Some Separation Axioms in Soft Biminimal Spaces

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Abstract

The aim of the present paper is to introduce and investigate some new separation axioms soft T_i-space (i=0,1,2) by using only pair of distinct soft points and we study some of their properties.
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1 Introduction

C. Boonpok [1] introduced the concept of biminimal structure space and studied $m_X^1 m_X^2$-open sets and $m_X^1 m_X^2$-closed sets in biminimal structure spaces. M. Tunapan [16] introduced some separation axiom in biminimal structure spaces such as CT_0-space, CT_1-space, C-Hausdorff space, C-regular space and C-normal space in a biminimal structure space. Russian researcher Molodtsov [9], initiated the concept of soft sets as a new mathematical tool to deal with problems in engineering physics, computer science, economics, social sciences and medical sciences. Later, he applied this theory to several directions [10] [11]. In this paper, we introduce and study some separation axioms in soft biminimal spaces.

2 Preliminaries

In this section, we recall the basic definitions and results of soft set. Throughout this paper U denotes initial universe, E denotes the set of all possible parameters, $P(U)$ is the power set of U, and A is the non-empty subset of E. That is $A \subseteq E$

Definition 2.1 [2] *A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here, f_A is called approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary, some of them may be empty, some may have non empty intersection.*

Note that the set of all soft sets over U will be denoted by S(U).

Definition 2.2 [2] *Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called an empty set,denoted by $F_\emptyset$. $f_A(x) = \emptyset$ means that there is no element in U related* *to the parameter $x \in E$. Therefore we do not display such elements in the soft set as it is meaningless to consider such parameters*

Definition 2.3 [2] *Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in E$,then F_A is called A-universal soft set,denoted by $F_{\tilde{A}}$. If $A = E$, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{E}}$.*

Definition 2.4 [2] *Let $F_A, F_B \in S(U)$. Then F_B is a soft subset of F_A (or F_A is a soft superset of F_B), denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.*

Definition 2.5 [2] *Let $F_A, F_B \in S(U)$. Then F_B and F_A are soft equal, denoted by $F_B = F_A$, $f_A(x) \subseteq f_B(x)$ for all $x \in E$.*

[2]

Definition 2.6 [2] *Let F_A,$F_B \in$ S(U). Then the soft union of F_A and F_B, denoted by $F_A \tilde{\cup} F_B$, is defined by the approximate functions $f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x)$.*

Definition 2.7 [2] *Let F_A,$F_B \in$ S(U). Then the soft intersection of F_A and F_B, denoted by $F_A \tilde{\cap} F_B$, is defined by the approximate functions $f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$.$F_A$ and F_B are soft disjoint is said to be if $F_A \cap F_B = F_\emptyset$*

Definition 2.8 [2] *Let F_A,$F_B \in$ S(U). Then the soft difference of F_A and F_B, denoted by $F_A \backslash F_B$, is defined by the approximate functions $f_{(A\backslash B)}(x) = f_A(x) \backslash f_B(x)$.*

Definition 2.9 [2] *Let $F_A \in S(U)$. Then the soft complement of F_A, denoted by $F_A^{\tilde{c}}$ is defined by the approximate function $f_{A^{\tilde{c}}}(x) = f_A^c(x)$, where $f_A^c(x)$ is complement of the set $f_A(x)$.that is, $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$. It is easy to see that $\left(F_A^{\tilde{c}}\right)^{\tilde{c}} = F_A$ and $F_\emptyset^{\tilde{c}} = F_{\tilde{E}}$*

Definition 2.10 [3] *Let $F_A \in S(U)$. Power soft set of F_A is defined by $\tilde{P}(F_A) = \left\{F_{A_i} \tilde{\subseteq} F_A : i \in I\right\}$ and its cardinality is defined by*

$$|\tilde{P}(F_A)| = 2^{\sum_{x\in E}|f_A(x)|}$$

where $|f_A(x)|$ is cardinality of $f_A(x)$.

Definition 2.11 [17] *Let (F, E) be a soft set over X and Y be a non-empty subset of X. Then the soft set (F, E) over Y denoted by $(^Y F, E)$, is defined as follows $^Y F(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$. In other words $(^Y F, E) = Y \cap (F, E)$*

Definition 2.12 [13] *The soft set $F_A \in S(U)$ is called a soft point in U_A, denoted by e_F, if for the element $e \in A$, $F(e) \neq \emptyset$ and $F(e^{'}) = \emptyset$ for all $e^{'} \in A - \{e\}$*

Definition 2.13 [6] *Let X be an initial universe set and E be the set of parameters. Let $(X, \tilde{m}_1, E)$ and $(X, \tilde{m}_2, E)$ be the two different soft minimals over X. Then $(X, \tilde{m}_1, \tilde{m}_2, E)$ or $(F_A, \tilde{m}_1, \tilde{m}_2)$ is called a soft biminimal spaces.*

Example 2.14 *Let* $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ *and*
$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. *Then*
$F_{A_1} = \{(x_1, \{u_1\})\}$, $F_{A_2} = \{(x_1, \{u_2\})\}$, $F_{A_3} = \{(x_1, \{u_1, u_2\})\}$,
$F_{A_4} = \{(x_2, \{u_1\})\}$, $F_{A_5} = \{(x_2, \{u_2\})\}$, $F_{A_6} = \{(x_2, \{u_1, u_2\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$, $F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$, $F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$, $F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$,
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$, $F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{15}} = F_A$, $F_{A_{16}} = F_\emptyset$ *are all soft subsets of* F_A

soft minimal $\tilde{m} = \{F_\emptyset, F_{A_2}, F_{A_5}, F_{A_7}, F_{A_{11}}, F_A\}$

Definition 2.15 [6] *Let* $(F_A, \tilde{m})$ *be a soft minimal space and* F_Y *be a soft subset of* F_A. *Define soft minimal set* $\tilde{m}_{F_Y}$ *on* F_Y *as follows:* $\tilde{m}_{F_Y} = \{F_B \tilde{\cap} F_Y | F_B \in \tilde{m}\}$. *Then* $(F_Y, \tilde{m}_{F_Y})$ *is called a soft minimal subspace of* $(F_A, \tilde{m})$.

3 Separation Axioms in Soft Biminimal Spaces

Definition 3.1 *A soft biminimal space* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is called a pairwise soft* T_0-*space if for each pair of distinct soft points* $(x, u), (y, u)$ *of* F_A, *there exist soft* $\tilde{m}_1$-*open set* U_B *and a soft* $\tilde{m}_2$-*open set* V_B *such that either* $(x, u) \notin V_B$ *or* $(y, u) \notin U_B$.

Definition 3.2 *A soft biminimal space* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is called a pairwise soft* T_1-*space if for each pair of distinct soft points* $(x, u), (y, u)$ *of* F_A, *there exist soft* $\tilde{m}_1$-*open set* U_B *and a soft* $\tilde{m}_2$-*open set* V_B *such that* $(x, u) \in U_B$, $(y, u) \notin U_B$ *and* $(x, u) \notin V_B$, $(y, u) \in V_B$.

Definition 3.3 *A soft biminimal space* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is called a pairwise soft* T_2-*space (pairwise soft Hausdroff space) if for each pair of distinct soft points* $(x, u), (y, u)$ *of* F_A, *there exist soft* $\tilde{m}_1$-*open set* U_B *and a soft* $\tilde{m}_2$-*open set* V_B *such that* $(x, u) \in U_B$, $(y, u) \in V_B$ *and* $U_B \cap V_B = F_\emptyset$.

Example 3.4 *Let us consider the soft subsets of* F_A *that are given in Example 2.15. Let* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *be a soft biminimal space where* $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$,
$\tilde{m}_1 = \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_5}, F_{A_7}, F_A\}$ *and* $\tilde{m}_2 = \{F_\emptyset, F_{A_2}, F_{A_6}, F_{A_{11}}, F_{A_{13}}, F_A\}$. *Then* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is a pairwise soft* T_0-*space, pairwise soft* T_1-*space and pairwise soft* T_2-*space.*

Proposition 3.5 *Let* $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ *be a soft biminimal subspace of* $(F_A, \tilde{m}_1, \tilde{m}_2)$. *If* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is a pairwise soft* T_0-*space then* $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ *is also a pairwise soft* T_0-*space.*

Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_0-space and $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ a soft subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. If $(x, u), (y, u) \in F_Y$ such that $(x, u) \neq (y, u)$, there exist soft $\tilde{m}_1$-open set U_B and soft $\tilde{m}_2$-open set V_B such that either $(x, u) \notin V_B$ or $(y, u) \notin U_B$. We defined an soft open set $M_B = U_B \cap F_Y \in \tilde{m}_{1_{F_Y}}$ and $N_B = V_B \cap F_Y \in \tilde{m}_{2_{F_Y}}$, hence $(x, u) \notin N_B$ or $(y, u) \notin M_B$. Thus $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is pairwise soft T_0-space. □

Proposition 3.6 *Let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft biminimal subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. If $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space then $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is also a pairwise soft T_1-space.*

Proof: The proof is similar to the proof of Proposition 3.5. □

Proposition 3.7 *Let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft biminimal subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. If $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_2-space then $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is also a pairwise soft T_2-space.*

Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_2-space and $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ a soft subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. If $(x, u), (y, u) \in F_Y$ such that $(x, u) \neq (y, u)$, there exist soft $\tilde{m}_1$-open set U_B and soft $\tilde{m}_2$-open set V_B such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $U_B \cap V_B = F_\emptyset$. We defined an soft open set $M_B = U_B \cap F_Y \in \tilde{m}_{1_{F_Y}}$ and $N_B = V_B \cap F_Y \in \tilde{m}_{2_{F_Y}}$, hence $(x, u) \in M_B$, $(y, u) \in N_B$ and $M_B \cap N_B = F_\emptyset$. Thus $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is pairwise soft T_2-space. □

Theorem 3.8 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space over X and $(x, u), (y, u) \in F_A$ be such that $(x, u) \neq (y, u)$. If there exist soft $\tilde{m}_1$-open set U_B such that $(x, u) \in U_B$ and $(y, u) \in (U_B)^c$ or a soft $\tilde{m}_2$-open set V_B such that $(y, u) \in V_B$ and $(x, u) \in (V_B)^c$. Then $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft T_0-space.*

Proof: Let $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, there is an soft open set $U_B \in \tilde{m}_1$ such that $(x, u) \in U_B$ and $(y, u) \in (U_B)^c$ or a soft $\tilde{m}_2$-open set V_B such that $(y, u) \in V_B$ and $(x, u) \in (V_B)^c$. If $(y, u) \in (U_B)^c$ then $(y, u) \notin (U_B^c)^c = U_B$. Similarly if $(x, u) \in (V_B)^c$, then $(x, u) \notin (V_B^c)^c = V_B$. Hence, there is an soft open set $U_B \in \tilde{m}_1$ and $V_B \in \tilde{m}_2$ such that $(x, u) \in U_B$ and $(y, u) \notin U_B$ or $(y, u) \in V_B$ and $(x, u) \notin V_B$. Hence $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft T_0-space. □

Theorem 3.9 *A soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft T_1 if and only if $(F_A, \tilde{m}_1)$ and $(F_A, \tilde{m}_2)$ are soft T_1.*

Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft T_1. Let $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. Since the space is pairwise soft T_1, there is a soft $\tilde{m}_1$-open set U_B such that $(x, u) \in U_B$, $(y, u) \notin U_B$. Thus $(F_A, \tilde{m}_1)$ is T_1. Similarly, $(F_A, \tilde{m}_2)$ is T_1. Conversely, $(F_A, \tilde{m}_1)$ and $(F_A, \tilde{m}_2)$ be T_1. Let $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. Since $(F_A, \tilde{m}_1)$ is T_1, there is a soft $\tilde{m}_1$-open set U_B such that $(x, u) \in U_B$, $(y, u) \notin U_B$. Since $(F_A, \tilde{m}_2)$ is T_1, there is a soft $\tilde{m}_2$-open set V_B such that $(x, u) \notin V_B$, $(y, u) \in V_B$. Hence, there is a soft $\tilde{m}_1$-open set U_B and a soft $\tilde{m}_2$-open set V_B such that $(x, u) \in U_B$, $(y, u) \notin U_B$ and $(x, u) \notin V_B$, $(y, u) \in V_B$. So that $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft T_1. □

Theorem 3.10 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space over X and $(x, u), (y, u) \in F_A$ be such that $(x, u) \neq (y, u)$. If there exist soft $\tilde{m}_1$-open set U_B such that $(x, u) \in U_B$ and $(y, u) \in (U_B)^c$ and a soft $\tilde{m}_2$-open set V_B such that $(y, u) \in V_B$ and $(x, u) \in (V_B)^c$, then $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft T_1-space.*

Proof: The proof is similar to the proof of Proposition 3.8. □

Theorem 3.11 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space over X. If $\{x, u\}$ is a soft closed set in $\tilde{m}_2$ for each $(x, u) \in F_A$ and $\{y, u\}$ is a soft closed set in $\tilde{m}_1$ for each $(y, u) \in F_A$, then $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space.*

Proof: Suppose that for each $(x, u) \in F_A$, $\{x, u\}$ is a soft closed set in $\tilde{m}_2$ then $\{x, u\}^c$ is soft open set in $\tilde{m}_2$. Let $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$. For each $(x, u) \in F_A$, $\{x, u\}^c$ is a soft open set in $\tilde{m}_2$ such that $(y, u) \in \{x, u\}^c$ and $(x, u) \notin \{x, u\}^c$. Similarly for each $(y, u) \in F_A$, $\{y, u\}$ is a soft closed set in $\tilde{m}_1$ then $\{y, u\}^c$ is soft open set in $\tilde{m}_1$ such that $(x, u) \in \{y, u\}^c$ and $(y, u) \notin \{y, u\}^c$. Thus $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space. □

Proposition 3.12 *Every pairwise soft T_1-space is pairwise soft T_0-space.*

Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, there is a soft open set $U_B \in \tilde{m}_1$ and $V_B \in \tilde{m}_2$ such that $(x, u) \in U_B$, $(y, u) \notin U_B$ and $(y, u) \in V_B$, $(x, u) \notin V_B$. Obviously then we have $(x, u) \in U_B$ and $(y, u) \notin U_B$ or $(y, u) \in V_B$ and $(x, u) \notin V_B$. Hence $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_0-space. □

Remark 3.13 *The converse of the above proposition 3.12. is not true in general. In other words,*

$$\textit{pairwise soft } T_0 \nRightarrow \textit{pairwise soft } T_1$$

Example 3.14 *Let us consider the soft subsets of F_A that are given in Example 2.15. Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space where $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$, $\tilde{m}_1 = \{F_\emptyset, F_{A_1}, F_{A_8}, F_{A_{12}}, F_A\}$ and $\tilde{m}_2 = \{F_\emptyset, F_{A_2}, F_{A_7}, F_{A_{11}}, F_A\}$. Then $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_0-space but not pairwise soft T_1-space.*

Proposition 3.15 *Every pairwise soft T_2-space is pairwise soft T_1-space.*

Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_2-space and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, there is a soft open set $U_B \in \tilde{m}_1$ and $V_B \in \tilde{m}_2$ such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $U_B \cap V_B = F_\emptyset$. Since $U_B \cap V_B = F_\emptyset$; $(x, u) \notin V_B$ and $(y, u) \notin U_B$. Thus $U_B \in \tilde{m}_1$ and $V_B \in \tilde{m}_2$ such that $(x, u) \in U_B$, $(y, u) \notin U_B$ and $(y, u) \in V_B$, $(x, u) \notin V_B$. Hence $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space. □

Remark 3.16 *The converse of the above proposition 3.15. is not true in general. In other words,*

$$\textit{pairwise } T_1 \nRightarrow \textit{pairwise } T_2.$$

Example 3.17 *Let us consider the soft subsets of F_A that are given in Example 2.15. Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space where $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$, $\tilde{m}_1 = \{F_\emptyset, F_{A_6}, F_{A_7}, F_{A_8}, F_A\}$ and $\tilde{m}_2 = \{F_\emptyset, F_{A_2}, F_{A_{10}}, F_{A_{11}}, F_A\}$. Then $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space but not pairwise soft T_2-space.*

Theorem 3.18 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space. If $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_1-space and for any $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, then there exist soft open set $U_B \in \tilde{m}_1$ and $V_B \in \tilde{m}_2$ such that $(x, u) \in U_B$, $(y, u) \notin U_B$ and $(y, u) \in V_B$, $(x, u) \notin V_B$ and $U_B \cup V_B = F_A$.*

Proof: Since $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft T_2-space and $(x, u), (y, u) \in F_A$ such that $(x, u) \neq (y, u)$, there exist soft open set $F_B \in \tilde{m}_1$, $G_B \in \tilde{m}_2$ such that $(x, u) \in F_B$ and $(y, u) \in G_B$ and $F_B \cap G_B = F_\emptyset$. Clearly, $F_B \tilde{\subseteq} (G_B)^c$ and $G_B \tilde{\subseteq} (F_B)^c$. Hence $(x, u) \in (G_B)^c$. Put $(G_B)^c = U_B$. This gives $(x, u) \in U_B$ and $(y, u) \notin U_B$. Also $(y, u) \in (F_B)^c$. Put $(F_B)^c = V_B$. This gives $(x, u) \notin V_B$ and $(y, u) \in V_B$. Therefore $(x, u) \in U_B$ and $(y, u) \in V_B$. Hence, $U_B \cup V_B = (G_B)^c \cup (F_B)^c = F_A$. □

Lemma 3.19 *Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ be a soft biminimal space and let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft closed subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. If U_B is soft open subset with respect to $\tilde{m}_i$, $i = 1, 2$ in $(F_A, \tilde{m}_1, \tilde{m}_2)$, then $U_B \cap F_Y$ is also a soft open subset with respect to $\tilde{m}_{iF_Y}$, $i = 1, 2$ in $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$.*

Proof: Let U_B be a soft open subset with respect to $\tilde{m}_i$, $i = 1, 2$ in $(F_A, \tilde{m}_1, \tilde{m}_2)$. Since $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft closed subset of $(F_A, \tilde{m}_1, \tilde{m}_2)$, then $(U_B)^c \cap F_Y$ is a soft closed subset with respect to $\tilde{m}_{iF_Y}$, $i = 1, 2$ in $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$. But $(U_B)^c \cap F_Y = F_Y - (U_B \cap F_Y)$. Therefore $F_Y - (U_B \cap F_Y)$ is a soft closed subset with respect to $\tilde{m}_{iF_Y}$, $i = 1, 2$, in $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$. Hence $(U_B \cap F_Y)$ is a soft open subset with respect to $\tilde{m}_{iF_Y}$, $i = 1, 2$, in $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$. □

Definition 3.20 *A soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is said to be pairwise soft semi Hausdroff space if for every $(x, u) \neq (y, u)$ in F_A, either there exist soft $\tilde{m}_1$-open set U_B such that $(x, u) \in U_B$ and $(y, u) \notin \tilde{m}_2Cl(U_B)$ or there exist an soft $\tilde{m}_2$-open set V_B such that $(y, u) \in V_B$ and $(x, u) \notin \tilde{m}_1Cl(V_B)$.*

Definition 3.21 *A soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is said to be pairwise soft Pseudo Hausdroff space if for every $(x, u) \neq (y, u)$ in F_A, there exist soft $\tilde{m}_1$-open set U_B such that $(x, u) \in U_B$ and $(y, u) \notin \tilde{m}_2Cl(U_B)$ and there exist an soft $\tilde{m}_2$-open set V_B such that $y \in V_B$ and $x \notin \tilde{m}_1Cl(V_B)$.*

Theorem 3.22 *Let soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft Pseudo Hausdroff, then every soft subspace $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ of $(F_A, \tilde{m}_1, \tilde{m}_2)$ is also pairwise soft pseudo Hausdroff.*

Proof: Let soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft pseudo Hausdroff. Let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. Since $(F_A, \tilde{m}_1, \tilde{m}_2)$ is a pairwise soft pseudo Hausdroff, then for every $(x, u) \neq (y, u)$, there exist soft $\tilde{m}_1$-open set U_B and soft $\tilde{m}_2$-open set V_B such that $(x, u) \in U_B$ and $(y, u) \notin \tilde{m}_2Cl(U_B)$ and $(y, u) \in V_B$ and $(x, u) \notin \tilde{m}_1Cl(V_B)$. Then $U_B \cap F_Y$ and $V_B \cap F_Y$ are $\tilde{m}_{1_{F_Y}}$ and $\tilde{m}_{2_{F_Y}}$ soft open sets respectively in F_Y such that $(x, u) \in U_B \cap F_Y$ and $(y, u) \notin \tilde{m}_{2_{F_Y}}Cl(U_B \cap F_Y)$ also $(y, u) \in V_B \cap F_Y$ and $(x, u) \notin \tilde{m}_{1_{F_Y}}Cl(V_B \cap F_Y)$. Therefore $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is pairwise soft pseudo Hausdroff. □

Definition 3.23 *A soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$ is said to be pairwise soft Uryshon space if given $(x, u) \neq (y, u)$ in F_A, there exist soft $\tilde{m}_1$-open set U_B and soft $\tilde{m}_2$-open set V_B such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $\tilde{m}_2Cl(U_B) \cap \tilde{m}_1Cl(V_B) = F_\emptyset$*

Theorem 3.24 *In a soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$, every soft subspace of a pairwise soft Uryshon space is pairwise soft Uryshon.*

Proof: Let $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft Uryshon space and let $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ be a soft subspace of $(F_A, \tilde{m}_1, \tilde{m}_2)$. Let $(x, u) \neq (y, u)$ in $F_Y \tilde{\subseteq} F_A$. Since $(F_A, \tilde{m}_1, \tilde{m}_2)$ is pairwise soft uryshon, there is a soft $\tilde{m}_1$-open set U_B and soft $\tilde{m}_2$-open set V_B such that $(x, u) \in U_B$, $(y, u) \in V_B$ and $\tilde{m}_2 Cl(U_B) \cap \tilde{m}_1 Cl(V_B) = F_\emptyset$. Now $U_B \cap F_Y$ and $V_B \cap F_Y$ are $\tilde{m}_{1_{F_Y}}$ and $\tilde{m}_{2_{F_Y}}$ soft open sets respectively in F_Y such that $(x, u) \in U_B \cap F_Y$ and $(y, u) \in V_B \cap F_Y$.
Consider,

$$\begin{aligned} \tilde{m}_{2_{F_Y}} Cl(U_B \cap F_Y) \cap \tilde{m}_{1_{F_Y}} Cl(V_B \cap F_Y) &= [\tilde{m}_{2_{F_Y}} Cl(U_B \cap F_Y)] \cap [\tilde{m}_{1_{F_Y}} Cl(V_B \cap F_Y)] \\ &= [\tilde{m}_{2_{F_Y}} Cl(U_B) \cap \tilde{m}_{1_{F_Y}} Cl(V_B)] \cap F_Y \\ &= [\tilde{m}_2 Cl(U_B) \cap \tilde{m}_1 Cl(V_B)] \cap F_Y \\ &= F_\emptyset \cap F_Y \\ &= F_\emptyset \end{aligned}$$

Hence $(F_Y, \tilde{m}_{1_{F_Y}}, \tilde{m}_{2_{F_Y}})$ is pairwise soft Uryshon space. □

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