NEUTROSOPHIC SOFT CUBIC SET

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Abstract :
This paper acts as a base for the amalgamation of the already existing cubic set, Neutrosophic cubic set and the theory of soft sets and named as Neutrosophic soft cubic set (NSCS). Here we define internal neutrosophic soft cubic set (INSCS) and external neutrosophic soft cubic set (ENSCS) and also propose the new idea of $\frac{1}{3}$INSCS (or $\frac{2}{3}$ ENSCS), $\frac{2}{3}$INSCS (or $\frac{1}{3}$ ENSCS). Further P-order, P-union, P-intersection as well as R-order, R-union, R-intersection are introduced for Neutrosophic soft cubic sets which acts as a tool to study some of their properties of newly introduced sets.

Keywords: Cubic set, cubic soft set, neutrosophic soft cubic set , internal (external) neutrosophic soft cubic set .

I. INTRODUCTION

The concept of Fuzzy sets were initiated by Zadeh [4]. In [14], Zadeh made an extension of the concept of a fuzzy set by an interval-valued fuzzy set, i.e., a fuzzy set with an interval-valued membership function. In traditional fuzzy logic, to represent, the expert's degree of certainty in different statements, numbers from the interval [0,1] are used. Interval-valued fuzzy sets have been actively used in real-life applications. The application of these sets is clearly dealt with in [11],[12],[2],[13],[5],which shows its importance
1999,[7] Molodsov initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. In [15] Maji et al. intiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union , intersection, complement of fuzzy soft set. Moreover in [16,17] Maji et al extended soft sets to intuitionistic fuzzy soft sets and Neutrosophic soft sets. Neutrosophic Logic has been proposed by Florentine Smarandache[9,10] which is based on nonstandard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.. Y. B. Jun et al[1]., introduced a new notion, called a cubic set by using a fuzzy set and an interval-valued fuzzy set, and investigated several properties.

This paper aims to introduce a new notion called neutrosophic soft cubic sets and using cubic sets, Neutrosophic cubic sets and soft sets.We also define some new notions such as internal (external) neutrosophic soft cubic sets. P-(R-)order, P-(R-)union, P-(R-)intersection are introduced, and related properties are investigated We also investigate some of the core properties of neutrosophic soft cubic set.

II. PRELIMINARIES

Definition:2. 1. [4] Let E be a universe. Then a fuzzy set μ over E is defined by X = { $\mu_x(x)$ / x: x ϵ E }where μ_x is called membership function of X and defined by $\mu_x : E \to [0,1]$. For each x E, the value $\mu_x(x)$ represents the degree of x belonging to the fuzzy set X.

Definition: 2.2 [5] Let E be a universe. Then, an interval valued fuzzy set A over E is defined by

$A = \{ [A^-(x), A^+(x)] / x: x \in E \}$ where $A^-(x)$ and $A^+(x)$ are referred to as the lower and upper degrees of membership $x \in E$ where $0 \le A^-(x) + A^+(x) \le 1$, respectively.

Definition: 2.3.[1] LetX be a non-empty set. By a cubic set, we mean a structure
$\Xi = \{\langle x, A(x), \mu(x)\rangle \mid x \in X\}$
in which A is an interval valued fuzzy set (IVF) and μ is a fuzzy set. It is denoted by$\langle A, \mu\rangle$.

Definition : 2. 4. [1] Let $\Xi_1 = \langle A_1, \mu_1\rangle$ and $\Xi_2 = \langle A_2, \mu_2\rangle$ be cubic sets in X. Then we define
1. (Equality)$\Xi_1 = \Xi_2$ if and only if $A_1 = A_2$ and $\mu_1 = \mu_2$
2. (P- Order) $\Xi_1 \subseteq_P \Xi_2$ if and only if $A_1 \subseteq A_2$ and $\mu_1 \leq \mu_2$
3. (R- Order) $\Xi_1 \subseteq_R \Xi_2$ if and only if $A_1 \subseteq A_2$ and $\mu_1 \geq \mu_2$

Definition : 2.5. [3] Let X be an universe. Then a neutrosophic (NS) set λ is an object having the form

$\lambda = \{< x : T(x),I(x),F(x) >: x \in X\}$

where the functions T, I, F : X →]⁻0, 1+[defines respectively the degree of Truth, the degree of indeterminacy, and the degree of Falsehood of the element x ∈ X to the set λ with the condition.

$$^{-}0 \leq T(x) + I(x) + F(x) \leq 3^{+}$$

For two NS, $\lambda_1 = \{< x, T_1(x), I_1(x), F_1(x) > | x \in X\}$ and $\lambda_2 = \{< x, T_2(x), I_2(x), F_2(x) > | x \in X\}$ the operations are defined as follows:

1. $\lambda_1 \subset \lambda_2$ if and only if

$T_1(x) \leq T_2(x), I_1(x) \geq I_2(x), F_1(x) \geq F_2(x)$

2. $\lambda_1 = \lambda_1$ if and only if, $T_1(x) = T_2(x), I_1(x) = I_2(x), F_1(x) = F_2(x)$

3. $\lambda_1{}^{c} = \{x, F_1(x), I_1(x), T_1(x) >: x \in X\}$

4. $\lambda_1 \cap \lambda_2 = \{< x, \min\{T_1(x), T_2(x)\}, \max\{I_1(x), I_2(x)\}, \max\{F_1(x), F_2(x)\} >: x \in X\}$

5. $\lambda_1 \cup \lambda_2 = \{< x, \max\{T_1(x), T_2(x)\}, \max\{I_1(x), I_2(x)\}, \min\{F_1(x), F_2(x)\} >: x \in X\}$

Definition : 2.6. [6] Let X be a non-empty set. An interval neutrosophic set (INS) A in X is characterized by the truth-membership function A_T, the indeterminacy-membership function A_I and the falsity-membership function A_F. For each point x ∈ X, $A_T(x), A_I(x), A_F(x) \subseteq [0,1]$.

For two INS

$A = \{<x, [A_T^-(x), A_T^+(x)], [A_I^-(x), A_I^+(x)], [A_F^-(x), A_F^+(x)]>: x \in X\}$

and

$B = \{<x, [B_T^-(x), B_T^+(x)], [B_I^-(x), B_I^+(x)], [B_F^-(x), B_F^+(x)]>: x \in X\}$

Then,

1. $A \tilde{\subseteq} B$ if and only if

$$A_T^-(x) \leq B_T^-(x), A_T^+(x) \leq B_T^+(x)$$
$$A_I^-(x) \geq B_I^-(x), A_I^+(x) \geq B_I^+(x)$$
$$A_F^-(x) \geq B_F^-(x), A_F^+(x) \geq B_F^+(x) \quad \text{for all } x \in X.$$

2. $A = B$ if and only if

$$A_T^-(x) = B_T^-(x), A_T^+(x) = B_T^+(x)$$
$$A_I^-(x) = B_I^-(x), A_I^+(x) = B_I^+(x)$$
$$A_F^-(x) = B_F^-(x), A_F^+(x) = B_F^+(x) \text{ for all } x \in X.$$

3. $A^{\tilde{C}} = \{< x, [A_F^-(x), A_F^+(x)], [A_I^-(x), A_I^+(x)], [A_T^-(x), A_T^+(x)] >: x \in X\}$

4. $A \tilde{\cap} B = \{< x, [\min\{A_T^-(x), B_T^-(x)\}, \min\{A_T^+(x), B_T^+(x)\}], [\max\{A_I^-(x), B_I^-(x)\}, \max\{A_I^+(x), B_I^+(x)\}], [\max\{A_F^-(x), B_F^-(x)\}, \max\{A_F^+(x), B_F^+(x)\}] >: x \in X\}$

5. $A \tilde{\bigcup} B = \{< x, [\max\{A_T^-(x), B_T^-(x)\}, \max\{A_T^+(x), B_T^+(x)\}], [\min\{A_I^-(x), B_I^-(x)\}, \min\{A_I^+(x), B_I^+(x)\}], [\min\{A_F^-(x), B_F^-(x)\}, \min\{A_F^+(x), B_F^+(x)\}] >: x \in X\}$

Definition: 2.7. [8]Let U be an initial universe set and E be a set of parameters. Consider A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F : A → P(U).

Definition: 2.8.[8] Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if A⊂ B, and $T_{F(e)}(x) \leq T_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$, $F_{F(e)}(x) \geq F_{G(e)}(x)$, ∀ e∈A , x∈U. We denote it by (F, A) ⊆ (G, B).

Definition: 2.9.[8] Complement of a neutrosophic soft set. The complement of a neutrosophic soft set (F, A) denoted by $(F,A)^c$ and is defined as $(F,A)^c = (F^c, ⌉A)$, where $F^c : ⌉A \to P(U)$ is a mapping given by $F^c(\alpha)$ = neutrosophic soft complement with $T_{Fc(x)} = F_{F(x)}, I_{Fc(x)} = I_{F(x)}$ and $F_{Fc(x)} = T_{F(x)}$.

Definition: 2.10. [10] The union of two neutrosophic soft sets (F,A)and (G,B) over (U,E) is neutrosophic soft set where C = A∪B, ∀e ∈ C

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$ and is

written as (F,A) ∪(G,B)= (H,C)

Definition :2.11 ([10]). The intersection of two neutrosophic soft sets (F,A) and (G,B) over (U,E) is neutrosophic soft set where C = A∩B ,∀e ∈ C , H(e) = F(e)∩G(e) and is written as (F,A) ∩ (G,B) = (H,C).

III. NEUTROSOPHIC SOFT CUBIC SET

Definition: 3.1

Let U be an initial universe set. Let NC(U) denote the set of all neutrosophic cubic sets and E be the set of parameters. Let $A \subset E$ then

$(P, A) = \{P(e_i) = \{<x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in U\} e_i \in I\}$, where

$A_{e_i}(x) = \{<x, A_{e_i}^T(x), A_{e_i}^I(x), A_{e_i}^F(x) > / x \in U\}$, is an interval neutrosophic set ,

$\lambda_{e_i}(x) = \{<x, (\lambda_{e_i}^T(x), \lambda_{e_i}^I(x), \lambda_{e_i}^F(x) > / x \in U\}$ is a neutrosophic set. The pair (P, A) is termed to be the neutrosophic soft cubic set over U where P is a mapping given by $P: A \rightarrow NC(U)$.

The sets of all neutrosophic soft cubic sets over U will be denoted by C_N^U.

Example:3.2

Let X = {x_1, x_2, x_3, x_4} be the set of cricket players under consideration and E = {e_1, e_2, e_3, e_4} be the set of parameters, where e_1, e_2, e_3, e_4 represent fitness, good current form, good domestic cricket record and good moral characters respectively. Let $A = \{e_1, e_2, e_3\} \subseteq E$. Then, the Neutrosophic soft cubic set

$$(P, A) = \{P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in X\} e_i \in I\}$$

i=1,2,3 in X is

X	$P(e_1)$		$P(e_2)$		$P(e_3)$	
	$\langle A_{e_1}(x)$	$\lambda_{e_1}(x) \rangle$	$\langle A_{e_2}(x)$	$\lambda(e_2) \rangle$	$\langle A_{e_3}(x)$	$\lambda_{e_3}(x) \rangle$
x_1	[0.2,0.4][0.4,0.7][0.3,0.6]	[0.2,0.5,0.45]	[0.3,0.6][0.3,0.5][0.5,0.7]	[0.3,0.4,0.5]	[0.4,0.7][0.5,0.6][0.3,0.6]	[0.5,0.55,0.45]
x_2	[0.4,0.7][0.3,0.6][0.5,0.8]	[0.5,0.4,0.7]	[0.5,0.7][0.4,0.6][0.3,0.7]	[0.6,0.5,0.6]	[0.3,0.6][0.5, 0.7][0.2,0.5]	[0.5,0.6,0.3]
x_3	[0.5,0.7][0.2,0.4][0.4,0.6]	[0.6,0.3,0.5]	[0.6,0.8][0.1,0.4][0.2,0.5]	[0.65,0.3,0.4]	[0.4,0.7][0.3,0.6][0.2, 0.5]	[0.3,0.7,0.6]
x_4	[0.6,0.8][0.3,0.6][0.2,0.5]	[0.7,0.4,0.4]	[0.4,0.7][0.3,0.6][0.5,0.8]	[0.7,0.5,0.65]	[0.5,0.9][0.2, 0.6][0.4,0.6]	[0.8,0.3,0.5]
x^5	[0.4,0.8][0.5,0.7][0.5,0.7]	[0.6,0.55,0.65]	[0.5,0.9][0.4,0.6][0.4,0.6]	[0.55,0.45,0.45]	[0.3,0.7][0.1, 0.4][0.4,0.7]	[0.45,0.35,0.55]

Definition:3.3

Let X be a non-empty set. A neutrosophic soft cubic set (P, A) in X is said to be

- truth-internal (briefly, T-internal) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (A_{e_i}^{-T}(x) \leq \lambda_{e_i}^{T}(x) \leq A_{e_i}^{+T}(x)), \tag{3.1}$$

- indeterminacy-internal (briefly, I-internal) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (A_{e_i}^{-I}(x) \leq \lambda_{e_i}^{I}(x) \leq A_{e_i}^{+I}(x)), \tag{3.2}$$

- falsity-internal (briefly, F-internal) if the following inequality is valid

$$(\forall x \in X, e_i \in E)(A_{e_i}^{-F}(x) \leq \lambda_{e_i}^{F}(x) \leq A_{e_i}^{+F}(x)).$$

(3.3)7

If a neutrosophic soft cubic set in X satisfies (3.1), (3.2) and (3.3) we say that (P, A) is an internal neutrosophic soft cubic in X.

Table 2. Tabular representation of (P, A)

X	$P[e1]$		$P[e2]$		$P[e3]$	
	$A_{e_1}(x)$	$\lambda_{e_1}(x)$	$A_{e_2}(x)$	$\lambda(e_2)$	$A_{e_3}(x)$	$\lambda(e_3)$
x_1	[0.4,0.6][0.3,0.5][0.2,0.6]	[0.5,0. 45,0.4]	[0.2,0.4][0.4, 0.7][0.3, 0.6]	[0. 3,0.5,0.55]	[0.3,0.6][0.3,0.5][0.5,0.7]	[0. 5,0.4,0.6]
x_2	[0.1,0.4][0.2,0.3][0.1,0.5]	[0. 3,0.25,0.4]	[0.4, 0.7][0.3,0.6][0.5,0.8]	[0. 5,0.45,0.6]	[0.5,0.7][0.4,0.6][0.3.0.7]	[0. 6,0.5,0.45]
x_3	[0.3,0.6][0.2,0.5][0.4,0.6]	[0. 4, 0.45, 0.5]	[0.5,0.7][0.2,0.4][0.4,0.6]	[0. 6,0.35,0.45]	[0.6,0.8][0.1,0.4][0.2,0.5]	[0. 65,0.3,0.4]
x_4	[0.2,0.5][0.1,0.4][0.3,0.8]	[0. 3, 0.3,0.5]	[0.6,0.8][0.3,0.6][0.2,0.5]	[0. 65,0.4,0.4]	[0. 2, 0. 6][0. 3, 0. 6][0. 4, 0. 8]	[0.55,0.45,0.65]
$x5$	[0.3,0.7][0.1,0.3][0.2,0.4]	[0.65,0.2,0.35]	[0.4,0.8][0.5, 0.7][0.5, 0.7]	[0.6,0.65,0.55]	[0. 3, 0. 6] [0. 2, 0. 7][0.4,0.7]	[0.4,0.5,0.6]

Definition: 3.4

Let X be a non-empty set. A neutrosophic soft cubic set (P, A) in X is said to be

- truth-external (briefly, T-external) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (\lambda_{e_i}^{T}(x) \notin (A_{e_i}^{-T}(x), A_{e_i}^{+T}(x))), \tag{3.4}$$

- indeterminacy-external (briefly, I-external) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (\lambda_{e_i}^{I}(x) \notin (A_{e_i}^{-I}(x), A_{e_i}^{+I}(x))), \tag{3.5}$$

- falsity-external (briefly, F-external) if the following inequality is valid

$$(\forall x \in X, e_i \in E)\ (\lambda_{e_i}^{F}(x) \notin (A_{e_i}^{-F}(x), A_{e_i}^{+F}(x))). \tag{3.6}$$

If a neutrosophic soft cubic set (P, A)) in X satisfies (3.4), (3.5) and (3.6), we say that (P, A) is an external neutrosophic soft cubic in X.

Example:3.5

Let X = {p_1, p_2, p_3, p_4} be the set of cricket players under consideration and E = {e_1, e_2, e_3, e_4} be the set of parameters, where e_1, e_2, e_3, e_4 represent fitness, good current form, good domestic cricket record and good moral character, resp. Let $I = \{e_1, e_2, e_3\} \subseteq E$. Then the NSCSS

$$(\tilde{P}, I) = \{P(e_i) = \{< p, A_{e_i}(p), \lambda_{e_i}(p) >: p \in X\}\ e_i \in I, i = 1,2,3\}$$

in X is external neutrosophic soft cubic set (ENSCS) in X.

Table 3. Tabular representation of (P, A)

X	$P[e1]$		$P[e2]$		$P[e3]$	
	$A_{e_1}(x)$	$\lambda_{e_1}(x)$	$A_{e_2}(x)$	$\lambda_{e_2}(x)$	$A_{e_3}(x)$	$\lambda_{e_3}(x)$
x_1	[0.4,0.6][0.3,0.5][0.2, 0.6]	[0.7,0.6,0.7]	[0.2,0.4][0.4,0.7][0.3,0.6]	[0.1,0.3,0.65]	[0.3,0.6][0.3,0.5][0.5,0.7]	[0.7,0.6,0.4]
x_2	[0.1,0.4][0.2,0.3][0.1,0.5]	[0.5,0.4,0.6]	[0.4, 0.7][0.3,0.6][0.5,0.8]	[0.3,0.2,0.4]	[0.5,0.7][0.4,0.6][0.3,0.7]	[0.4,0.7,0.2]
x_3	[0.3,0.6][0.2,0.5][0.4,0.6]	[0.7, 0.6, 0.2]	[0.5,0.7][0.2,0.4][0.4,0.6]	[0.4,0.5,0.35]	[0.6,0.8][0.1,0.4][0.2,0.5]	[0.5,0.5,0.6]
x_4	[0.2,0.5][0.1,0.4][0.3,0.8]	[0.6, 0.5,0.2]	[0.6,0.8][0.3,0.6][0.2,0.5]	[0.5,0.2,0.6]	[0.2, 0.6][0.3, 0.6][0.4, 0.8]	[0.7,0.25,0.35]
x_5	[0.5,0.7][0.1,0.3][0.2,0.4]	[0.45,0.4,0.5]	[0.4,0.8][0.5, 0.7][0.5, 0.7]	[0.3,0.45,0.4]	[0.3, 0.6][0.2, 0.7][0.4,0.7]	[0.2,0.8,0.3]

Theorem: 3.6

Let

$(P, A) = \{ P(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in X\}\ e_i \in I \}$

be a neutrosophic soft cubic set in X which is not an ENSCS. Then, there exists at least one $e_i \in I$ for which there exists some $x \in X$ such that

$\lambda^T_{e_i}(x) \in (A^{-T}_{e_i}(x), A^{+T}_{e_i}(x)),$

$\lambda^I_{e_i}(x) \in (A^{-I}_{e_i}(x), A^{+I}_{e_i}(x)),$

$\lambda^T_{e_i}(x) \in (A^{-F}_{e_i}(x), A^{+F}_{e_i}(x))$

Proof: Straightforward

Theorem: 3.7

Let (P, A) be a neutrosophic soft cubic set in X. If (P, A) is both T-internal and T-external in X, then

$(\forall x \in X, e_i \in E)\ (\lambda^T_{e_i}(x) \in \{A^{-T}_{e_i}(x) / x \in X, e_i \in E\} \cup \{A^{+T}_{e_i}(x) / x \in X, e_i \in E\})$

(3.7)

Proof.

Consider the conditions (3.1) and (3.4) which implies that $A^{-T}_{e_i}(x) \le \lambda^T_{e_i}(x) \le A^{+T}_{e_i}(x)$ and $\lambda^T_{e_i}(x) \notin (A^{-T}_{e_i}(x), A^{+T}_{e_i}(x)$ for all $x \in X$, $e_i \in E$.

Then it follows that $\lambda^T_{e_i}(x) =$

$A^{-T}_{e_i}(x)\ or\ \lambda^T_{e_i}(x) = A^{+T}_{e_i}(x),$

And hence

$\lambda^T_{e_i}(x) \in \{A^{-T}_{e_i}(x) / x \in X, e_i \in E\} \cup \{A^{+T}_{e_i}(x) / x \in X, e_i \in E\}.$

Hence Proved.

Similarly, the following propositions hold for the indeterminate and falsity values.

Theorem: 3.8

Let (P, A) be a neutrosophic soft cubic set in a non-empty set X. If (P, A) is both I-internal and I-external, then

$(\forall x \in X, e_i \in E)$

$(\lambda^I_{e_i}(x) \in \{A^{-I}_{e_i}(x) / x \in X, e_i \in E\} \cup \{A^{+I}_{e_i}(x) / x \in X, e_i \in E\})$

Theorem: Let (P, A) be a neutrosophic soft cubic set in a non-empty set X. If (P, A) is both F-internal and F-external, then

$(\forall x \in X, e_i \in E)$

$(\lambda^F_{e_i}(x) \in \{A^{-F}_{e_i}(x) / x \in X, e_i \in E\} \cup \{A^{+F}_{e_i}(x) / x \in X, e_i \in E\})$

Definition: 3.10

Let $\Im = (P, A) \in C^X_N$. If

$A^{-T}_{e_i}(x) \le \lambda^T_{e_i}(x) \le A^{+T}_{e_i}(x)$,

$A^{-I}_{e_i}(x) \le \lambda^I_{e_i}(x) \le A^{+I}_{e_i}(x)$ and

$\lambda^F_{e_i}(x) \notin (A^{-F}_{e_i}(x), A^{+F}_{e_i}(x)$ or

$A^{-T}_{e_i}(x) \le \lambda^T_{e_i}(x) \le A^{+T}_{e_i}(x)$,

$A^{-F}_{e_i}(x) \le \lambda^F_{e_i}(x) \le A^{+F}_{e_i}(x)$ and

$\lambda^I_{e_i}(x) \notin (A^{-I}_{e_i}(x), A^{+I}_{e_i}(x)$ or

$A^{-F}_{e_i}(x) \le \lambda^F_{e_i}(x) \le A^{+F}_{e_i}(x)$,

$A^{-I}_{e_i}(x) \le \lambda^I_{e_i}(x) \le A^{+I}_{e_i}(x)$ and

$\lambda^T_{e_i}(x) \notin (A^{-T}_{e_i}(x), A^{+T}_{e_i}(x)$ for all $x \in X$

corresponding to each $e_i \in I$. Then $\Im$ is called an external neutrosophic soft cubic set $\frac{2}{3}$ INSCS or $\frac{1}{3}$ ENSCS.

Example: 3.11

Let $\Im = (P, A) \in C^X_N$. If

$(P, A) = P(e) =$

$\{< x, ([0.2,0.5],[0.5,0.7],[0.3,0.5]), (0.3,0.4,0.4) >\}$

all $x \in X$ corresponding to each $e_i \in I$ .Then

$\Im = (P, A)$ is a $\frac{2}{3}$ INSCS.

Definition: 3.12

Let $\Im = (P, A) \in C^X_N$. If

$A^{-T}_{e_i}(x) \le \lambda^T_{e_i}(x) \le A^{+T}_{e_i}(x)$,

$\lambda^I_{e_i}(x) \notin (A^{-I}_{e_i}(x), A^{+I}_{e_i}(x)$ and

$\lambda^F_{e_i}(x) \notin (A^{-F}_{e_i}(x), A^{+F}_{e_i}(x)$

or , $A^{-F}_{e_i}(x) \le \lambda^F_{e_i}(x) \le A^{+F}_{e_i}(x)$,

$\lambda^T_{e_i}(x) \notin (A^{-T}_{e_i}(x), A^{+T}_{e_i}(x)$ and

$\lambda^I_{e_i}(x) \notin (A^{-I}_{e_i}(x), A^{+I}_{e_i}(x)$ or

$A^{-I}_{e_i}(x) \le \lambda^I_{e_i}(x) \le A^{+I}_{e_i}(x)$,

$\lambda^F_{e_i}(x) \notin (A^{-F}_{e_i}(x), A^{+F}_{e_i}(x)$ and

$\lambda^T_{e_i}(x) \notin (A^{-T}_{e_i}(x), A^{+T}_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in I$. Then $\Im$ is called an external neutrosophic soft cubic set $\frac{1}{3}$ INSCS or $\frac{2}{3}$ ENSCS.

Example: 3.13

Let $\Im = (P, A) \in C^X_N$. If

$(P, A) = P(e) =$

$\{< x, ([0.2,0.5],[0.5,0.7],[0.3,0.5]), (0.3,0.4,0.6) >\}$

where all $x \in X$ corresponding to each $e_i \in I$ .Then $\Im = (P, A)$ is a $\frac{1}{3}$ INSCS.

Theorem: 3.14

Let $\Im = (P, A) \in C^X_N$ .Then

i. *Every INSCS is a generalization of the ICS*

ii. *Every ENSCS is a generalization of the ECS.*

iii. *Every NSCS is the generalization of cubic set.*

Proof. The proof is direct from the above definitions.

Definition 3.15.

Let

$(P, I) = \{ P(e_i) =$

$\{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in X\} \; e_i \in I\}$

and

$(Q, J) = \{ Q(e_i) =$

$\{< x, B_{e_i}(x), \mu_{e_i}(x) >: x \in X\} \; e_i \in I\}$

be two neutrosophic soft cubic sets in X. Let I and J be any two subsets of E (set of parameters), then we have the following

1. $(P, I) = (Q, J)$ if and only if the following conditions are satisfied
   a) I = J and
   b) $P(e_i) = Q(e_i)$ for all $e_i \in I$ if and only if $A_{e_i}(x) = B_{e_i}(x)$ and $\lambda_{e_i}(x) = \mu_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in I$.
2. (P, I) and (Q, J) are two neutrosophic soft cubic set then we define and denote P- order as $(P, I) \subseteq_P (Q, J)$ if and only if the following conditions are satisfied
   c) I $\subseteq$ J and
   d) $P(e_i) \le_P Q(e_i)$ for all $e_i \in I$ if and only if $A_{e_i}(x) \subseteq B_{e_i}(x)$ and $\lambda_{e_i}(x) \le \mu_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in I$.
3. (P, I) and (Q, J) are two neutrosophic soft cubic set then we define and denote P- order as $(P, I) \subseteq_R (Q, J)$ if and only if the following conditions are satisfied
   e) I $\subseteq_R$ J and
   f) $P(e_i) \le_R Q(e_i)$ for all $e_i \in I$ if and only if $A_{e_i}(x) \subseteq B_{e_i}(x)$ and $\lambda_{e_i}(x) \ge \mu_{e_i}(x)$ for all $x \in X$ corresponding to each $e_i \in I$.

We now define the P-union, P-intersection, R-union and R-intersection of neutrosophic cubic soft sets as follows:

Definition: 3.16

Let (F, I) and (G, J) be two neutrosophic soft cubic sets (NSCS) in X where I and J are any two subsets of the parameteric set E. Then we define **P-union** as $(F, I) \cup_p (G, J) = (H, C)$ where $C = I \cup J$

$H(e_i)$ =

$$\begin{cases} F(e_i) & If\ e_i \in I - J \\ G(e_i) & If\ e_i \in J - I \\ F(e_i) \vee_P G(e_i) & If\ e_i \in I \cap J \end{cases}$$

where $F(e_i) \vee_P G(e_i)$ is defined as

$F(e_i) \vee_P G(e_i)$ =

$$\{<x, \max\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \vee \mu_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

where $A_{e_i}(x), B_{e_i}(x)$ represent interval neutrosophic sets. Hence

$F^T(e_i) \vee_P G^T(e_i)$ =

$$\{<x, \max\{A^T_{e_i}(x), B^T_{e_i}(x)\}, (\lambda^T_{e_i} \vee \mu^T_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

,

$F^I(e_i) \vee_P G^I(e_i)$ =

$$\{<x, \max\{A^I_{e_i}(x), B^I_{e_i}(x)\}, (\lambda^I_{e_i} \vee \mu^I_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

,

$F^F(e_i) \vee_P G^F(e_i)$ =

$$\{<x, \max\{A^F_{e_i}(x), B^F_{e_i}(x)\}, (\lambda^F_{e_i} \vee \mu^F_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

.

Definition: 3.17

Let (F,I) and (G,J) be two neutrosophic soft cubic sets (NSCS) in X where I and J are any subsets of parameter's set E.

Then we define **P-intersection** as

$(F,I) \cap_p (G,J) = (H,C)$ where $C = I \cap J$,

$H(e_i) = F(e_i) \wedge_P G(e_i)\ and\ e_i \in I \cap J.$

Here $F(e_i) \wedge_P G(e_i)$ is defined as

$F(e_i) \wedge_P G(e_i) = H(e_i)$ =

$$\{<x, \min\{A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \wedge \mu_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

.

where $A_{e_i}(x), B_{e_i}(x)$ represent interval neutrosophic sets. Hence

$F^T(e_i) \wedge_P G^T(e_i)$ =

$$\{<x, \min\{A^T_{e_i}(x), B^T_{e_i}(x)\}, (\lambda^T_{e_i} \wedge \mu^T_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

,

$F^I(e_i) \wedge_P G^I(e_i)$ =

$$\{<x, \min\{A^I_{e_i}(x), B^I_{e_i}(x)\}, (\lambda^I_{e_i} \wedge \mu^I_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

,

$F^F(e_i) \wedge_P G^F(e_i)$ =

$$\{<x, \min\{A^F_{e_i}(x), B^F_{e_i}(x)\}, (\lambda^F_{e_i} \wedge \mu^F_{e_i})(x)>: x \in X\}\ e_i \in I \cap J$$

.

Example: 3.18 (P-ORDER)

Let X={x_1,x_2,x_3,x_4,x_5} be initial universe, I = J= {e_1,e_2} are any subsets of parameter's set E={e_1,e_2,e_3}.

Let (F,I) be NSCS defined as

$$(F,I) = \{F(e_i) = \{<x, A_{e_i}(x), \lambda_{e_i}(x)>: x \in X\}\ e_i \in I\}$$

is

X	F[e1]		F[e2]	
	$A_{e_1}(x)$	$\lambda_{e_1}(x)$	$A_{e_2}(x)$	$\lambda(e_2)$
x_1	[0.5,0.6][0.3,0.8][0.3,0.4]	[0.4,0.2,0.5]	[0.2,0.6][0.1,0.3][0.2,0.8]	[0.7,0.5,0.6]
x_2	[0.2,0.5][0.4,0.7][0.5,0.6]	[0.3,0.5,0.4]	[0.4,0.5][0.3,0.5][0.2,0.4]	[0.6,0.4,0.5]
x_3	[0.3,0.4][0.7,0.9][0.1,0.2]	[0.5,0.6,0.4]	[0.2,0.3][0.1,0.3][0.4,0.5]	[0.5,0.3,0.4]
x_4	[0.1,0.7][0.2,0.4][0.6,0.7]	[0.6,0.3,0.6]	[0.5,0.6][0.4,0.5][0.3,0.4]	[0.8,0.5,0.5]
x^5	[0.4,0.5][0.3,0.5][0.2,0.4]	[0.7,0.4,0.2]	[0.3,0.6][0.2,0.3][0.5,0.6]	[0.4,0.3,0.3]

Let (G,J) be NSCS defined as

$$(G,J) = \{G(e_i) = \{<x, B_{e_i}(x), \mu_{e_i}(x)>: x \in X\}\ e_i \in J\}$$

is

X	G[e1]		G[e2]	
	$B_{e1}(x)$	$\mu_{e1}(x)$	$B_{e2}(x)$	$\mu_{e2}(x)$
x_1	[0.3,0.4][0.7,0.9][0.1,0.2]	[0.5,0.6,0.4]	[0.4,0.6][0.7,0.8][0.1,0.4]	[0.8,0.6,0.5]
x_2	[0.6,0.8][0.3,0.4][0.1,0.7]	[0.4,0.5,0.6]	[0.1,0.5][0.4,0.7][0.5,0.6]	[0.5,0.3,0.4]
x_3	[0.3,0.6][0.4,0.7][0.3,0.4]	[0.3,0.4,0.6]	[0.4,0.7][0.1,0.3][0.2,0.4]	[0.6,0.3,0.2]
x_4	[0.6,0.7][0.3,0.4][0.2,0.4]	[0.6,0.5,0.5]	[0.3,0.4][0.7,0.9][0.1,0.2]	[0.4,0.6,0.6]
x^5	[0.2,0.6][0.2,0.4][0.3,0.5]	[0.3,0.2,0.3]	[0.5,0.6][0.6,0.7][0.3,0.4]	[0.6,0.7,0.5]

Then P- union is denoted by $(F,I) \cup_p (G,J)$ and defined as

X	F ∪ G(e1)		F ∪ G[e2]	
	< A U B e1(x)	λUμ e1(x) >	< A U B e2(x) ,	λUμ e2(x) >
x_1	[0.5,0.6][0.7,0.9][0.3,0.4]	[0.5,0.6,0.5]	[0.4,0.6][0.7,0.8][0.2,0.8]	[0.8,0.6,0.6]
x_2	[0.6,0.8][0.4,0.7][0.5,0.6]	[0.4,0.5,0.6]	[0.4,0.5][0.4,0.7][0.5,0.6]	[0.6,0.4,0.5]
x_3	[0.3,0.6][0.7,0.9][0.3,0.4]	[0.5,0.6,0.6]	[0.4,0.7][0.1,0.3][04,0.5]	[0.6,0.3,0.4]
x_4	[0.6,0.7][0.3,0.4][0.6,0.7]	[0.6,0.5,0.6]	[0.5,0.6][0.7,0.9][0.3,0.4]	[0.8,0.6,0.6]
x^5	[0.4,0.5][0.3,0.5][0.3,0.5]	[0.3,0.2,0.3]	[0.5,0.6][0.6,0.7][0.5,0.6]	[0.6,0.7,0.5]

Then P- intersection denoted by $(F,I) \cap_p (G,J)$ and defined as

X	F ∩ G(e1)		F∩ G(e2)	
	< A ∩ B e1(x)	λ∩μ e1(x) >		λ∩μ e2(x) >
x_1	[0.3,0.4][0.3,0.8][0.1,0.2]	[0.4,0.2,0.4]	[0.2,0.6][0.10.3][0.1,0.4]	[0.7,0.5,0.5]
x_2	[0.2,0.4][0.3,0.4][0.1,0.7]	[0.3,0.5,0.6]	[0.1,0.5][0.3,0.5][0.2,0.4]	[0.5,03,0.4]
x_3	[0.3,0.4][0.2,0.7][0.1,0.2]	[0.3,0.4,0.4]	[0.2,0.3][0.1,0.3][0.2,0.4]	[0.5,0.3,0.2]
x_4	[0.1,0.7][0.2,0.4][0.2,0.4]	[0.6,0.3,0.5]	[0.3,0.4][0.4,0.5][0.1,0.2]	[0.4,0.5,0.05]
x^5	[0.2,0.6][0.2,0.4][0.2,0.4]	[0.3,0.2,0.2]	[0.3,0.6][0.2,0.3][0.3,0.4]	[0.4,0.3,0.3]

Definition: 3.18

Let (F,I) and (G,J) be two neutrosophic soft Subic sets (NSCS) in X where I and J are any subsets of parameter's set E.

Then we define **R-union** as

$(F,I) \cup_R (G,J) = (H,C)$ where $C = I \cup J$

$H(e_i)$ =

$$\begin{cases} F(e_i) & if\ e_i \in I - J \\ G(e_i) & if\ e_i \in J - I \\ F(e_i) \vee_R G(e_i) & if\ e_i \in I \cap J \end{cases}$$

where $F(e_i) \vee_R G(e_i)$ is defined as

$F(e_i) \vee_R G(e_i)$ =

$\{< x, \max\{ A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \wedge \mu_{e_i})(x) >: x \in X\}\ e_i \in I \cap J$

where $A_{e_i}(x), B_{e_i}(x)$ represent interval neutrosophic sets.

Hence $F^T(e_i) \vee_R G^T(e_i)$ =

$\{< x, \max\{ A^T_{e_i}(x), B^T_{e_i}(x)\}, (\lambda^T_{e_i} \wedge \mu^T_{e_i})(x) >: x \in X\}\ e_i \in I \cap J$

,

$F^I(e_i) \vee_R G^I(e_i)$ =

$\{< x, \max\{ A^I_{e_i}(x), B^I_{e_i}(x)\}, (\lambda^I_{e_i} \wedge \mu^I_{e_i})(x) >: x \in X\}\ e_i \in I \cap J$

,

$F^F(e_i) \vee_R G^F(e_i)$ =

$\{< x, \max\{ A^F_{e_i}(x), B^F_{e_i}(x)\}, (\lambda^F_{e_i} \wedge \mu^F_{e_i})(x) >: x \in X\}\ e_i \in I \cap J$

Definition: 3.19

Let (F, I) and (G, J) be two neutrosophic soft cubic sets (NSCS) in X where I and J are any subsets of parameter's set E.

Then we define **R-intersection** as

$(F, I) \cap_R (G, J) = (H, C)$ where $C = I \cap J$,

$H(e_i) = F(e_i) \wedge_R G(e_i)$ *and* $e_i \in I \cap J$.

Here $F(e_i) \wedge_R G(e_i)$ is defined as

$F(e_i) \wedge_R G(e_i) = H(e_i)$ =

$\{< x, \min\{ A_{e_i}(x), B_{e_i}(x)\}, (\lambda_{e_i} \vee \mu_{e_i})(x) >: x \in X\}\ e_i \in I \cap J\}$

.

Example 3.21: (R –ORDER)

Let X={x_1,x_2,x_3,x_4,x_5} be initial universe, I = J= {e_1,e_2} are any subsets of parameter's set E={e_1,e_2,e_3}.

Let (F,I) be NSCS defined as

$(F, I) = \{ F(e_i) = \{< x, A_{e_i}(x), \lambda_{e_i}(x) >: x \in X\}\ e_i \in I\}$

X	$P(e_1)$		$P(e_2)$	
	$A_{e_1}(x)$	$\lambda_{e_1}(x)$	$A_{e_2}(x)$	$\lambda(e_2)$
x_1	[0.2,0.3][0.3,0.5][0.3,0.5]	[0.35,0.35,0.4]	[0.3,0.7][0.3,0.5][0.1,0.5]	[0.25,0.25,0.6]
x_2	[0.4,0.7][0.1,0.4][0.2,0.4]	[0.35,0.5,0.3]	[0.5,0.8][0.5,0.6][0.2,0.5]	[0.45,0.3,0.4]
x_3	[0.6,0.9][0.0,0.2][0.3,0.4]	[0.5,0.6,0.55]	[0.4,0.9][0.4,0.7][0.3,0.5]	[0.35,0.2,0.45]

is

Let (G,J) be NSCS defined as

$(G, J) = \{ G(e_i) = \{< x, B_{e_i}(x), \mu_{e_i}(x) >: x \in X\}\ e_i \in J\}$

is

X	G[e1]		G[e2]	
	< B e1(x)	μe1(x) >	< B e2(x)	μe2(x) >
x_1	[0.3,0.5][0.4,0.6][0.4,0.5]	[0.35,0.3,0.5]	[0.1,0.3][0.5,0.7][0.3,0.5]	[0.4,0.6,0.8]
x_2	[0.2,0.5][0.5,0.7][0.3,0.5]	[0.3,0.4,0.6]	[0.4,0.7][0.4,0.6][0.5,0.8]	[0.5,0.3,0.6]
x_3	[0.6,0.8][0.4,0.7][0.7,0.9]	[0.7,0.3,0.4]	[0.7,0.9][0.4,0.5][0.7,0.9]	[0.5,0.45,0.2]

Then R-union denoted by $(F, I) \cup_R (G, J)$ and defined as

X	F ∪ G(e1)		F ∪ G[e2]	
	A U B e1(x)	λ∪μ e1(x) >	< A U B e2(x) ,	λ∪μ e2(x) >
x_1	[0.3,0.5][0.4,0.6][0.4,0.5]	[0.35,0.25,0.4]	[0.3,0.7][0.5,0.7][0.3,0.5]	[0.25,0.25,0.6]
x_2	[0.4,0.7][0.5,0.7][0.3,0.5]	[0.3,0.4,0.3]	[0.5,0.8][0.5,0.6][0.5,0.8]	0.45,0.3,0.4]
x_3	[0.6,0.9][0.4,0.7][0.7,0.9]	[0.5,0.3,0.4]	[0.7,0.9][0.4,0.7][0.7,0.9]	[0.35,0.2,0.2]

Then R- intersection is denoted by $(F, I) \cap_R (G, J)$ and defined as

X	F ∩ G(e1)		F∩ G(e2)	
	< A ∩ B e1(x)	λ∩μ e1(x) >	< A∩B e2(x)	λ∩μ e2(x) >
x_1	[0.2,0.3][0.3,0.5][0.3,0.5]	[0.35,0.3,0.5]	[0.1,0.3][0.3,0.5][0.1,0.5]	[0.4,0.6,0.8]
x_2	[0.2,0.5][0.1,0.4][0.2,0.4]	[0.35,0.5,0.6]	[0.4,0.7][0.4,0.6][0.2,0.5]	[0.5,0.3,0.6]
x_3	[0.6,0.8][0.0,0.2][0.3,0.4]	[0.7,0.6,0.55]	[0.4,0.9][0.4,0.5][0.3,0.5]	[0.5,0.45,0.45]

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