# Creep and Rupture Time response in a transversely isotropic rotating disc

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Abstract: The steady states creep and rupture time response in a transversely isotropic FG rotating disc has been investigated. The disc under investigation is made of FGM containing non-linear distribution of silicon carbide particle (SiCp) in a matrix of pure aluminum along the radial distance. The stresses and strain rates in the FGM disc have been estimated for different values of anisotropic constant ( $\alpha$ ) varying between 0.5 to 1.5. It is observed that radial stress in the disc decreases a little by increasing ( $\alpha$ ) from 0.5 to 1.5. However, the tangential stress in the disc increases near the inner radius, but decreases towards the outer radius when increases from 0.5 to 1.5. The radial as well as tangential strain rates in the FGM disc are significantly reduce with the increase in extent of anisotropy from 0.5 to 1.5.

Keywords: Creep, Anisotropy, Variable thickness

# 1. INTRODUCTION

Rotating disc is a widely used component in engineering and structural applications like, flywheels, compressors and turbines rotors. A number of researchers have investigated the problem of creep deformations in a rotating disc. FGMs are the materials in which contents of reinforcement are tailored with respect to position coordinates. These are designed especially for high-temperature applications [1-3]. In short fiber reinforced composite, material flow during processes, thus resulting in nisotry in mechanical properties [4]. Bhatnagar *et al.* [5] observed creep behavior in an orthotropic rotating disc using Norton's power law. Jain *et al.* [6] studied a constant thickness composite disc by tailoring the anisotropic elastic constants along the radial direction.

Singh and Ray [7] investigated the effect of anisotropy on steady state creep in a rotating composite disc by using Norton's power law. It is concluded that the presence of anisotropy significantly affect the creep behavior. Alexandrova and Alexandrov [8] studied the influence of anisotropy on the stress distribution in rotating disc using Hill's quadratic yield criterion. The results obtained were compared with those obtained by FEM for disc made of anisotropic and isotropic materials.

Several investigators [3, 9-10] conclude that the creep strain rates in rotating disc having variable thickness are much lower than those observed in a uniform thickness disc. The literature consulted so far reveals that the study pertaining to effect of anisotropy on creep behavior of variable thickness rotating FGM disc is not available. The present study aims to investigate the effect of anisotropy on creep behavior of a rotating FGM disc having non-linear distribution of reinforcement. The thickness profile of the disc is assumed to decrease from the inner to outer radius. The results obtained are compared with a similar FGM disc having isotropic properties.

# 2. DISC PROFILE AND DISTRIBUTION OF REINFORCEMENT

Considering a Al-SiCp composite disc of inner and outer radii a (=31.75 mm) and b (=152.4 mm), respectively. The thickness h(r) of the disc is assumed to vary radially as given below [10],

$$h(r) = h_b + \frac{(h_a - h_b)}{(b - a)}(b - r)$$
(1)

Where  $h_a$  (= 43.22 mm) and  $h_b$  (=13.97 mm) are disc thickness at the inner and outer radii.

The distribution of  $SiC_p$  in the FGM disc is given by [11],

$$V(r) = V_{\max} - \frac{(r-a)^2}{(b-a)^2} (V_{\max} - V_{\min})$$
<sup>(2)</sup>

where  $V_{max}$  (= 25 vol. %) and  $V_{min}$  (= 10.78 vol. %) are respectively the SiC<sub>p</sub> content at the inner and outer radii respectively.

The density  $\rho(r)$  of the FGM disc at any radius r is estimated by using the rule of mixture as given below,

$$\rho(r) = \rho_m + \frac{(\rho_d - \rho_m)V(r)}{100} = A_\rho - B_\rho (r - a)^2$$
(3)

where,

$$A_{\rho} = \rho_m + (\rho_d - \rho_m) \frac{V_{\text{max}}}{100} \text{ and } B_{\rho} = \frac{(\rho_d - \rho_m)(V_{\text{max}} - V_{\text{min}})}{100(b-a)^2}$$

where  $\rho_m$  (= 2698.9 kg/m<sup>3</sup>) and  $\rho_d$  (= 3210 kg/m<sup>3</sup>) are respectively the densities of pure Al matrix and SiC<sub>p</sub> reinforcement [12-13].

#### 3. CREEP LAW

The steady state creep behavior of the composite disc is described by threshold stress ( $\sigma_0$ ) based law [14] given by,

$$\dot{\overline{\varepsilon}} = \left[ M(r) \{ \overline{\sigma} - \sigma_0(r) \} \right]^n \tag{4}$$

The value of true stress exponent (*n*) in Eq. (4) is kept equal to 5. The creep parameters M(r) and  $\sigma_0(r)$  are dependent on particle size (*P*), operating temperature (*T*) and SiC<sub>p</sub> content *V*(*r*). The following regression equations are developed to estimate the values of creep parameters,

$$M(r) = 0.0288 - \frac{0.0088}{P} - \frac{14.0267}{T} + \frac{0.0322}{V(r)}$$
(5)

$$\sigma_0(r) = -0.084 P - 0.023T + 1.185 V(r) + 22.207 \tag{6}$$

#### 4. ANALYSIS OF CREEP IN FGM DISC

The analysis carried out in this study is based on the following assumptions:

- 1. The disc is made of transversely isotropic material and elastic deformations in the disc are small and hence neglected as compared to creep deformations.
- 2. Steady state condition of stress and plane stress condition ( $\sigma_z = 0$ ) in the disc are assumed.

The effective stress ( $\overline{\sigma}$ ) in a transversely isotropic rotating disc under biaxial state of stress (*i.e.*  $\sigma_z = 0$ ) is given by Hill's yield criterion as,

$$\overline{\sigma} = \frac{1}{\sqrt{G+H}} \left[ (G+H)\sigma_r^2 + (H+F)\sigma_\theta^2 - 2H\sigma_r\sigma_\theta \right]^{1/2} = \frac{\left[ (2\alpha)\sigma_r^2 + (1+\alpha)\sigma_\theta^2 - 2\alpha\sigma_r\sigma_\theta \right]^{1/2}}{\sqrt{2\alpha}}$$
(7)

where,  $\alpha = G/F = H/F$  is the coefficient or extent of anisotropy.

The generalized constitutive equations for creep in a transversely isotropic composite under biaxial state of stress takes the following form when the reference frame is along the principal directions r,  $\theta$  and z [1],

$$\dot{\varepsilon}_{r} = \frac{\left[(G+H)\sigma_{r} - H\sigma_{\theta}\right]\dot{\overline{\varepsilon}}}{(G+H)\overline{\sigma}} = \frac{\left[2\sigma_{r} - \sigma_{\theta}\right]\dot{\overline{\varepsilon}}}{2\overline{\sigma}}$$

$$\dot{\varepsilon}_{\theta} = \frac{\left[(H+F)\sigma_{\theta} - H\sigma_{r}\right]\dot{\overline{\varepsilon}}}{(G+H)\overline{\sigma}} = \frac{\left[(1+\alpha)\sigma_{\theta} - \alpha\sigma_{r}\right]\dot{\overline{\varepsilon}}}{(2\alpha)\overline{\sigma}}$$

$$\dot{\varepsilon}_{z} = \frac{\left[-\alpha\sigma_{r} - \sigma_{\theta}\right]\dot{\overline{\varepsilon}}}{(2\alpha)\overline{\sigma}} - (\dot{\varepsilon}_{r} + \dot{\varepsilon}_{\theta})$$
(8)

where,  $\dot{\varepsilon}_r, \dot{\varepsilon}_{\theta}, \dot{\varepsilon}_z$  and  $\sigma_r, \sigma_{\theta}, \sigma_z$  are respectively the strain rates and the stresses in the disc along  $r, \theta$  and z directions.

Considering the force equilibrium equation for rotating disc with variable thickness may be written as [15],

$$\frac{d}{dr}[h(r)r\sigma_r] - h(r)\sigma_\theta + \rho(r)\omega^2 r^2 h(r) = 0$$
<sup>(9)</sup>

The disc is assumed to be fitted on a splined shaft where small axial movement is permitted. Therefore, the following free-free boundary conditions apply [2],

$$\sigma_r = 0 \text{ at } r = a \text{ and } \sigma_r = 0 \text{ at } r = b$$
 (10)

The equilibrium Eq. (9) is solved along with set of constitutive Eq. (8) under the imposed boundary conditions given in Eq. (10) to obtain the distribution of stresses and strain rates in the FGM disc.

## 5. **RESULTS AND DISCUSSIONS**

Based on analysis presented in the previous section, a computer code has been developed to compute stresses and strain rates in the rotating FGM disc. The thickness of the disc is linearly decreasing as given by Eq. (1). The SiC<sub>p</sub> content in the FGM disc decreases non-linearly from the inner to outer radius, as given by Eq. (2). In the present study, we have assumed that the FGM disc contains  $V_{max}(= 25 \text{ vol. }\%)$  and  $V_{min}(= 10.78 \text{ vol. }\%)$  of SiC<sub>p</sub> content at the inner and outer radii respectively. The effect of variation in anisotropic constant ( $\alpha$ ) is studied on the creep behavior of the FGM disc with linearly decreasing thickness. The results obtained are compared with a similar isotropic FGM disc ( $\alpha = 1$ ).

Before discussing the results obtained in the present study, it is necessary to check the validity of analysis and the computer code developed. For this purpose, the radial strain has been computed for a rotating steel disc, by using the current analytical scheme. The results obtained are compared with the experimental results reported in literature for steel disc (Wahl *et al.*, 1954). The operating conditions, disc dimensions and creep parameters of the steel disc, used for validation, are reported in Table 1. Figure 1 shows good agreement between the results obtained by the procedure outlined here and the experimental results reported for steel disc [Wahl *et al.* (1954)], which validates the analysis carried out and the computer code developed in this study.

#### 5.1 Distribution of stresses and strain rates

The effect of anisotropy on the creep behavior of a variable thickness FGM disc is shown in Figs. (1-4). The results for creep stresses and strain rates are calculated for FGM discs having different values of anisotropic coefficient ( $\alpha = 0.5, 0.75, 1.25, 1.5$ ). The results obtained for transversely isotropic FGM disc are compared with those obtained for a similar FGM disc but having isotropic properties ( $\alpha = 1$ ). The effect of varying  $\alpha$  on radial stress is shown in Fig. 2. The radial stress in all the discs increases from zero at the inner radius, reaches a maximum, before decreasing to zero again at the outer radius, under the imposed boundary conditions given in Eq. 10. The radial stress in the disc decreases with the increase in extent of anisotropy from 0.5 to 1.5. The radial stress in the FGM disc is the highest for  $\alpha = 0.5$  and lowest for  $\alpha = 1.5$ . The change observed in radial stress is significant in the middle region of the disc but negligible near the inner and outer radii.

The effect of varying  $\alpha$  on tangential stress is shown in Fig. 3. As compared to isotropic FGM disc ( $\alpha = 1$ ), the tangential stress increases near the inner radius but decreases towards the outer radius for FGM disc having  $\alpha < 1$ . The effect of  $\alpha$  on tangential stress is just opposite when it's value is greater than 1 (*i.e.*  $\alpha > 1$ ). The effect of varying  $\alpha$  on tangential stress is more near the inner radius than that observed near the outer radius. It is observed that the strain rates, radial as well as tangential, decrease significantly over the entire radius, with the increase in  $\alpha$  from 0.5 to 1.0 (Figs. 4-5). The change observed in strain rates is more near the inner radius than that observed to any other FGM disc. Thus, it is evident that by employing FGM disc with higher strength along the radial and axial directions compared to tangential direction, *i.e.*  $\alpha > 1$ , the effective stress and strain rates in the disc are significantly reduced.

# 5.2 Effect of anisotropic coefficient on rupture time

In this section, the effect of varying the value of anisotropic coefficient ( $\alpha$ ) has been calculated on the creep rupture time of the FGM discs. With the increase in ( $\alpha$ ) from 0.5 to 2, both the strain rates are observed to reduce drastically by about 100 times of magnitude (Figs. 4-5). On the basis of maximum strain rates (tangential), at the inner disc radius, the creep life (defined in terms of the creep rupture time) has been investigated for the FGM disc with different values of anisotropic coefficient ( $\alpha$ ). The rupture strain for Al-SiC composites is observed in the range of 2 to 4% [(Nieh (1984); Orlando and Filho (2004)]. Therefore, the rupture time has been estimated for different values of ( $\alpha$ ) by taking 2%, 3% and 4% strain as the rupture strain. The rupture time is observed to increase slightly (by few hours) as the value of exponent anisotropic coefficient ( $\alpha$ ) increases from 0.5 to 1.0 (Fig. 6). On increasing anisotropic coefficient ( $\alpha$ ) above 1.0, the rupture time is observed to increase significantly (by several years). For an example, the rupture time increases by around 100 times of magnitude when the value of anisotropic coefficient ( $\alpha$ ) increases from 1.0 to 2. By increasing ( $\alpha$ ), rupture time can be increased significantly leads to reduction for creep damage in the FGM disc.

# 6. CONCLUSIONS

The present study has led to the following conclusions:

- > In the presence of anisotropy, the radial as well as tangential stresses in FGM disc are affected significantly. In anisotropic FGM disc having  $\alpha < 1$ , the radial stress increases a little over the entire disc whereas the tangential stress increases slightly near the inner radius but decreases slightly towards the outer radius, when compared with isotropic FGM disc ( $\alpha = 1$ ). The effect of anisotropy on radial and tangential stresses in transversely isotropic FGM disc having  $\alpha > 1$  is opposite of that observed for FGM disc  $\alpha < 1$ .
- With the increase of anisotropic constant, the strain rates in the rotating FGM disc reduces significantly with the increase in extent of anisotropy from 0.5 to 1.5. Thus, the FGM disc with the extent of anisotropy ( $\alpha$ )>1 have lesser chances of distortion.
- The study also reveals that amongst several FGM discs with different values of ( $\alpha$ ), the FGM disc with higher value of  $\alpha$  exhibits the maximum creep life. By increasing  $\alpha$  from 1.0 to 2, the creep life of the disc could be increased from few hours to thousand hours.

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Fig. 1 Comparison of experimental and estimated radial strain in steel disc



Fig. 2: Effect of anisotropy on radial stress



Fig. 4: Effect of anisotropy on radial strain rate



Fig. 3: Effect of anisotropy on tangential stress







Parameters	Density of disc material $(\rho) = 7,823.18 \text{ kg/m}^3$ and Stress exponent $(n) = 5$
	Disc radii: $a = 31.75 mm$ , $b = 152.4 mm$ and Disc Thickness: $t = 25.4 mm$ ,
	Creep parameters: $M = 2.0408 \times 10^{-4}  s^{-1/5} / MPa$ , $\sigma_0 = 37.178  MPa$
Operating conditions	Disc $rpm = 15,000$ , Creep duration = 180 hrs, Temperature = 810.78 K

 Table 1: Parameters and operating conditions for steel disc