Properties of Connected Semirings and B-lattice Semirings

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Abstract — *This paper contains some results on connected semirings and b- lattice semirings. We consider a connected semiring (s,+,.,0) satisfying the identity 1+y=y+1 =1 for all y in s in which s is a variant of semi group (or) a-connected semigroup then it is proved that (s,a) is I- Medial, I –Semi Medial, Quasive separative.weakly separative. (S,.) is singular if (S,.) is rectangular band.Again we consider the same identity with (S,.) is commutative if (S,a) is L- Commutative. On the other hand (S,+,.) be a b-lattice semiring then(S,+) is diagonal if (S,+) is singular and the b-lattice semiring satisfying the identity a+b+ab=ab for all a,b in S then (S,+) is semimedial if (S,+) ,(S,.) are singular*

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I. Introduction

The notation of semiring was introduced by ***vandiver*** in 1934.The structure of semirings has been studied by so many authors. The theory of semirings and the theory of semigroups have considerable impact on the developments of the theory of semirings. It is observed that many researchers studied on different structures of semigroups. First of all we study variants of semigroup,it is developed by ***HickeyJ.B*** [2] and extend this vital information into the semirings. This approach construct a new algebraic structure of connected semirings. In this paper we presented that results on connected semirings and b- lattice semirings . The motivation of this paper due to the results of ***HickeyJ.B[2], Howie J.M. [3],Golan.J.S[1]..***

I.PRILIMINARIES

1.1 Definition : Let (S, .) be a semigroup and for any a in S. we define a binary operation (sandwich operation) '**o**' on the set S by x**o**y = xay where x, y$\in$S. Then S becomes a semigroup with respect to this operation. We denote it by (S, a) and we refer to (S, a) as a variant of (S, .) (or) a-connected semigroup.

1.2 Definition : Let (S,+,.)be a semiring. For any a$\in$S consider the semigroup (S, a) defined as xoy = xay for x,y$\in$ S then (S, +, ., o) is a semiring If (1) (S, a) is semigroup.
(2) 'o' distributes over addition i.e., xo(b+c) = xob+xoc and (b+c)ox = box+cox
Here (S, +, ., o) is called a connected semiring.

1.3 Definition : A semigroup (S, .) is said to be I-semimedial if aabc = abac for all a, b, c$\in$S

1.4 Definition : A semigroup (S, .) is said to be L-commutative if abc = acb for all a, b, c$\in$S

1.5 Definition : A semigroup (S, .) is said to be I-medial if abcd = acbd for all a, b, c, d$\in$S

1.6 Definition : A semigroup S is called quasi separative if x^2 =xy=yx= y^2 implies x=y for all x,y in S**.**

1.7 Definition : A semigroup is called weakly separative if for any x,y $\in$S x^2 =xy= y^2 implies x=y**.**

1.8 Definition: A semiring (s,+,.) is called a b-lattice, if(s,**.**) is a band and (s,+) is a semilattice

1.9 Definition : A semigroup (S, .) is said to be rectangular band if it satisfies the identity aba=
a for all a,b in S.

1.10 Theorem : Let (S, +, ., o) be a connected semiring and satisfies the identity 1+y = y+1 = y. If (S,.) is a rectangular band then (S, a) is I-medial and I-semi medial.

Proof : Let (S, +, ., o) be a connected semiring in which (S, .) is rectangular band. Then aba = a for all a, b $\in$ S
Since S satisfies the condition 1+y = y+1 = y for y $\in$ S
We have 1+y = y
$\Rightarrow$ xo(1+y) = xoy
$\Rightarrow$ xo1+xoy = xoy
$\Rightarrow$ xa+xay = xay
Now we prove that (S, a) is I-medial then
xoyozot = xozoyot
$\therefore$ xoyozot = xayazat
= xaya(1+z)a(1+t)
= xay(a+az)(a+at)
= (xaya+xayaz)(a+at)
= (xa+xaz) (a+at) (S is rectangular)
= xaz(a+at)
= xaza+xazat
= xazaya+xazayat
= xazaya(1+t)
= xazayat = xozoyot
$\therefore$ xoyozot = xozoyot
Hence (S, a) is I-medial

(2) To prove that (S, a) is I-semi medial.
i.e xoxoyoz = xoyoxoz
now xoxoyoz = xaxayaz
= xayaz
= xa(1+y)a(1+z)
= (xa+xay)(a+az)
= xay(a+az)
= xaya+xayaz
= xayaxa+xayaxaz
= xayaxa(1+z)
= xayaxaz
$\therefore$ xoxoyoz = xoyoxoz
Hence (S,a) is I-semi medial.

1.11 Theorem : Let (S, +, ., o) be a connected semiring satisfies the identity 1+y = y+1 = y and (S, .) is commutative then (S, a) is L-commutative.

Proof: Given that (S, +, ., o) is a connected semiring with the property
1+y = y+1 = y for all y $\in$ S
Let (S, .) be a commutative.
To prove that (S,a) is L-commutative for any a $\in$ S i.e xoyoz = xozoy
Consider
xoyoz = xayaz
= xazay
= xa(1+z)ay
= xa(1+z) a(1+y)
= (xa+xaz) (a+ay)
= xaz (a+ay)
= xaza+xazay
= xaza (1+y)
= xazay

= xozoy
∴ xoyoz = xozoy
Hence (S, a) is L-commutative.

Note : Similarly we prove (S, a) is R-commutative.

1.12 Theorem : Let (S,+,.,o) be a connected semiring satisfying 1+y = y+1 = y for $y \in S$. If (S, .) is rectangular band then
(1) (S, .) is singular.
(2) (S, a) is quasi separative and
(3) (S, a) is weakly separative.

Proof : Assume that S satisfies the identity 1+y = y+1 = y for $y \in S$
Let (S, +, ., o) be a semiring in which (S, .) be a rectangular band
Now 1+y = y
x(1+y) = xy
$\Rightarrow$ y(x+xy) = yxy
$\Rightarrow$ yx+yxy = yxy
$\Rightarrow$ yx+y = y
$\Rightarrow$ yx = y
Again 1+y = y
$\Rightarrow$ (1+y)x = yx
$\Rightarrow$ x+yx = yx
$\Rightarrow$ xy+yxy = yxy
$\Rightarrow$ xy+y = y
$\Rightarrow$ xy = y
∴ (S, .) is left and right singular.
Hence (S, .) is singular.

Now we show that (S, a) is weakly separative
xox = xoy $\Rightarrow$ xax = xay $\Rightarrow$ x = xy
xoy = yoy $\Rightarrow$ xay = yay $\Rightarrow$ xy = y
∴ xox = xoy = yoy
Now we prove that (S, a) is quasi separative
clearly xox = xoy $\Rightarrow$ xax = xay
$\Rightarrow$ x = xy
$\Rightarrow$ x = y
Now yoy = yox $\Rightarrow$ yay = yax
$\Rightarrow$ y = yx
$\Rightarrow$ y = x
∴ xox = xoy = yox= yoy $\Rightarrow$ x = y
Hence (S, a) is quasi separative

1.13Theorem: Let (S, +, . ,o) be a connected semiring satisfies the identity x+y+xoy = x. if (S, +) is singular then (S, a) is singular.

Proof : Given that (S, +, ., o) is a connected semiring.
Now S satisfies the identity x+y+xoy = x
Clearly (S, +) is singular ie x+y = x
If x+y+xoy = x
x+y+xay = x
x+xay = x
xay = x (1+ay = ay)
xoy = x (x+xay = xay)

Hence (S, a) is singular

1.14 Theorem : Let (S, +, .) be a b-lattice semiring satisfying the identity $a + b + ab = a$ for all a,b in S. If (S, +), (S, .) are singular then (S, +) is semi medial.

Proof: Given that S be a b-lattice semiring then (S, +) is a semilattice and (S, .) is a band.

Consider $a+b+ab = a$

$\Rightarrow$ $a+b+a = a$ ($\therefore$ (S, .) is singular)

$\Rightarrow$ $a+b = a$

$\Rightarrow$ $(a+b)\, a = a^2$

$\Rightarrow$ $a^2+ ba = a^2$

$\Rightarrow$ $a^2+ab+b+c = a^2+b+c$

$\Rightarrow$ $a^2+ab+b+a+c = a^2+b+c$

$\Rightarrow$ $a^2+b+b+a+c = a^2+b+c$

$\Rightarrow$ $a+b+a+c= a^2+b+c$ ($a^2 = a$, $b+b = b$)

$\therefore$ (S, +) is semi media

1.15 Theorem: Let (S, +, .) be a b-lattice semiring. If (S, +) is singular then (S,+) is diagonal.

Proof: Given that (S, +, .) be a b-lattice semiring.

To show that (S, +) is diagonal.

For this we have to show that $a+a = a$ and $a+b+c = a+c$ for all a,b,c in S.

Since S is b-lattice semiring we have $a+a = a$

$a+b+c = a+(c+b)$) ($\therefore$ (S, +) commutative)

$= a+c$ ($\therefore$ (S, +) is singular)

$\Rightarrow$ $a+b+c = a+c$

$\therefore$ (S, +) is diagonal.

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