Contra Harmonic Mean Labeling of Some Disconnected Graphs

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Abstract:

A graph G(V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct element f(x) from 0,1,…,q in such a way that when each edge e = uv is labeled with

$$f(e = uv) = \left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil \text{ or } \left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$$ with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Key words: *Graph, Contra Harmonic mean labeling, Contra Harmonic mean graphs, Path, Cycle, Comb.*

1. INTRODUCTION

All graph in this paper are simple, finite, undirected. Let G be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notation we follow Harary [2]. S. Somasundaram and R.Ponraj introduced mean labeling for some standard graphs in 2013 [3]. S. Somasundaram and S.S. Sandhya introduced Harmonic mean labeling of graph [4]. We have introduced Contra Harmonic mean labeling in [5].The Contra Harmonic mean labeling of disconnected graphs are introduced in [6]. In this paper we investigate the Contra Harmonic mean labeling behaviour of some more disconnected graphs. The following definition are useful for our present study.

Definition 1.1

A graph G (V,E) is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0,1,…,q in such a way that when each edge e = uv is labeled with

$$f(e=uv) = \left\lceil \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rceil \text{ or } \left\lfloor \frac{f(u)^2+f(v)^2}{f(u)+f(v)} \right\rfloor$$ with distinct edge labels. Here f is called a Contra Harmonic mean labeling of G.

Definition 1.2: The union of two graphs $G_1 = (V_1 E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$.

Definition 1.3: The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.4: A graph G is called a (nxm) -flower graph if it has n vertices which form a n –cycle and n sets of m-2 vertices which form m-cycles around the n-cycle on a single edge. This graph is denoted by f_{nx3} .

Theorem 1.5: Any Path is a Contra Harmonic mean graph.

Theorem 1.6: Any Cycle is a Contra Harmonic mean graph.

Theorem 1.7: Any Comb is a Contra Harmonic mean graph.

Theorem 1.8: Any Crown is a Contra Harmonic mean graph.

2. Main Results

Theorem: 2.1 $(C_m \odot K_3) \cup P_n$ is a Contra Harmonic mean graph for $m \geq 3$.

Proof: Let $u_1u_2\ldots u_m$ be the cycle C_m. Let v_i,w_i be the vertices of K_3 that are joined to the vertex u_i of C_m, $1\le i\le m$.

Let P_n be a path with vertices $z_1\ldots z_n$.

Let $G = (C_m \odot K_3) \cup P_n$.

Define $f: V(G) \to \{0,1,\ldots,q\}$ by

$f(u_i) = 4i-3,\ 1\le i\le m-1$, $f(u_m) = 4m-2$

$f(v_1) = 2,\ f(v_i) = 4i-4,\ 2\le i\le m$

$f(w_i) = 4i-1$, $1\le i\le m-1,\ f(w_m) = 4m$

$f(z_1) = 0,\ f(z_i) = 4m+i-1,\ 2\le i\le n$

Then the distinct edge labels are

$f(u_iu_{i+1}) = 4i,\ 1\le i\le m-1,\ f(u_mu_1) = 4m-2$

$f(u_iv_i) = 4i-3,\ 1\le i\le m$

$f(u_iw_i) = 4i-1,\ 1\le i\le m-1,\ f(u_mw_m) = 4m$

$f(v_iw_i) = 4i-2$, $1\le i\le m-1$, $f(v_mw_m) = 4m-1$

$f(z_i z_{i+1}) = 4m+i,\ 1\le i\le n-1$

Clearly, $(C_m \odot K_3)\cup P_n$ is a Contra Harmonic mean graph.

The Contra Harmonic mean labeling of $(C_5 \odot K_3) \cup P_5$ is

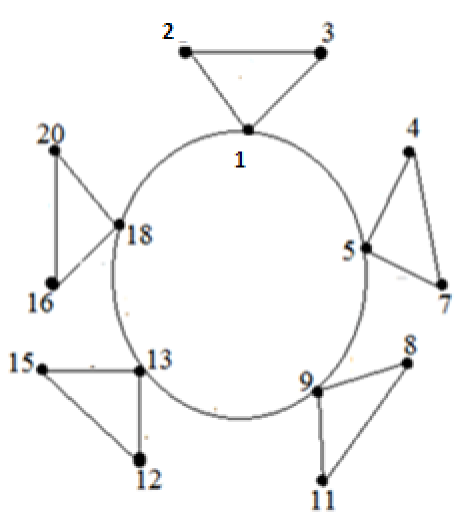


Figure: 1

Theorem 2:2

$(C_m \odot K_3) \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph, for $m\ge 3$. .

Proof: Let $u_1\ldots\ldots u_m$ be the cycle C_m. Let v_i,w_i be the vertices of K_3 that are joined to the vertex u_i of C_m. The resultant graph is $(C_m \odot K_3)$.

Let P_n be a path with vertices $t_1\ldots t_n$ and let s_i be the vertex that are joined to the vertex t_i, $1\le i\le n$ of P_n. The resultant graph is $(P_n \odot K_1)$.

Let $G = (C_m \odot K_3) \cup (P_n \odot K_1)$.

Define $f: V(G) \to \{0,1,\ldots,q\}$ by

$f(u_i) = 4i-3,\ 1\le i\le m-1$,$f(u_m) = 4m-2$

$f(v_1) =2,\ f(v_i) = 4i-4,\ 2\le i\le m$

$f(w_i) = 4i-1,\ 1\le i\le m-1,\ f(w_m) = 4m$

$f(t_1) = 0,\ f(t_i) = 4m+2i-2,\ 2\le i\le n$

$f(s_i) = 4m+2i-1,\ 1\le i\le n$

Then the distinct edge labels are

$f(u_iv_i) = 4i-3,\ 1\le i\le m$

$f(v_iw_i) = 4i-2,\ 1\le i\le m-1$

$f(v_mw_m) = 4m-1,$

$f(u_iw_i) = 4i-1,\ 1\le i\le m-1,\ f(u_mw_m) = 4m$

$f(u_i u_{i+1}) = 4i,\ 1\le i\le m-1,\ f(u_mu_1) = 4m-2$

$f(t_i t_{i+1}) = 4m+2i,\ 1\le i\le n-1$

$f(t_is_i) = 4m+2i-1,\ 1\le i\le n$

Clearly, f is a Contra Harmonic mean graph of G.

The Contra Harmonic mean labeling of

$(C_5 \odot K_3) \cup (P_6 \odot K_1)$

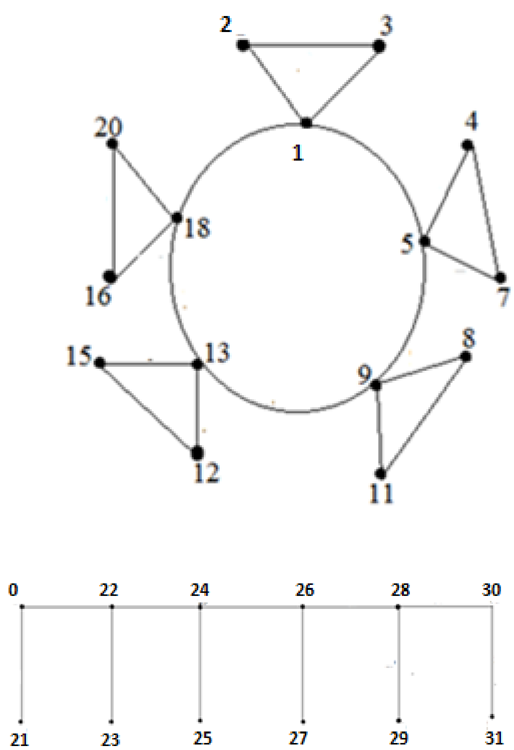


Figure: 2

Theorem 2.3

$(C_m \odot K_3) \cup (P_n \odot K_3)$ is a Contra Harmonic mean graph, for $m \geq 3$. .

Proof:

Let $u_1 \ldots . u_m$ be the cycle C_m. Let, v_i, w_i, be the vertices of K_3 that are joined to the vertex u_i of C_m. The resultant graph is $(C_m \odot K_3)$.

Let P_n be a path and let s_i, x_i be the vertices of K_3 that are joined with the vertices t_i of P_n, $1 \leq i \leq n$.

The resultant graph is $(P_n \odot K_3)$.

Let $G = (C_m \odot K_3) \cup (P_n \odot K_3)$

Define a function $f : V(G) \rightarrow \{0,1,\ldots,q\}$ by

$f(u_1) = 0$, $f(u_i) = 4i-3$, $2 \leq i \leq m-1$, $f(u_m) = 4m-2$

$f(v_1) = 1$, $f(v_i) = 4i-4$, $2 \leq i \leq m$

$f(w_i) = 4i-1$, $1 \leq i \leq m-1$, $f(w_m) = 4m$

$f(t_i) = 4m+4i-3$, $1 \leq i \leq n$

$f(s_i) = 4m+4i-2$, $1 \leq i \leq n$

$f(x_i) = 4m+4i-1$, $1 \leq i \leq n$

Then the distinct edge labels are

$f(u_1v_1) = 1$, $f(u_2v_2) = 4$, $f(u_iv_i) = 4i-3$, $3 \leq i \leq m$

$f(v_iw_i) = 4i-2$, $1 \leq i \leq m-1$ and $f(v_mw_m) = 4m-1$,

$f(u_iw_i) = 4i-1$, $1 \leq i \leq m-1$, $f(u_m, w_m) = 4m$

$f(u_1u_2) = 5$, $f(u_iu_{i+1}) = 4i$, $2 \leq i \leq m-1$,

$f(u_mu_1) = 4m-2$

$f(t_it_{i+1}) = 4m+4i$, $1 \leq i \leq n-1$

$f(t_is_i) = 4m+4i-3$, $1 \leq i \leq n$

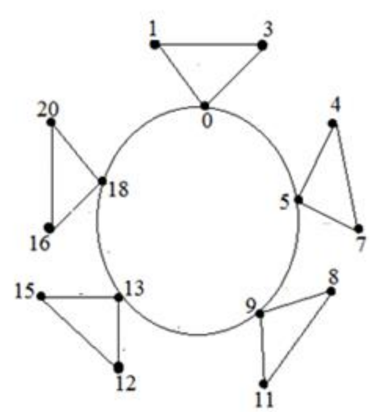
$f(t_ix_i) = 4m+4i-1$, $1 \leq i \leq n$

$f(s_ix_i) = 4m+4i-2$, $1 \leq i \leq n$

Clearly, f is a Contra Harmonic mean graph of G.

The Contra Harmonic mean labeling of

$(C_5 \odot K_3) \cup (P_4 \odot K_3)$ is



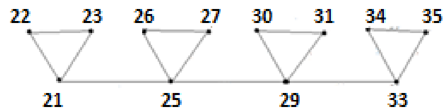


Figure: 3

Theorem2.4: ((C_m⊙K_1) ⊙ K_3) ∪ P_n is a Contra Harmonic mean graph, for m≥ 3. .

Proof:

Let $u_1u_2….u_m$ be the cycle C_m. Let v_i be the vertex which is adjacent to u_i, 1≤*i*≤m. The resultant graph is C_m⊙K_1. Let x_i, y_i be the vertices of K_3 which are attached to each of the vertex v_i. The resultant graph is ((C_m⊙K_1) ⊙ K_3).

Let P_n path with vertices $t_1..t_n$.

Let G = ((C_m⊙K_1) ⊙ K_3) ∪P_n

Define f: V(G) →{0, 1,…,q} by

$f(u_1) = 1$, $f(u_i) = 5i-1$, $2\le i\le m-1$, $f(u_m) = 5m$

$f(v_1) = 2$, $f(v_i) = 5i-2$, $2\le i\le m$

$f(x_1) = 3$, $f(x_i) = 5i-5$, $2\le i\le m$

$f(y_1) = 4$, $f(y_i) = 5i-4$, $2\le i\le m$

$f(t_1) = 0$, $f(t_i) = 5m+i-1$, $2\le i\le n$

Then the distinct edge labels are

$f(u_1u_2) = 9$, $f(u_iu_{i+1}) = 5i+2$, $2\le i\le m-2$

$f(u_{m-2}\, u_{m-1}) = 5(m-1)+3$,

$f(u_{m-1}\, u_m) = 5m-1$

$f(u_1v_1) =1$, $f(v_2v_2) = 8$, $f(u_iv_i) = 5i-1$, $3\le i\le m-1$,

$f(u_mv_m) =5m$

$f(v_1x_1) = 2$, $f(v_ix_i) = 5i-4$, $2\le i\le m$.

$f(v_1y_1) = 4$, $f(v_2y_2) = 7$, $f(v_iy_i) = 5i-2$, $3\le i\le m-1$

$f(v_my_m) = 5m-3$,

$f(x_1y_1) =3$, $f(x_iy_i) = 5i-5$, $2\le i\le m$

$f(t_it_{i+1}) = 5m+i$, $1\le i\le n-1$

Clearly, f is a Contra Harmonic mean graph of G.

The Contra Harmonic mean labeling of

(C_5⊙K_1) ⊙K_3)∪P_7 is

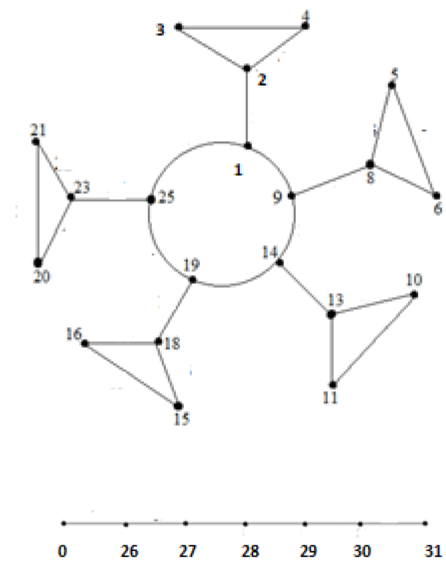


Figure 4

Theorem 2.5: ((C_m⊙K_1) ⊙K_3) ∪ (P_n⊙K_1) is a Contra Harmonic mean graph, for m≥ 3..

Proof:

Let $u_1u_2….u_m$ be the cycle C_m. Let v_i be the vertex which is adjacent to u_i, 1≤*i*≤m. The resultant graph is C_m⊙K_1. Let x_i, y_i be the vertices of K_3 which are attached to each of the vertex v_i. The resultant graph is **((C_m⊙K_1) ⊙K_3)** .

Let t_1 ….t_n be a path P_n.

Let s_i be the vertex that is joined to t_i of the path P_n, $1 \le i \le n$. The resultant graph is $(P_n \odot K_1)$.

Let G =$((\mathbf{C_m} \odot \mathbf{K_1}) \odot \mathbf{K_3}) \cup (P_n \odot K_1)$

Define f: V(G) $\to$ {0,1,…,q} by

$f(u_1) = 1$, $f(u_i) = 5i-1$, $2 \le i \le m-1$, $f(u_m) = 5m$

$f(v_1) = 2$, $f(v_i) = 5i-2$, $2 \le i \le m$.

$f(x_1) = 3$, $f(x_i) = 5i-5$, $2 \le i \le m$

$f(y_1) = 4$, $f(y_i) = 5i-4$, $2 \le i \le m$

$f(t_1) = 0$, $f(t_i) = 5m+2i-2$, $2 \le i \le n$

$f(s_i) = 5m+2i-1$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_1u_2) = 9$, $f(u_iu_{i+1}) = 5i+2$, $2 \le i \le m-2$

$f(u_{m-2}u_{m-1}) = 5(m-1)+3$,

$f(u_{m-1}u_m) = 5m-1$

$f(u_1v_1) = 1$, $f(u_2v_2) = 8$, $f(u_iv_i) = 5i-1$, $3 \le i \le m-1$

$f(u_mv_m) = 5m$

$f(v_1x_1) = 2$, $f(v_ix_i) = 5i-4$, $2 \le i \le m$

$f(v_1y_1) = 4$, $f(v_2y_2) = 7$, $f(v_iy_i) = 5i-2$, $3 \le i \le m-1$

$f(v_my_m) = 5m-3$,

$f(x_1y_1) = 3$, $f(x_iy_i) = 5i-5$, $2 \le i \le m$

$f(t_it_{i+1}) = 5m+2i$, $1 \le i < n-1$

$f(t_is_i) = 5m+2i-1$, $1 \le i \le n$

Clearly, f is a Contra Harmonic mean labeling of G.

The Contra Harmonic mean labeling of

$((C_5 \odot K_1) \odot K_3) \cup (P_7 \odot K_1)$ is

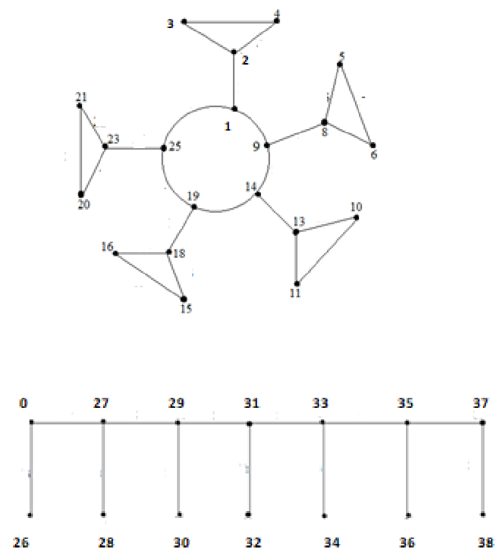


Figure 5

Theorem 2.6: $((\mathbf{C_m} \odot \mathbf{K_1}) \odot \mathbf{K_3}) \cup \mathbf{C_m}$ is a Contra Harmonic mean graph for $m \ge 3$, $n \ge 3$.

Proof:

Let $u_1u_2 \ldots u_m$ be the cycle C_m. Let v_i be the vertex which is adjacent to u_i, $1 \le i \le m$. The resultant graph is $C_m \odot K_1$. Let x_i, y_i be the vertices of K_3 which are attached to each of the vertex v_i. The resultant graph is $((\mathbf{C_m} \odot \mathbf{K_1}) \odot \mathbf{K_3})$.

Let $t_1 \ldots t_n$ be a cycle.

Let G = $((\mathbf{C_m} \odot \mathbf{K_1}) \odot \mathbf{K_3}) \cup \mathbf{C_n}$

Define f: V(G) {0,1,…,q} by

$f(u_1) = 1$, $f(u_i) = 5i-1$, $2 \le i \le m-1$, $f(u_m) = 5m$

$f(v_1) = 2$, $f(v_i) = 5i-2$, $2 \le i \le m$

$f(x_1) = 3$, $f(x_i) = 5i-5$, $2 \le i \le m$

$f(y_1) = 4$, $f(y_i) = 5i-4$, $2 \le i \le m$

$f(t_1) = 0$, $f(t_i) = 5m+i-1$, $2 \le i \le n-1$, $f(t_n) = 5m+n$

Then the distinct edge labels are

$f(u_1u_2) = 9$, $f(u_i\, u_{i+1}) = 5i+2$, $2 \le i \le m-2$

$f(u_{m-2}\, u_{m-1}) = 5(m-1)+3$,

$f(u_{m-1}u_m) = 5m-1$

$f(u_1v_1) = 1$, $f(u_2v_2) = 8$, $f(u_iv_i) = 5i-1$, $3 \le i \le m-1$

$f(u_mv_m) = 5m$

$f(v_1x_1) = 2$, $f(v_ix_i) = 5i-4$, $2 \le i \le m$

$f(v_1y_1) = 4$, $f(v_2y_2) = 7$, $f(v_iy_i) = 5i-2$, $3 \le i \le m-1$

$f(v_my_m) = 5m-3$,

$f(x_1y_1) = 3$, $f(x_iy_i) = 5i-5$, $2 \le i \le m$

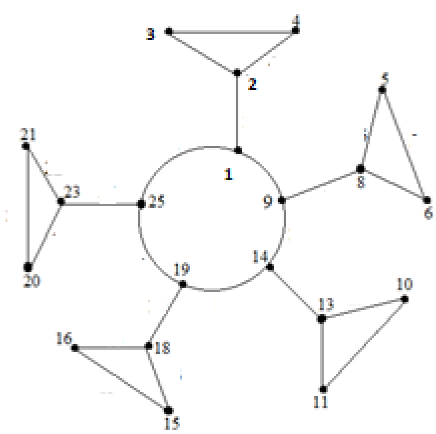
$f(t_it_{i+1}) = 5m+i$, $1 \le i \le n-1$

$f(t_n\, t_1) = 5m+n$

Clearly, f is a Contra Harmonic mean labeling of G.

The Contra Harmonic mean labeling of

$((C_5 \odot K_1) \odot K_3) \cup C_6$ is



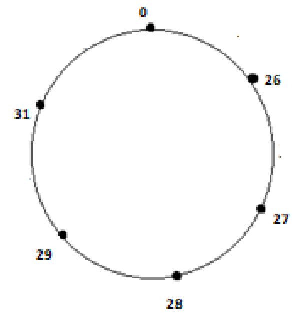


Figure 6

Theorem 2.7 : $((C_m \odot K_1) \odot K_3) \cup (P_n \odot K_3)$ is a Contra Harmonic mean graph for $m \ge 3$..

Proof: Let $u_1u_2\ldots.u_m$ be the cycle C_m. Let v_i be the vertex which is adjacent to u_i, $1 \le i \le m$. The resultant graph is $C_m \odot K_1$. Let x_i, y_i be the vertices of K_3 which are attached to each of the vertex v_i. The resultant graph is **$(C_m \odot K_1) \odot K_3)$** .

Let $t_1\ldots\ldots t_n$ be the path P_n. Let s_i, w_i be the vertex of K_3 that are joined with the vertices t_i of P_n, $1 \le i \le n$.

The resultant graph is $P_n \odot K_3$.

Let G = **$((C_m \odot K_1) \odot K_3)$**$\cup(P_n \odot K_3)$

Define f: $V(G) \rightarrow \{0,1,\ldots,q\}$ by

$f(u_1) = 0$, $f(u_i) = 5i-1$, $2 \le i \le m-1$,$f(u_m) = 5m$

$f(v_1) = 1$, $f(v_i) = 5i-2$, $2 \le i \le m$

$f(x_1) = 2$, $f(x_i) = 5i-5$, $2 \le i \le m$

$f(y_1) = 4$, $f(y_i) = 5i-4$, $2 \le i \le m$

$f(t_i) = 5m+4i-3$, $1 \le i \le n$

$f(s_i) = 5m+4i-2$, $1 \le i \le n$

$f(w_i) = 5m+4i-1$, $1 \le i \le n$

Then the distinct edge labels are

$f(u_1u_2)=9$, $f(u_iu_{i+1}) = 5i+2$, $2\le i\le m-2$

$f(u_{m-2}\, u_{m-1}) = 5(m-1)+3$,

$f(u_{m-1}\, u_m) = 5m$

$f(u_1v_1)=1$, $f(u_2v_2)=8$, $f(u_iv_i)=5i-1$, $3\le i\le m$

$f(v_1x_1)=2$, $f(v_ix_i) = 5i-4$, $2\le i\le m$

$f(v_1y_1)=4$, $f(v_2y_2)=7$, $f(u_iy_i) = 5i-2$, $3\le i\le m-1$

$f(v_my_m) = 5m-3$,

$f(x_1y_1)=3$, $f(x_iy_i) = 5i-5$, $2\le i\le m$

$f(t_i\, t_{i+1}) = 5m+4i$, $1\le i\le n-1$

$f(t_is_i) = 5m+4i-3$, $1\le i\le n$

$f(t_iw_i) = 5m+4i-1$, $1\le i\le n$

$f(s_iw_i) = 5m+4i-2$, $1\le i\le n$

Then f is a Contra Harmonic mean labeling of G.

The Contra Harmonic mean labeling of

$((C_5\odot K_1)\odot K_3)\cup(P_5\odot K_3)$ is

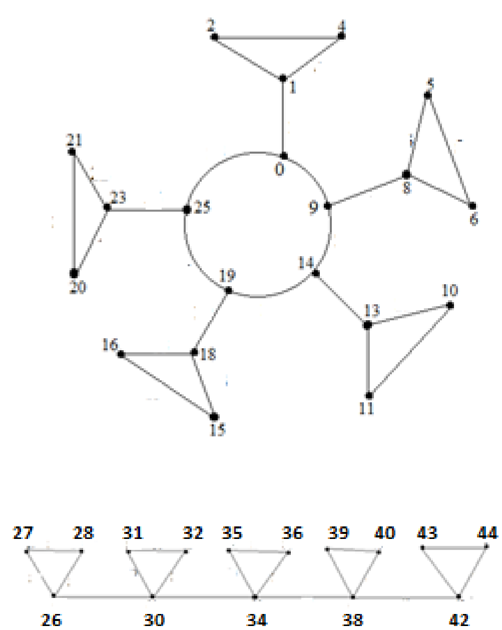


Figure 7

Theorem2.8: $f_{mx3}\cup P_n$ is a Contra Harmonic mean graph for $m\ge 3$..

Proof:

Let f_{mx3} be a flower graph. Let P_n be the path with vertices t_1…….t_n.

Let $G = f_{mx3}\cup P_n$

Define $f: V(G)\to\{0,1,\ldots,q\}$ by

$f(v_i) = 2i$, for $i=1,2$, $f(v_i) = 3i-2$, $3\le i\le m-1$,

$f(v_m) = 3m$

$f(u_1) = 1$, $f(u_i) = 3i-3$, $2\le i\le m-1$, $f(u_m)=3m-2$,

$f(t_1) = 0$, $f(t_i) = 3m+i-1$, $2\le i\le n$

Then distinct edge labels are

$f(u_1u_2)=3$, $f(u_i\, u_{i+1}) = 3i-1$, $2\le i\le m-1$

$f(u_mu_1) = 3m-2$

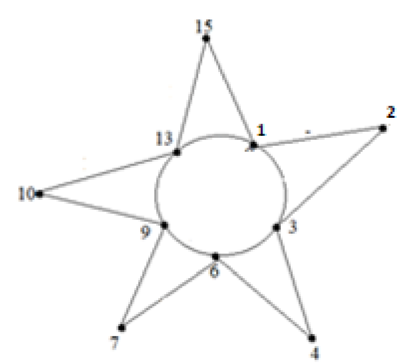
$f(u_iv_i) = 3i-2$, $1\le i\le m-1$, $f(u_mv_m) = 3m-1$

$f(u_1v_2) = 2$, $f(u_iv_{i+1}) = 3i$, $2\le i\le m$

$f(t_it_{i+1}) = 3m+i$, $1\le i\le n-1$

Clearly, f is a Contra Harmonic mean graph.

The Contra Harmonic mean labeling of $f_{5x3}\cup P_7$ is



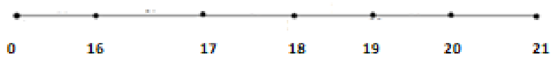


Figure 8

Theorem 2.9: $f_{mx3} \cup (P_n \odot K_1)$ is a Contra Harmonic mean graph, for $m \geq 3$.

Proof:

Let f_{mx3} be a a flower graph. Let $t_1 \ldots.. t_n$ be the path P_n. Let s_i be the vertex that is joined to t_i of the path P_n $1 \leq i \leq n$.

Let $G = f_{mx3} \cup P_n \odot K_1$

Define $f : V(G) \rightarrow \{0,1,\ldots,q\}$ by

$f(v_i) = 2i$, for $i = 1,2$

$f(v_i) = 3i-2$, $3 \leq i \leq m-1$, $f(v_m) = 3m$

$f(u_1) = 1$, $f(u_i) = 3i-3$, $2 \leq i \leq m-1$, $f(u_m) = 3m-2$

$f(t_1) = 0$, $f(t_i) = 3m+2i-2$, $2 \leq i \leq n$

$f(s_i) = 3m+2i-1$, $1 \leq i \leq n$

Then the distinct edge labels are

$f(u_1u_2) = 3$, $f(u_iu_{i+1}) = 3i-1$, $2 \leq i \leq m-1$

$f(u_mu_1) = 3m-2$

$f(u_iv_i) = 3i-2$, $1 \leq i \leq m-1$, $f(u_mv_m) = 3m-1$

$f(u_1v_2) = 2$, $f(u_iv_{i+1}) = 3i$, $2 \leq i \leq m$

$f(t_it_{i+1}) = 3m+2i$, $1 \leq i \leq n-1$

$f(t_is_i) = 3m+2i-1$, $1 \leq i \leq n$

Clearly, f is a Contra Harmonic mean graph of G.

The Contra Harmonic mean labeling of $f_{5x3} \cup (P_6 \odot K_1)$ is

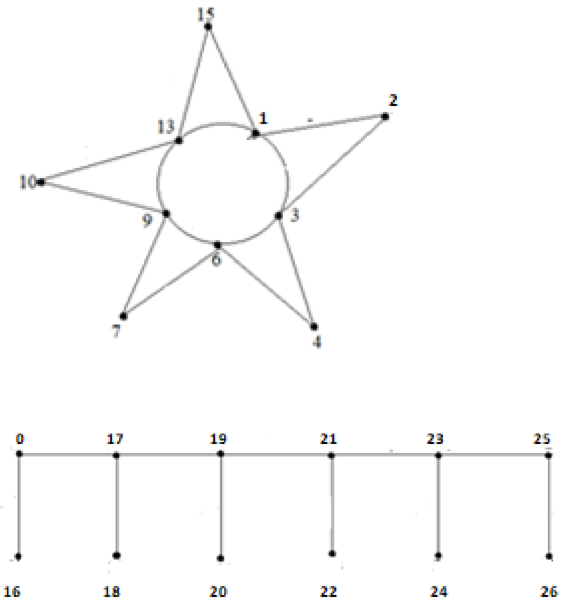


Figure 9

Theorem 2.10: $f_{mx3} \cup C_n$ Contra Harmonic mean graph for $m \geq 3$, $n \geq 3$.

Proof: Let f_{mx3} be a flower graph.

Let $t_1 \ldots\ldots t_n$ be a cycle C_n.

Let $G = f_{mx3} \cup C_n$

Define $f: V(G) \rightarrow \{0,1,\ldots,q\}$ by

$f(v_i) = 2i$, for $i = 1,2$

$f(v_i) = 3i-2$, $3 \leq i \leq m-1$, $f(v_m) = 3m$

$f(u_1) = 1$, $f(u_i) = 3i-3$, $2 \leq i \leq m-1$, $f(u_m) = 3m-2$

$f(t_1) = 0$, $f(t_i) = 3m+i-1$, $2 \leq i \leq n-1$, $f(t_n) = 3m+n$

Then the distinct edge labels are

$f(u_1u_2) = 3$, $f(u_iu_{i+1}) = 3i-1$, $2 \leq i \leq m-1$

$f(u_mu_1) = 3m-2$

$f(u_iv_i) = 3i-2$, $1 \leq i \leq m-1$

$f(u_mv_m) = 3m-1$

$f(u_1v_2) = 2$, $f(u_iv_{i+1}) = 3i$, $2 \le i \le m$

$f(t_it_{i+1}) = 3m+i$, $1 \le i \le n-1$

$f(t_nt_1) = 3m+n$

Clearly, f is a Contra Harmonic mean graph of G.

The Contra Harmonic mean labeling of $f_{5x3} \cup C_6$

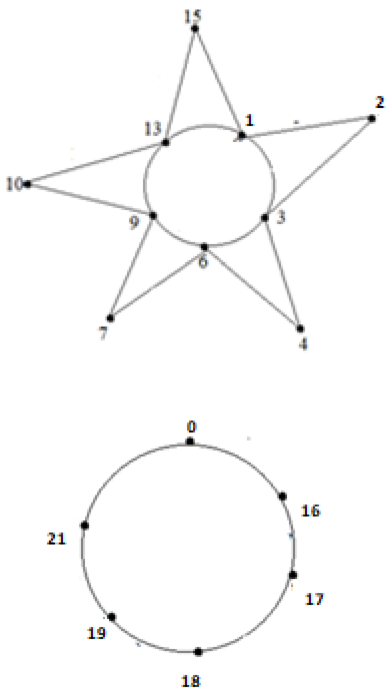


Figure 10

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