ -0.2,\ -0.4,\ -0.6>,$

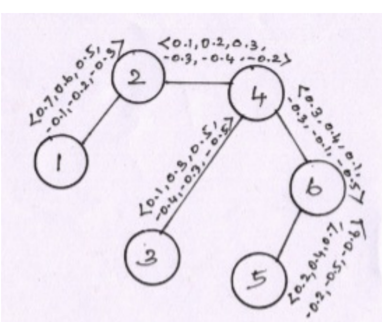
$< 0.4,\ 0.5,\ 0.2,\ -0.3,\ -0.4,\ -0.6 >\}$
$=< 0.3,\ 0.4,\ 0.1,\ -0.3,\ -0.1,\ -0.5 >$.

Iteration 5:
Let $\tilde{C}_5 = \{1,\ 2,\ 3,\ 4,\ 5\}$ and $\overline{\tilde{C}_5} = \{6\}$



$s(< 0.4,\ 0.3,\ 0.6,\ -0.2,\ -0.4,\ -0.6 >)$
$= (\frac{1}{6}) \times (4.4 - 1.1)$
$= 0.55$
$s(< 0.4,\ 0.5,\ 0.2,\ -0.3,\ -0.4,\ -0.6 >)$
$= (\frac{1}{6}) \times (4.4 - 1)$
$= 0.566$
$s(< 0.2,\ 0.4,\ 0.7,\ -0.2,\ -0.5,\ -0.6 >)$
$= (\frac{1}{6}) \times (4.3 - 1.3)$
$= 0.5$
minimum$\{< 0.4,\ 0.3,\ 0.6,\ -0.2,\ -0.4,\ -0.6 >$,
$< 0.4,\ 0.5,\ 0.2,\ -0.3,\ -0.4,\ -0.6 >$,
$< 0.2,\ 0.4,\ 0.7,\ -0.2,\ -0.5,\ -0.6 >\}$
$=< 0.2,\ 0.4,\ 0.7,\ -0.2,\ -0.5,\ -0.6 >$.

Finally, Bipolar Neutrosophic Minimal spanning tree is



Hence,
$< 0.7,\ 0.6,\ 0.5,\ -0.1,\ -0.2,\ -0.3 > +$
$< 0.1,\ 0.2,\ 0.3,\ -0.3,\ -0.4,\ -0.2 >+$
$< 0.3,\ 0.4,\ 0.1,\ -0.3,\ -0.1,\ -0.5 >+$
$< 0.1,\ 0.3,\ 0.5,\ -0.4,\ -0.3,\ -0.5 >+$
$< 0.2,\ 0.4,\ 0.7,\ -0.2,\ -0.5,\ -0.6 >$

$=< 0.8299,\ 0.0144,\ 0.0375,\ -0.0036,\ -0.6976,\ -0.86 >+$
$< 0.2,\ 0.4,\ 0.7,\ -0.2,\ -0.5,\ -0.6 >$
$=< 0.86392,\ 0.00576,\ 0.02625,\ -0.00072,\ -0.8488,\ -0.944 >.$

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References

[1] Atanassov. K, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, vol. 20, 1986, pp. 87-96.

[2] Broumi. S, Bakali. A, Talea. M and Smarandache. F, *Computation of Shortest Path Problem in a Network with Single Valued Neutrosophic Number Based on Ranking Method*, 2016 (submitted).

[3] Broumi. S, Bakali. A, Talea. M and Smarandache. F, *Shortest Path Problem Under Interval Valued Neutrosophic Setting*, 2016 (submitted).

[4] Broumi. S, Bakali. A, Talea. M and Smarandache. F, Ali. M, *Shortest Path Problem Under Bipolar Neutrosophic Setting*, 2016 (submitted).

[5] Broumi. S, Bakali. A, Talea. M and Smarandache. F, *An Introduction to Bipolar Single Valued Neutrosophic Graph Theory*, Applied Mechanics and Materials, vol.841, 2016, pp.184-191.

[6] Broumi. S, Smarandache. F, Talea. M and Bakali. A, *Decision-Making Method Based On the Interval Valued Neutrosophic Graphs*, FTC 2016-Future Technologies Conference 2016, In press.

[7] Broumi. S, Talea. M, Bakali. A and Smarandache. F, "On Bipolar Single Valued Neutrosophic Graphs," Journal of New Theory, N11, 2016, pp.84-102.

[8] Broumi. S, Talea. M, Smarandache. F and Bakali. A, *Single Valued Neutrosophic Graphs: Degree, Order and Size*. IEEE World Congress on Computation Intelligence, 2016, 8 pages, in press.

[9] Deli. I, Ali. M, Smarandache. F, *Bipolar neutrosophic sets and their application besed on multi-criteria decision making problems*, Advanced Mechatronic Systems (ICAMechS), 2015 International Conference, 2015, pp.249-254.

[10] Hamdy A. Taha, *Operations Research: An Introduction*, Eighth Edition.

[11] Jayagowri and Geetha Ramani. G, *Using Trapezoidal Intuitionistic Fuzzy Number to Find Optimized Path in a Network*, Volume 2014, Advances in Fuzzy Systems, 2014, 6 pages.

[12] Kumar. G, Bajaj. R. K and Gandotra. N, *Algorithm for shortest path problem in a network with interval valued intuitionistic trapezoidal fuzzy number*,Procedia Computer Science 70, 2015, pp.123-129.

[13] Kumar. A and Kaur. M, *A New Algorithm for Solving Shortest Path Problem on a Network with Imprecise Edge Weight*, Applications and Applied Mathematics, Vol. 6, Issue 2, 2011, pp.602-619.

[14] Kumar. A and Kaur. M, *Solution of fuzzy maximal flow problems using fuzzy linear programming*, World Academy of Science and Technology. 87, 2011, pp.28-31.

[15] Majumder. S and Pal. A, *Shortest Path Problem on Intuitionstic Fuzzy Network*, Annals of Pure and Applied Mathematics, Vol. 5, No. 1, November 2013.

[16] Ngoor. A and Jabarulla. M. M, *Multiple labeling Approch For Finding shortest Path with Intuitionstic Fuzzy Arc Length*, International Journal of Scientific and Engineering Research, V3, Issue 11, 2012, pp.102-106.

[17] Smarandache. F, *A geometric interpretation of the neutrosophic set - A generalization of the intuitionistic fuzzy set*, Granular Computing (GrC). 2011 IEEE International Conference, 2011, pp.602-606.

[18] Smarandache. F, *Neutrosophic set - A generalization of the intuitionistic fuzzy set*, Granular Computing, 2006 IEEE International Conference, 2006, pp. 38-42.

[19] Smarandache. F, *Symbolic Neutrosophic Theory*, Europanova asbl, Brussels, 2015, 195p.

[20] Smarandache. F, *Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Application in Technology,"* seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania 06 June 2015

[21] VasanthaKandasamy, Ilanthenral. K and Smarandache. F, *Neutrosophic graphs*: A New Dimension to Graph Theory. Kindle Edition. USA. 2015, 127p.

[22] Wang. H, Smarandache. F, Zhang. Y and Sunderraman, *Single valued Neutrosophic Sets*, Multisspace and Multistructure 4, 2010, pp.410-413.

[23] Zadeh. L, *Fuzzy sets*, Inform and Control, 8, 1965, pp.338-353.

[24] http://fs.gallup.unm.edu/NSS.