On Some Basic Properties of Geometric Real Sequences

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Abstract—Objective of this paper is to introduce geometric real sequence and discuss their basic properties.
Keywords *—Non-Newtonian Calculus, geometric Arithmetic, geometric integers, geometric real numbers.*

1. INTRODUCTION

In mid-Seventeenth century, Newton and Leibniz created classical calculus considering the deviations by difference, i.e. as $(x + h) - f(x)$. In 1967 Michael Grossman and Robert Katz created the first system of non-Newtonian calculus [10] and is called "Multiplicative Calculus" or exponential calculus considering the deviation by ratio, i.e. as $\frac{f(x+a)}{f(a)}$. The operations of multiplicative calculus are called as multiplicative derivative and multiplicative integral. In 1970 they had created infinite family of non-Newtonian calculi, each of which are different from the classical calculus of Newton and Leibniz.

After Grossman and Katz's pioneering creation of non-Newtonian calculus, many researches have been developing the field of non-Newtonian calculus and its applications. We refer Grossman and Katz [10] , Stanley [24] , Bashirov et al. [2, 3], Grossman [9] , K. Boruah and B. Hazarika [13, 14, 15, 16, 17, 18] for elements of multiplicative calculus and its applications. An extension of multiplicative calculus to functions of complex variables is handled in Bashirov and Riza [1], Uzer [27] ,Bashirov et al. [3] , Çakmak and Başar[5], Tekin and Başar [25], Türkmen and Başar [26] . Kadak and Özlük studied the generalized Runge-Kutta method with respect to non-Newtonian calculus. Kadak et al [28] studied certain new types of sequence spaces over the Non-Newtonian Complex Field.

Geometric calculus is also a type of non-Newtonian calculus. It provides differentiation and integration tools based on multiplication instead of addition. Every property in Newtonian calculus has an analog in multiplicative calculus. Generally speaking multiplicative calculus is a methodology that allows one to have a different look at problems which can be investigated via calculus. In some cases, for example for growth related problems, the use of multiplicative calculus is advocated instead of a traditional Newtonian one.

The main aim of this paper is to discuss about the basic properties of geometric sequences. Before we establish new results, we recall the construction of arithmetics generated by different generators and the geometric arithmetic, which is the keyword of the whole article.

2. α −GENERATOR AND GEOMETRIC REAL FIELD

A *generator* is a one-to-one function whose domain is the set of real numbers, $\mathbb{R}$ and range is a set $B \subset \mathbb{R}$. For example, the identity function I and the exponential function exp are generators. We consider any generator α with realm i.e. domain, say, A and range B, by $\alpha - arithmetic$, we mean the arithmetic whose operations and ordering relation are defined as follows:

$$
\begin{array}{lll}
\alpha - \text{addition} & x \dot{+} y & = \alpha[\alpha^{-1}(x) + \alpha^{-1}(y)] \\
\alpha - \text{subtraction} & x \dot{-} y & = \alpha[\alpha^{-1}(x) - \alpha^{-1}(y)] \\
\alpha - \text{multiplication} & x \dot{\times} y & = \alpha[\alpha^{-1}(x) \times \alpha^{-1}(y)] \\
\alpha - \text{division} & x \dot{/} y & = \alpha[\alpha^{-1}(x)/\alpha^{-1}(y)] \\
\alpha - \text{order} & x \dot{<} y & \Leftrightarrow \alpha^{-1}(x) < \alpha^{-1}(y).
\end{array}
$$

for $x, y \in A$, where A is a domain of the function α.

It is to be noted that each generator generates exactly one arithmetic and each arithmetic is generated by exactly one generator. For example, if we choose the exponential function exp as an $\alpha - generator$ defined by $\alpha(x) = e^z$ for $x \in \mathbb{R}$ and hence $\alpha^{-1}(x) = \ln x$, then $\alpha - arithmetic$ turns out to geometric arithmetic as follows:

geometric addition	$x \oplus y$	$= \alpha[\alpha^{-1}(x) + \alpha^{-1}(y)]$	$= e^{(\ln x + \ln y)}$	$= x.y$
geometric $-$ subtraction	$x \ominus y$	$= \alpha[\alpha^{-1}(x) - \alpha^{-1}(y)]$	$= e^{(\ln x - \ln y)}$	$= \dfrac{x}{y}, \quad y \neq 0$
geometric $-$ multiplication	$x \odot y$	$= \alpha[\alpha^{-1}(x) \times \alpha^{-1}(y)]$	$= e^{(\ln x \times \ln y)}$	$= x^{\ln y}$
geometric $-$ division	$x \oslash y$	$= \alpha[\alpha^{-1}(x)/\alpha^{-1}(y)]$	$= e^{(\ln x \div \ln y)}$	$= x^{\frac{1}{\ln y}}, y \neq 1.$

Similarly, the identity function generates classical arithmetic.

It is obvious that $\ln(x) < \ln(y)$ if $x < y$ for $x, y \in \mathbb{R}^+$. That is, $x < y \Leftrightarrow \alpha^{-1}(x) < \alpha^{-1}(y)$. So, without loss of generality, we use $x < y$ instead of the geometric order $x \overset{.}{<} y$.
C. Türkmen and F. Başardefined the sets of geometric integers, geometric real numbers and geometric complex numbers $\mathbb{Z}(G), \mathbb{R}(G)$ and $\mathbb{C}(G)$, respectively, as follows:

$$\begin{aligned} \mathbb{Z}(G) &= \{e^x : x \in \mathbb{Z}\} \\ \mathbb{R}(G) &= \{e^x : x \in \mathbb{R}\} = \mathbb{R}^\wedge + \setminus \{0\} \\ \mathbb{C}(G) &= \{e^z : z \in \mathbb{C}\} = \mathbb{C} \setminus \{0\}. \end{aligned}$$

If we take extended real number line, then $\mathbb{R}(G) = [0, \infty]$.
$(\mathbb{R}(G), \oplus, \odot)$ is a field with geometric zero 1 and geometric identity e, since

(1). $(\mathbb{R}(G), \oplus)$ is a geometric additive Abelian group with geometric zero 1,

(2). $(\mathbb{R}(G) \setminus \{1\}, \odot)$ is a geometric multiplicative Abelian group with geometric identity e,

(3). $\odot$ is distributive over $\oplus$.

But $(\mathbb{C}(G), \oplus, \odot)$ is not a field, however, geometric binary operation $\odot$ is not associative in $\mathbb{C}(G)$. For, we take $x = e^{1/4}, y = e^4$ and $z = e^{(1 + i\pi/2)} = ie$. Then
$$(x \odot y) \odot z = e \odot z = z = ie$$
But$x \odot (y \odot z) = x \odot e^4 = e.$

Let us define geometric positive real numbers and geometric negative real numbers as follows:
$$\begin{aligned} \mathbb{R}^+(G) &= \{x \in \mathbb{R}(G) : x > 1\} \\ \mathbb{R}^-(G) &= \{x \in \mathbb{R}(G) : x < 1\}. \end{aligned}$$

2.1 *Some Useful Relations between Geometric Operations and Ordinary Arithmetic Operations:*
For all $x, y \in \mathbb{R}(G)$

- $x \oplus y = xy$

- $x \ominus y = x/y$

- $x \odot y = x^{\ln y} = y^{\ln x}$

- $\left(\dfrac{a}{b}\right) \odot y = \left(\dfrac{a}{b}\right)^{\ln y} = \dfrac{a^{\ln y}}{b^{\ln y}} = \dfrac{a \odot y}{b \odot y}$

- $\dfrac{a}{b} \odot \dfrac{c}{d} = \left(\dfrac{a}{b}\right)^{\ln\left(\frac{c}{d}\right)} = \left(\dfrac{b}{a}\right)^{-\ln\left(\frac{c}{d}\right)} = \left(\dfrac{b}{a}\right)^{\ln\left(\frac{d}{c}\right)} = \dfrac{b}{a} \odot \dfrac{d}{c}$

- $x \oslash y$ or $\dfrac{x}{y} G = x^{\frac{1}{\ln y}}, y \neq 1$

- $x^{2_G} = x \odot x = x^{\ln x}$

- $x^{p_G} = x^{\ln^{p-1} x}$

- $\sqrt{x}^{G} = e^{(\ln x)^{\frac{1}{2}}}$

- $x^{-1_G} = e^{\frac{1}{\log x}}$

- $x \odot e = x$ and $x \oplus 1 = x$

- $e^n \odot x = x \oplus x \oplus \ldots \ldots$ (upto n numberof x) $= x^n$

$$|x|^G = \begin{cases} x, & \text{if} & x > 1 \\ 1, & \text{if} & x = 1 \\ \dfrac{1}{x}, & \text{if} & 0 < x < 1. \end{cases}$$

Thus $|x|^G \geq 1$.

- $\sqrt{x^{2_G}}^G = |x|^G$

- $|e^y|^G = e^{|y|}$

- $|x \odot y|^G = |x|^G \odot |y|^G$

- $|x \oplus y|^G \leq |x|^G \oplus |y|^G$

- $|x \oslash y|^G = |x|^G \oslash |y|^G$

- $|x \ominus y|^G \geq |x|^G \ominus |y|^G$

- $0_G \ominus 1_G \odot (x \ominus y) = y \ominus x$, $i.e.$ in short $\ominus (x \ominus y) = y \ominus x$.

Further $e^{-x} = \ominus e^x$ holds for all $x \in \mathbb{Z}^+$. Thus the set of all geometric integers turns out to the following:

$$\mathbb{Z}(G) = \{\ldots, e^{-3}, e^{-2}, e^{-1}, e^0, e^1, e^2, e^3, \ldots\} = \{\ldots, \ominus e^3, \ominus e^2, \ominus e, 1, e, e^2, e^3, \ldots\}.$$

3. MAIN RESULTS: GEOMETRIC REAL SEQUENCE

A function whose domain is the set $\mathbb{N}$ of natural numbers and range a set of geometric real numbers is called a geometric real sequence. We denote geometric real sequence as $S : \mathbb{N} \to \mathbb{R}(G)$.

Since, we will discuss about geometric real sequences only, we shall use the term *geometric sequence* to denote *geometric real sequence*.

A geometric sequence will be denoted by $\{S_n\}$ or $\{S_1, S_2, S_3, \ldots, S_n, \ldots\}$ where $n \in \mathbb{N}$. Numbers $S_1, S_2, S_3, \ldots$ will be called first term, second term, third term, of the sequence, respectively. As well as the ordinary real sequence, all the terms of geometric sequence will be treated as distinct terms although some terms are equal in different situations.

Example of geometric sequence:

1. $\{S_n\} = \{e^{(-1)^n}, n \in \mathbb{N}\}$.
2. $\{S_n\} = \{e^n, n \in \mathbb{N}\}$.
3. $\{S_n\} = \{e^{1+(-1)^n}, n \in \mathbb{N}\}$.

3.1 Bounds of Geometric Sequence:

It is obvious that the set $\mathbb{R}(G) = \mathbb{R}^+ \setminus \{0\}$ of geometric real numbers is bounded bellow. So, geometric sequences are always bounded bellow. A geometric sequence is said to be bounded if it is bounded above. That is, a sequence $\{S_n\}$ is bounded if there exists a real number K such that $S_n \leq K, \forall n \in \mathbb{N}$.

3.2 G-Convergence of Geometric Sequences:

Definition:3.1. A geometric sequence $\{S_n\}$ is said to be convergent to a real number $l > 0$ if for $\varepsilon > 1$, there exists a positive integer m depending on ε such that $|S_n \ominus l|^G < \varepsilon$, for all $n \geq m$. In ordinary sense $\frac{l}{\varepsilon} < S_n < l\varepsilon$ for all $n \geq m$.

In other word, terms of the sequence approach the value l by ratio as n becomes larger and larger. Symbolically, we write

$$\underset{x \to a}{_G \lim} S_n = l \quad \text{or} \quad S_n \overset{G}{\to} l$$

---

and we say that the sequence geometrically converges to l or G-convergent to l.

Since, $\frac{l}{\varepsilon} < S_n < l\varepsilon$, for all $n \geq m$, hence infinite number of terms lie to the left of $l\varepsilon$ and infinite number of terms lie to the right of $\frac{l}{\varepsilon}$.

Theorem 3.1. *G-convergent sequence are bounded.*

Proof. Let the geometric sequence $\{S_n\}$ converge to l. Then for given $\varepsilon > 1$, there exists a positive integer m such that $\frac{l}{\varepsilon} < S_n < l\varepsilon$, for all $n \geq m$. Now, since $\varepsilon > 1$ and $l > 0$, hence $\frac{l}{\varepsilon}$ and $l\varepsilon$ are finite numbers. Let

$$L = \min\{\frac{l}{\varepsilon}, S_1, S_2, .., S_{m-1}\}.$$
$$U = \max\{l\varepsilon, S_1, S_2, .., S_{m-1}\}.$$

Then, $L \leq S_n \leq U$, for all $n \geq m$. Hence the sequence is bounded.

Theorem 3.2. *Boundedness need not imply geometric convergence.*

Proof. Let us consider the sequence $\{S_n\}$, where $S_n = e^{(-1)^n}$, $n \in \mathbb{N}$ such that $S_n \overset{G}{\to} l$. Then for $\varepsilon = e, \exists m \in \mathbb{N}$, such that

$$\frac{l}{\varepsilon} < S_n < l\varepsilon \forall n \geq m$$

$$\Rightarrow \qquad \frac{l}{e} < e^{(-1)^n} < le$$

$$\Rightarrow \quad \frac{l}{e} < e^{(-1)^{2m}} < le \quad \text{and} \quad \frac{l}{e} < e^{(-1)^{2m-1}} < le$$

$$\Rightarrow \qquad \frac{l}{e} < e < le \quad \text{and} \quad \frac{l}{e} < e^{-1} < le$$

$$\Rightarrow \qquad \frac{l}{e^2} < 1 < l \quad \text{and} \quad l < 1 < le^2$$

$$\Rightarrow \qquad 1 < l \quad \text{and} \quad l < 1$$

which is impossible at the same time. Hence, the sequence is not convergent.

Also, it can be proved that a geometric sequence can not G-converse to more than one limits.

3.3 G-Limit of a Function:

According to Grossman and Katz [10], geometric limit of a positive valued function defined in a positive interval is same to the ordinary limit. We have defined G-limit of a function with the help of geometric arithmetic in as follows:

A function f, which is positive in a given positive interval, is said to tend to the limit $l > 0$ as x tends to $a \in \mathbb{R}$, if, corresponding to any arbitrarily chosen number $\varepsilon > 1$, however small(but greater than 1), there exists a positive number $\delta > 1$, such that

$$1 < |f(x) \ominus l|^G < \varepsilon$$

for all values of x for which $1 < |x \ominus a|^G < \delta$. We write

$$_G \lim_{x \to a} f(x) = l \quad \text{or} \quad f(x) \overset{G}{\to} l.$$

Here,

$$|x \ominus a|^G < \delta \qquad \Rightarrow \left|\frac{x}{a}\right|^G < \delta$$

$$\Rightarrow \frac{1}{\delta} < \frac{x}{a} < \delta$$

$$\Rightarrow \frac{a}{\delta} < x < a\delta.$$

Similarly, $|f(x) \ominus l|^G < \varepsilon \Rightarrow \frac{l}{\varepsilon} < f(x) < l\varepsilon$.

Thus, in ordinary sense, $f(x) \overset{G}{\to} l$ means that for any given positive real number $\varepsilon > 1$, no matter however closer to 1, $\exists$ a finite number $\delta > 1$ such that $f(x) \in]\frac{l}{\varepsilon}, l\varepsilon[$ for every $x \in]\frac{a}{\delta}, a\delta[$. It is to be noted that lengths of the open intervals $]\frac{a}{\delta}, a\delta[$ and $]\frac{l}{\varepsilon}, l\varepsilon[$ decreases as δ and ε respectively decreases to 1. Therefore, as ε

decreases to $1, f(x)$ becomes closer and closer to l, as well as x becomes closer and closer to a as δ decreases to 1. Hence, l is also the ordinary limit of $f(x)$. i.e. $f(x) \xrightarrow{G} l \Rightarrow f(x) \to l$. In other words, we can say that G-limit and ordinary limit are same for bipositive functions whose functional values as well as arguments are positive in the given interval. Only difference is that in G-calculus we approach the limit geometrically, but in ordinary calculus we approach the limit linearly.

A function f is said to tend to limit l as x tends to a from the left, if for each $\varepsilon > 1$ (however small), there exists $\delta > 1$ such that $| f(x) \ominus l |^G < \varepsilon$ when $a/\delta < x < a$. In symbols, we write
$$\underset{x \to a-}{{}_G \lim} f(x) = l \text{ or } f(a-1) = l.$$
Similarly, a function f is said to tend to limit l as x tends to a from the right, if for each $\varepsilon > 1$ (however small), there exists $\delta > 1$ such that $| f(x) \ominus l |^G < \varepsilon$ when $a < x < a\delta$. In symbols, we then write
$$\underset{x \to a+}{{}_G \lim} f(x) = l \text{ or } f(a+1) = l.$$
If $f(x)$ is negative valued in a given interval, it will be said to tend to a limit $l < 0$ if for $\varepsilon > 1, \exists \delta > 1$ such that $f(x) \in]l\varepsilon, \frac{l}{\varepsilon}[$ whenever $x \in]\frac{a}{\delta}, a\delta[$.

3.4 *G-Continuity of a Function*[16]:

A function f is said to be G-continuous at $x = a$ if

1. $f(a)$ i.e., the value of $f(x)$ at $x = a$, is a definite number,

2.   the G-limit of the function $f(x)$ as $x \xrightarrow{G} a$ exists and is equal to $f(a)$.

Alternatively, a function f is said to be G-continuous at $x = a$, if for arbitrarily chosen $\varepsilon > 1$, however small, there exists a number $\delta > 1$ such that
$$|f(x) \ominus f(a)|^G < \varepsilon$$
for all values of x for which, $|x \ominus a|^G < \delta$.

On comparing the above definitions of limits and continuity, we can conclude that a function f is G-continuous at $x = a$ if
$$\lim_{x \to a} \frac{f(x)}{f(a)} = 1.$$

3.5 G-Limit Point of a Geometric Sequence:

A real number ξ is said to be a G-limit point of a geometric sequence $\{S_n\}$ if for a given $\varepsilon > 1$ however small but greater than 1, $S_n \in]\frac{\xi}{\varepsilon}, \xi l[$ for an infinite number of values of $n \in \mathbb{N}$. Thus, ξ is a limit point of the sequence if every neighborhood of ξ contains an infinite number of members of the sequence.

It can be easily proved that ***Bolzano-Weierstrass Theorem*** is also valid for infinite set of geometric real numbers and geometric sequences.

Theorem 3.1 *(Cauchy's general principle of geometric convergence). A necessary and sufficient condition for the convergence of a geometric sequence $\{S_n\}$ is that, for $\varepsilon > 1$ there exists a positive integer m_0 such that*
$$|S_n \ominus S_m|^G < \varepsilon, \forall n, m \geq m_0.$$
That is,
$$|S_{m+1} \oplus S_{m+2} \oplus \cdots \oplus S_n|^G < \varepsilon, \forall n, m \geq m_0$$
$$\Rightarrow \qquad |S_{m+1} S_{m+2} \cdots S_n|^G < \varepsilon.$$

Equivalently, we can say that the sequence $\{S_n\}$ is convergent if and only if for $\varepsilon > 1$ there exists a positive integer m such that
$$|S_{n+p} \ominus S_n|^G < \varepsilon, \forall n \geq m, p \geq 1.$$

Theorem 3.2 *If $\{a_n\}$ and $\{b_n\}$ be two convergent sequences such that ${}_G \lim a_n = a$, ${}_G \lim b_n = b$, then for all $n \in \mathbb{N}$*

1. ${}_G \lim(a_n \oplus b_n) = a \oplus b,$

2. ${}_G \lim(a_n \ominus b_n) = a \ominus b,$

3. ${}_G \lim(a_n \odot b_n) = a \odot b,$

4. $\quad _G\lim(a_n \oslash b_n) = a \oslash b.$

Proof. (i) We have discussed that in case of bi-positive functions, geometric limit and ordinary limit are same. Since terms of a geometric sequence are always positive, hence $\quad _G\lim a_n = a \Rightarrow \lim a_n = a$ and $\quad _G\lim b_n = b \Rightarrow \lim b_n = b$. Now,

$$_G\lim(a_n \oplus b_n) \underset{G}{=} \lim(a_n b_n)$$
$$= \lim(a_n b_n)$$
$$= \lim(a_n)\lim(b_n) = ab.$$

(ii)
$$_G\lim(a_n \ominus b_n) \underset{G}{=} \lim(a_n / b_n)$$
$$= \lim(a_n / b_n)$$
$$= \lim(a_n)/\lim(b_n)$$
$$= a/b = a \ominus b.$$

(iii)
$$_G\lim(a_n \odot b_n) \underset{G}{=} \lim\big[(a_n)^{\ln(b_n)}\big]$$
$$= \lim\big[(a_n)^{\ln(b_n)}\big]$$
$$= \lim\big[e^{\ln(a_n).\ln(b_n)}\big]$$
$$= e^{\lim[\ln(a_n).\ln(b_n)]}$$
$$= e^{[\ln a.\ln b]}$$
$$= a^{\ln(b)} = a \odot b.$$

(iv) Proof is similar.

Theorem3.3 *If $\quad _G\lim a_n = l$, such that $l > 0$, then*
$$_G\lim_{n\to\infty}\left(\frac{a_1 \oplus a_2 \oplus \ldots \oplus a_n}{e^n}G\right) = l.$$

Proof. We know that, if a positive term series $\{a_n\}$ converges to positive number l, then $\lim_{n\to\infty}(a_1 a_2 \ldots a_n)^{\frac{1}{n}} = l$. Since, terms of a geometric sequence age always positive and $\quad _G\lim a_n = l \Rightarrow \lim a_n = l$, hence

$$_G\lim_{n\to\infty}\left(\frac{a_1 \oplus a_2 \oplus \ldots \oplus a_n}{e^n}G\right) \underset{G}{=} \lim_{n\to\infty}(a_1 a_2 \ldots a_n)^{1/\ln(e^n)}$$
$$\underset{G}{=} \lim_{n\to\infty}(a_1 a_2 \ldots a_n)^{1/n}$$
$$= \lim_{n\to\infty}(a_1 a_2 \ldots a_n)^{1/n}$$
$$= l.$$

5. ACKNOWLEDGMENT

6. CONCLUSION

Here we have discussed about some basic properties of geometric real sequences. In [13] we have introduced geometric difference sequence spaces which are based on geometric arithmetic. Here we have just introduce geometric real sequences, G-limit, G-continuity and their basic properties which will be helpful for the development of the Geometric Calculus.

REFERENCES

[1]   Bashirov, M. Riza, *On Complex multiplicative differentiation*, TWMS J. App. Eng. Math. 1(1)(2011), 75-85.
[2]   A. E. Bashirov, E. Misirli, Y. Tandoğdu, A. Özyapici, *On modeling with multiplicative differential equations*, Appl. Math. J. Chinese Univ., 26(4)(2011), 425-438.
[3]   A. E. Bashirov, E. M. Kurpinar, A. Özyapici, *Multiplicative Calculus and its applications*, J. Math. Anal. Appl., 337(2008), 36-48.
[4]   F. Başar,, *Summability Theory and Its Applications*, Bentham Science Publishers, e-books, Monographs, xi+405 sf., Istanbul-2012, ISBN:978-1-60805-420-6.
[5]   A. F. Çakmak, F. Başar, *On Classical sequence spaces and non-Newtonian calculus*, J. Inequal. Appl. 2012, Art. ID 932734, 12pp.

[6]   Richard G. Cooke, *Infinite Matrices and Sequence Spaces*, Macmillan and Co., London, 1950. U.

[7]   Kadak, *Determination of Köthe-Toeplitz duals over non-Newtonian Complex Field*, The Scientific World Journal, Volume 2014, Article ID 438924, 10 pages.

[8]   D. J. H. Garling, *The β- and γ-duality of sequence spaces*, Proc. Camb. Phil. Soc., 63(1967), 963-981.

[9]   M. Grossman, *Bigeometric Calculus: A System with a scale-Free Derivative*, Archimedes Foundation, Massachusetts, 1983.

[10]   M. Grossman, R. Katz, *Non-Newtonian Calculus*, Lee Press, Piegon Cove, Massachusetts, 1972.

[11]   J. Grossman, M. Grossman, Katz, *The First Systems of Weighted Differential and Integral Calculus*, University of Michigan.

[12]   Jane Grossman,*Meta-Calculus: Differential and Integral*, University of Michigan.

[13]   K. Boruah and B. Hazarika, *Application of Geometric Calculus in Numerical Analysis and Difference Sequence Spaces*, Journal of Mathematical Analysis and Applications, SCI, Elsevier Publication 449(2)(2017), 1265-1285.

[14]   K. Boruah and B. Hazarika, *On Some Generalized Geometric Difference Sequence Spaces*, Proyecciones Journal of Mathematics (accepted).

[15]   K. Boruah and B. Hazarika, *Some Basic Properties of $G-Calculus$ and Its Applications in Numerical Analysis*, Numerical Functional Analysis and Optimization (under review).

[16]   K. Boruah and B. Hazarika, *$G-Calculus$*, TWMS Journal of Applied and Engineering Mathematics (accepted), ESCI.

[17]   K. Boruah and B. Hazarika, *Solution of Bigeometric-Differential Equations by Numerical Methods*, Numerical Functional Analysis and Optimization (under review).

[18]   K. Boruah and B. Hazarika, *Bigeometric Integral Calculus*, TWMS Journal of Applied and Engineering Mathematics (accepted), ESCI.

[19]   H. Kizmaz, *On Certain Sequence Spaces*, Canad. Math. Bull., 24(2)(1981), 169-176.

[20]   G. Köthe, Toplitz, *Vector Spaces I*, Springer-Verlag, 1969.

[21]   G. Köthe, O. Toplitz, *Linear Raumemitunendlichenkoordinaten und Ring unendlichenMatrizen*, J. F. Reine u. angew Math., 171(1934), 193-226.

[22]   I.J. Maddox, *Infinite Matrices of Operators*, Lecture notes in Mathematics, 786, Springer-Verlag(1980).

[23]   MikailEt and RifatÇolak, *On Some Generalized Difference Sequence Spaces*, Soochow J. Math., 21(4), 377-386.

[24]   D. Stanley, *A multiplicative calculus, Primus* IX 4 (1999) 310-326.

[25]   S. Tekin, F. Başar, *Certain Sequence spaces over the non-Newtonian complex field*, Abstr. Appl. Anal., 2013. Article ID 739319, 11 pages.

[26]   CengizTürkmen and F. Başar, *Some Basic Results on the sets of Sequences with Geometric Calculus*, Commun. Fac. Fci. Univ. Ank. Series A1. Vol G1. No 2(2012) Pages 17-34.

[27]   A. Uzer, *Multiplicative type Complex Calculus as an alternative to the classical calculus*, Comput. Math. Appl., 60(2010), 2725-2737.

[28]   Uğur Kadak and Hakan Efe, *Matrix Transformation between Certain Sequence Spaces over the Non-Newtonian Complex Field*,Hindawi Publishing Corporation, The Scientific World Journal, Volume 2014, Article ID 705818, 12 pages.