Unpreserved Inventory Models with Inflation Induced Demand under Progressive Credit Limit

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ABSTRACT
Inventory control has one of the most important tasks faced by modern manager. The investment in inventories for most form their assets committed to inventories. Further inventories one often the least stable and difficult to manage type of assist. Rapid change in level of business activities effect on inventories. In recent year, change in interest rate effect the inventories. Employ and customer theft has also led to increased cost of maintaining inventories. But carrying inventory is a costly thing as the storage cost, stock out cost, capacity related cost, item cost, ordering cost, deterioration and expiration of the product etc. must be taken in to account. Some policies, procedures and techniques employed in maintaining the optimum number of amount of each inventory item is the inventory management. While inventory is an asset, it is a non productive asset since it earns no interest but costs an organization in handling insurance, taxes, shrinkage and space. Careful inventory management can make a huge difference in the profitability of a firm.

Keywords: Inventory, modern manager, policies

I. INTRODUCTION

The effect of deterioration of physical goods can not be disregarded in many inventory systems. Deterioration is defined as decay, damage or spoilage. Food items, photographic films, drugs, chemicals, electronic components and radioactive substances are some examples of items in which sufficient deterioration may occur during the normal storage period of the units and consequently this loss must be taken into account while analyzing the inventory system.

An order level inventory model for deteriorating items with inflation induced demand and shortage has been developed. Since most decision makers think that inflation does not have significant influence on the inventory policy, the effects of inflation are not considered in some inventory models. However, from a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm' s limited capital funds. Thus, it is necessary to consider the effects of inflation on the inventory system, as many countries experience high annual inflation rate. The whole environment of business dealing has been assumed to be progressive credit period. Further, we use a numerical example to illustrate the model and sensitivity analysis on some parameters is made.

II. ASSUMPTION AND NOTATIONS

The following assumptions are used to develop a foresaid model:

- Shortages are allowed
- If the retailer pays by M. then the supplier does not charge to the retailer. If the retailer pays after M and before N (N>M), he can keep the difference in the unit sale price and unit purchase price in an interest bearing account at the rate of Ic/Unit/Year. During [M,N], the supplier charges the retailer an interest rate of IC_1/Unit/Year on unpaid balance. If the retailer pays after N, then supplier charges the retailer an interest rate of IC_2/Unit/Year ($IC_1>IC_2$) on unpaid balance.

The notations are as follows

- s = selling price /unit
- C_0 = the unit purchase cost with $C_0<s$
- M = the first offered credit period in selling the account without any charges,
- N = the second permissible credit period in settling the account with interest charge IC_2 on unpaid balance and N>M
- IC_1 = the interest charged per $ in stock per year by the supplier when retailer pays during [M,N]
- IC_2 = the interest charged per $ in stock per year by the supplier when retailer pays during [N,T]($IC_1>IC_2$)
- Ie = the interest earned / $ / year
- r = discount rate $r>\alpha$
- IE = the interest earned / time unit
- IC = the interest charged /time unit
- T = length of replenishment cycle.
- The demand rate is exponentially increasing and $D(t)=\lambda_0 e^{\alpha t}$ where $0\le\alpha\le1$ a

constant inflation rate is and λ_0 is initial demand rate.

- A_0 = ordering cost / order
- C_{10} = carrying cost / unit time
- C_{20} = shortage cost / unit time
- θt = variable deterioration rate
- A discounted cash flow (DCF) approach is used to consider the various costs at various times $(r > \alpha)$ is discount rate.
- L is the length of finite planning horizon.

III. MATHEMATICAL FORMULATION

Assuming continuous compounding of inflation, the ordering cost, unit cost of the item, out of pocket inventory carrying cost and storage cost at any time t are

$$A(t) = A_0 e^{\alpha t}$$

$$C(t) = C_0 e^{\alpha t}$$

$$C_1(t) = C_{10} e^{\alpha t}$$

and $$C_2(t) = C_{20} e^{\alpha t}$$

the planning horizon L has been discarded into n equal cycles of length T(i.e. T= L/n) let us consider the itch cycle i.e. $t_{i-1} \le t \le t_i$ where $t_0 = 0$, $t_n = L$, $t_i - t_{i-1} = T$ and $t_i = it$ (i = 1,2,.......n). At the beginning of itch cycle a batch of q_i units enters the inventory system from which s_i units are delivered towards backorders leaving a balance of I_{0i} units as the initial inventory level of itch cycle $q_i = I_{0i} + s_i$. there after as time passes, the inventory level gradually decreasing mainly due to demand and partially due to deterioration and reaches zero at time t_{i1} (Fig.1) further demands during the remaining period of the cycle from t_{i1} to t_i are backlogged and are of fulfilled by a new procurement.

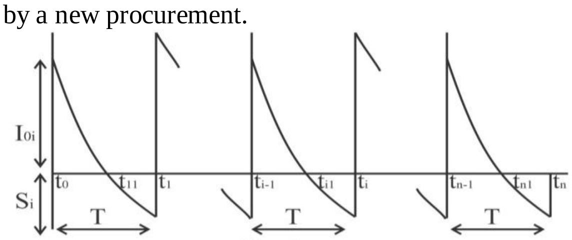


Fig. 1

Now

$$t_{i1} = t_i - kt = (i-k)\,L/n \quad i = (1,2,.......n)\ (0 \le k \le 1)$$

where kt is the fraction of the cycle having shortages. Let $I_i(t)$ be the inventory level of the ith cycle at time $t\ (t_{i-1} \le t \le t_i,\ i = 1,2,.......n)$. Now at the beginning of each cycle there will be cash out flow of ordering cost and purchase cost. Further since the inventory carrying cost is proportional to the value of the inventory, the out of pocket (Physical storage) inventory carrying cost per unit time at time t is $I(t)C_1(t)$. Similarly the shortage cost can also be obtained. The inventory level is represented by the following differential equations:

$$\frac{dI_i(t)}{dt} + \theta t I_i(t) = -\lambda(t) = -\lambda_0 e^{\alpha t} \qquad t_{i-1} \le t \le t_{i1} \quad i = 1,2,.....n \qquad ....(1.1)$$

$$\frac{dI_i(t)}{dt} = -\lambda(t) = -\lambda_0 e^{\alpha t} \qquad t_{i1} \le t \le t_i \quad i = 1,2,......n$$

The solution of the above differential equation along with the boundary condition $I(t_{i-1}) = I_{0i}$ and $I_i(t_{i1}) = 0$ is

$$I_i(t) = I_{0i}\, e^{\theta/2 \left(t_{i-1}^2 - t^2\right)} + \lambda_0 \left[(t_{i-1} - t) + \frac{\alpha}{2}(t_{i-1}^2 - t^2) + \frac{(\theta + \alpha^2)}{6}(t_{i-1}^3 - t^3) \right] e^{-\theta t^2/2} \quad ...\ (1.3a)$$

The solution of (2) is

$$I_i(t) = -\frac{\lambda_0}{\alpha}\left(e^{\alpha t} - e^{\alpha t_{i1}}\right) \qquad t_{i1} \le t \le t_i \qquad i = 1,2,.........n$$

Since $I_i(t_{i1}) = 0$ and $I_i(t_i) = -s_i$

Now put $I_i(t_{i1}) = 0$ in (3a) then

$$I_{0i} = -\lambda_0 \left[(t_{i-1} - t_{i1}) + \frac{\alpha}{2}(t_{i-1}^2 - t_{i1}^2) + \frac{(\theta + \alpha^2)}{6}(t_{i-1}^3 + t_{i1}^3) \right] e^{-\theta t_{i-1}^2} \quad i = 12,......n$$

... (1.4)

Now put $I_i(t_i) = -s_i$ in (3b) then

$$s_i = \frac{\lambda_0}{\alpha}\left(e^{\alpha t_i} - e^{\alpha t_{i1}}\right) \qquad i = 12,.........n$$

Now we put the value of I_{0i} in (3a) then

$$I_i(t) = \lambda_0 e^{-\theta t^2/2} \left[(t_{i1} - t) + \frac{\alpha}{2}(t_{i1}^2 - t^2) + \frac{(\theta + \alpha^2)}{6}(t_{i1}^3 - t^3) \right] \quad t_{i-1} \le t \le t_{i1} \quad i = 1,2,$$

...(1.6)

Further batch size qi for the ith cycles is :

$$q_i = I_{0i} + s_i$$

$$q_i = -\lambda_0 \left[(t_{i-1} - t_{i1}) + \frac{\alpha}{2}(t_{i-1}^2 - t_{i1}^2) + \frac{(\theta + \alpha^2)}{6}(t_{i-1}^3 - t_{i1}^3) \right] \times e^{-\theta/2\, t_{i-1}^2} + \frac{\lambda_0}{\alpha}\left(e^{\alpha t_i} - e^{\alpha t_{i1}}\right) \quad i = 1,2,.........n$$

(1) Present worth of ordering cost for the ith cycle A_i is –

$$A_i = A(t_{i-1}) e^{-rt_{i-1}} = A_0\, e^{(\alpha - r)t_{i-1}} \quad i = 1,2,.....................n$$

...(1.8)

(2) Present worth of the purchase cost for the ith cycle P_i is -

$$P_i = q_i C(t_{i-1}) e^{-rt_{i-1}} = q_i C_0\, e^{(\alpha - r)t_{i-1}} \quad i = 1,2,.....................n$$

...(1.9)

(3) Present worth of the inventory carrying cost for the ith cycle H_i is

$$H_i = C_1(t_{i-1})e^{-rt_{i-1}} \int_{t_{i-1}}^{t_{i1}} I_i(t)e^{-rt}dt$$

$$H_i = C_{10}\lambda_0 e^{(\alpha-r)t_{i-1}} \int_{t_{i-1}}^{t_{i1}} e^{-\theta t^2/2}\left[(t_{i1}-t)+\frac{\alpha}{2}(t_{i1}^2-t^2)+\frac{(\theta+\alpha^2)}{6}(t_{i1}^3-t^3)\right]e^{-rt}dt$$

…(1.10)

(4) Present worth of the shortage cost for the ith cycle is –

$$\pi_i = C_2(t_{i-1})e^{-rt_{i-1}} \int_{t_{i1}}^{t_i} I_i(t)e^{-rt}dt$$

$$= C_{20}e^{(\alpha-r)t_{i-1}} \frac{\lambda_0}{\alpha} \int_{t_{i1}}^{t_i} (e^{\alpha t}-e^{-\alpha t_{i1}})e^{-rt}dt$$

$$= \lambda_0 C_{20}\left[\frac{e^{(\alpha-r)t_i}-e^{(\alpha-r)t_{i1}}}{(\alpha-r)}+\frac{e^{\alpha t_i}}{r}(e^{-rt_i}-e^{-rt_{i1}})\right]\times e^{(\alpha-r)t_{i-1}} \quad i=1,2,\ldots\ldots\ldots n$$

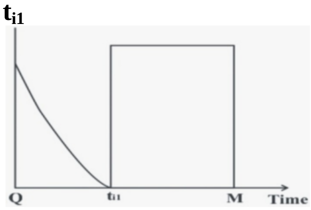
Therefore the present worth of the total variable cost for the ith cycle Pw_i is the sum of the ordering cost A_i purchase cost P_i, inventing carrying cost (H_i) and shortage cost (π_i) i.e.

$$Pw_i = A_i + P_i + H_i + \pi_i$$

The present worth of the total variable cost of the system during the entire time horizon L is given by –

$$PW_L(k,n) = \sum_{i=1}^{n} PW_i = \sum_{i=1}^{n}(A_i + P_i + H_i + \pi_i)$$

Case I $M \geq t_{i1}$



Inventory level Fig. 2 $M \geq t_{i1}$

In the first case, retailer does not pay any interest to the supplier. Here retailer sells I_s units during $(0, t_{i1})$ time internal and paying for CI_s units in full to the supplier at time $M \geq t_{i1}$ so interest charges are zero i.e.

$$IC_1 = 0$$

Retailers deposits the revenue in an interest bearing account at the rate of Ie/$/year.

Therefore, interest earned IE_1, per year is

$$IE_1 = \frac{sI_e}{T_2}\left[\int_0^{t_{i1}} D(t)t\,dt + (M-t_{i1})\int_0^{t_{i1}} D(t)\,dt\right]$$

Total cost per unit time of an inventory system is –

$$T[PW_L(k,n)] = \sum_{i=1}^{n} PWi + IC_1 - IE_1$$

$$= \sum_{i=1}^{n}(A_i + P_i + H_i + \pi_i) + IC_1 - IE_1$$

Case II – $M < t_{i1} < N$

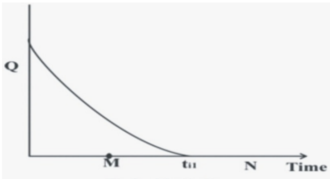


Fig 3 M<ti1<N

In the second case, supplier charges interest at the rate IC_1 on unpaid balance –

Interest earned, IE_2 during [0,M] is

$$IE_2 = sIe\int_0^M D(t)t\,dt$$

…. (1.11)

Retailer pay Is units purchased at time t = 0 at the rate of C/$/unit to the supplier during [0,M]. The retailer sells D(M).M units at selling price s/unit.

So, he has generated revenue of s D(M).M + IE_2.

Then two sub cases may be arises.

Sub Case 2.1 –

Let SD (M).M + $IE_2 \geq$ CIs retailer has enough money to settle, his account for all is units procured at time t =0 then interest charge will be

$$IC_{2.1} = 0$$

And interest earned (1.12)

$$IE_{2.1} = \frac{IE_2}{T_2}$$

So the total cost $T_{2.1}[PW_L(k,n)]$ per unit time of inventory system is …(1.13)

$$T_{2.1}[PW_L(k,n)] = \sum_{L=1}^{n}(A_i + P_i + H_i + \pi_i) + IC_{2.1} - IE_{2.1}$$

Sub Case 2.2 –

Let SD (M).M + IE_2 < CIs here retailer will have to pay interest on unpaid balance U_1= CIs – (sD(M).M + IE_2) at the rate of IC_1 at time M to the supplier. Then interest paid per unit time us given by –

$$IC_{2.2} = \frac{U_1^2 IC_1}{I_s}\int_M^{t_i} I_i(t)\,dt$$

and interest earned

$$IE_{2.2} = \frac{IE_2}{T_2}$$

... (1.14)

So the total cost $T_{2.2}[PW_L(k,n)]$ per unit time of inventory system is

$$T_{2.2}[PW_L(k,n)] = \sum^{n}(A_i + P_i + H_i + \pi_i) + IC_{2.2} - IE_{2.2}$$ ---(1.23)

… (1.15)

Case III $t_{i1} > N$

Inventory level

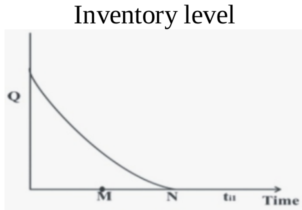


Fig 4 ti1≥N

1.16)

In the final case, retailer pays interest at the rate of IC_2 to the supplier. Based on the total purchased cost

CIs, the total money sD(M).M + IE_2 in account at M and total money SD(N).N+IE_2 at N, there are three sub cases may arise.

This case is same as sub case 2.1 have 3.1 designate decision variables and objective function.

Sub case 3.2 – Let sD(M).M+IE_2 < CI_s and

$$sD(N-M)(N-M)+sI_e\int_M^N D(t)dt \geq CI_s(sD(M).M+IE_2)$$

Here, retailer does not have enough money to pay off total purchase dost at N. He will not pay money sD(M).M+IE_2 at M and $sD(N-M)(N-M)+sI_e\int_M^N D(t)dt$ at N. That' s why we has to pay interest on unpaid balance $U_1 = CI_s - (sD(M).M + IE_2)$ with IC_1 interest rate during (M, N) and $U_2 = U_1 - sD(N-M)(N-M) + sI_e\int_M^N D(t)dt$ with interest rate IC_2 during (N,t_{i1}).

Therefore, total interest charged on retailer $IC_{3.3}$ per unit time is –

$$IC_{3.3} = \frac{U_1 IC_1 (N-M)}{T_2} + \frac{U_2^2 IC_1}{PI_s}\int_N^{t_i} I_i(t)\,dt \quad \text{-----(1.24)}$$

And interest earned per unit time is

$$IE_{3.3} = \frac{IE_2}{T_2} \quad \text{-----(1.25)}$$

So total cost $T_{3.3}[PW_L(k,n)]$ per unit time of inventory system is

$$T_{3.3}[PW_L(k,n)] = \sum_{i=1}^{n}(A_i + P_i + H_i + \pi_i) + IC_{3.3} - IE_{3.3}$$

-----(1.26)

IV. NUMERICAL ILLUSTRATIONS

The preceding theory can be illustrated by the following numerical example where the parameters are given as follows:

Demand parameters, λ_0 = 420, α = 0.02
Selling price, s = 23
Buyer's purchased cost, C_b = 27
Buyer's percentage holding cost per year per dollar, C_{bh} = 0.03
Buyer's ordering cost per order, C_{bs} = 410
Buyer's shortage cost, S_b = 10
Vendor's unit cost, C_v = 18
Vendor's percentage holding cost per year per dollar, C_{vh} = 0.2
Vendor's setup cost per order, C_{vs} = 550
Vendor's production rate per year, K = 2
Deterioration rate, θ = 0.02
First delay period, M= 0.06
Second delay period, N= 0.2
The interest earned, Ie = 0.04
The interest charged, Ic1 = 0.20
The interest charged, Ic2 = 0.16 (Ic1 > Ic2)

Table 1.1: Retailer does not pay any interest to the Supplier

N	T_2	t_1	TC
1	0.754546	0.116863	1284.561
2	0.786092	0.118116	1289.345
3	0.806756	0.118522	1292.717
4	0.836491	0.118723	1331.382
5	0.865356	0.118844	1348.485

Table 1.2: Supplier charges interest but Retailer has enough money to settle his account

N	T_2	t_1	TC
1	0.764371	0.124818	1615.17
2	0.787892	0.125711	1531.16
3	0.815642	0.125993	1518.25
4	0.826030	0.126018	1487.83
5	0.857485	0.126614	1411.19

Table 1.3: Retailer will have to pay interest on unpaid balance at the rate of interest Ic_1; Retailer does not have enough money to pay off at M

N	T_2	t_1	TC
1	0.764371	0.124818	1513.88
2	0.787892	0.125711	1478.79
3	0.815642	0.125993	1445.51
4	0.826030	0.126018	1431.04
5	0.857485	0.126614	1401.45

Table 1.4: Retailer pays interest at the rate of Ic_2 to the Supplier; Retailer does not have enough money to pay off at N

N	T_2	t_1	TC
1	1.218450	0.896290	1553.54
2	1.297826	0.758734	1497.97
3	1.433762	0.689980	1411.22

4	1.563398	0.549718	1351.21
5	1.784529	0.427600	1314.65

V. CONCLUSION

The goal of this work is to develop an inventory model with shortages, in which units are, deteriorate with time dependent rates and the demand rate is increasing exponentially due to inflation under trade credit. Most products experience a period of rapid demand increase during the introduction phase of product life cycle, level off in demand after reaching their maturity period, and will enter a period of sales decline due to new competing products or changes in consumer preference. An inventory control is an intriguing yet practicable issue of decision science when inflation induced demand is involved. The effect of inflation on an inventory system has been taken into consideration. Cost minimization technique is used to get the expressions for total cost and other parameters. A numerical assessment of the theoretical model has been done to illustrate the theory. The whole combination of the setup is very unique and more practical.

VI. REFERENCES

1) Begum, R., Sahoo, R. R., & Sahu, S. K. (2012). A replenishment policy for items with price dependent demand, time-proportional deterioration and no shortages. International Journal of Systems Science, 43, 903– 910.
2) Benkherouf, L. (2014), " On an inventory model for deteriorating items with stock dependent and time varying demand rates" , Computers & Operations Research, 31, 223-240.
3) Bhunia, A. K., Pal, P., Chattopadhyay, S., & Medya, B. K. (2011). An inventory model of two – warehouse system with variable demand dependent on instantaneous displayed stock and marketing decision via hybrid RCGA. International Journal of Industrial Engineering Computation, 23, 51– 68.
4) Chen, L.H. and Kang, F.S. (2012), " Integrated vendor-buyer cooperative inventory models with permissible delay in payments" , E.J.O.R, 183 (2), 658-673.
5) Deane, J., & Agarwal, A. (2012). Scheduling online advertisements to maximize revenue under variable display frequency. Omega, 40, 562– 570.
6) Dutta, T.K. and Pal, A.K. (2010), " Order level inventory system with power demand pattern for items with variable rate of deterioration" , Indian Journal of Pure and Applied Mathematics (I.J.P.A.M.), 19(11), 1043-1053.
7) Guria, A., Das, B., Mondal, S., & Maiti, M. (2013). Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. Applied Mathematical Modelling, 37, 240– 257
8) Huang, Y.F. (2013), " The optimal cycle time for EPQ inventory model under permissible delay in payments" , International Journal of Production Economics, 84, 3, 307-318.
9) Huang, Y.F. (2015), The optimal inventory policies under permissible delay in payments depending on the ordering quantity, *International Journal of Production Economics*, *95, 2, 203-213*.
10) Musa, A., & Sani, B. (2012). Inventory ordering policies of delayed deteriorating items under permissible delay in payments. International Journal of Production Economics, 136, 75– 83.
11) Shah, N. H., Gor, A. S., & Wee, H. M. (2010). An integrated approach for optimal unit price and credit period for deteriorating inventory system when the buyer' s demand is price sensitive. American Journal of Mathematical and Management Sciences, 30, 317– 330.
12) Shah, N. H., Soni, H. N., & Patel, K. A. (2012). Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. Omega, 41, 421– 430.
13) Widyadana, G. A., & Wee, H. M. (2012). An economic production quantity model for deteriorating items with multiple production setups and rework. International Journal of Production Economics, 138, 62– 67.
14) Yang, H. L., & Chang, C. T. (2013). A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. Applied Mathematical Modelling, 37, 2717– 2726.