Generalized Hyper Connected Space in bigeneralized topological space

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Abstract

In this paper we define $\tau_1\tau_2$* dense set and generalized hyper connected space. Also we study their properties.

I. Introduction:

In 1963, J.C. Kelly [3] defined bitopological spaces. The concepts of generalized topology was introduced by A. csaszar [2] in 2002. In 2010, C. Boonpok [1] introduced the concepts of bigeneralized topological spaces. In 2006, G. Navalagi, M.L. Thivagar, R.R. Rajeswari and S.A. Ponmani [5] introduced the concepts of (1,2) α - hyper connected spaces. In this paper, we define $\tau_1\tau_2$* dense set and generalized hyper connected space and study their properties.

II. Preliminaries

Definition 2.1[4]

Let X be a non – empty set and τ be a collection of subsets of X. Then τ is called a generalized topology on X if $\phi\in\tau$ and arbitrary union of elements of τ in τ . A space with a generalized topology is called a generalized topological space.

Definition 2.2[4]

Let X be a non – empty set and $\tau_{1,}\tau_2$ be generalized topologies on X. The triple (X, $\tau_{1,}\tau_2$) is called a bigeneralized topological space.

Definition: 2.3 [4]

Let (X, τ_1,τ_2) be a bigeneralized topological space. A subset A is said to be $\tau_1\tau_2$* closed if $\tau_1$cl A $\cap$ $\tau_2$clA = A. The smallest $\tau_1\tau_2$* closed set containing A is called the $\tau_1\tau_2$* closure of A and is denoted by $\tau_1\tau_2$* cl (A).

Definition: 2.4 [4]

Let (X, τ_1,τ_2) be a bigeneralized topological space. A subset A of X is said to be $\tau_1\tau_2$* open if its complement is $\tau_1\tau_2$* closed.

Result: 2.5 [4]

Let (X, τ_1,τ_2) be a bigeneralized topological space. A is a subset of X

i) If A is τ_1 closed then A is $\tau_1\tau_2$* closed.

ii) If A is τ_1 open then A is $\tau_1\tau_2$* open.

iii) If A is τ_2 closed then A is $\tau_1\tau_2$* closed.

iv) If A is τ_2 open then A is $\tau_1\tau_2$* open.

Theorem: 2.6 [4]

Let (X, τ_1,τ_2) be a bigeneralized topological space and let A be a subset of X. Then $\tau_1\tau_2$* cl(A) = $\tau_1$cl A $\cap$ $\tau_2$clA.

Definition: 2.7 [4]

Let (X, τ_1,τ_2) be a bigeneralized topological space. A subset A of X is said to be $\tau_1\tau_2$* dense if $\tau_1\tau_2$*cl(A) = X.

III. Generalized hyper connected space

Definition: 3.1

A bigeneralized topological space (X, τ_1,τ_2) is said to be generalized hyper connected space if every non empty $\tau_1\tau_2$* open sets is $\tau_1\tau_2$* dense in X. Equivalently any non-empty $\tau_1\tau_2$* open sets intersects.

Theorem: 3.2

If X is generalized hyper connected space then (X, τ_1) is hyper connected.

Proof:

Let X be generalized hyper connected and let A be τ_1 open set in X. Then A is $\tau_1\tau_2$* open set. Since X be generalized hyper connected, $\tau_1\tau_2$* cl(A) = X .This implies $\tau_1$clA $\cap$ $\tau_2$clA = X. Therefore, $\tau_1$clA = X. This implies A is dense in (X, τ_1). Hence (X, τ_1) is hyper connected.

Result: 3.3

The converse is not true. If (X, τ_1) is hyper connected then (X, τ_1, τ_2) is not generalized hyper connected. For example,

Let X = {a,b,c,d}τ_1 = {∅, X, {a,b}, {a,b,c}, {b,c,d}, {a,c}} τ_2= {∅, X, {a,b}, {c}, {a,b,c}, {a,d}, {a,b,d}, {a,c,d}}. The $\tau_1\tau_2$* open sets are ∅, X, {a,b} {a,b,c}, {b,c,d}, {a,c}, {c}, {a,d}, {a,b.d} and {a,c,d}. Clearly (X, τ_1) is hyper connected. But (X, τ_1, τ_2) is not generalized hyper connected space.

Theorem: 3.4

If X is generalized hyper connected space then (X, τ_2) is hyper connected space.

Proof:

Similar to theorem3.2.

Result: 3.5

The converse is not true.

Theorem: 3.6

If X is generalized hyper connected then (X, τ_1) is hyper connected and (X, τ_2) is hyper connected.

Proof:

If follows from previous Theorem3.2 and 3.4.

Result: 3.7

The converse is not true. If (X, τ_1) is hyper connected and (X, τ_2) is hyper connected then (X, τ_1, τ_2) is not generalized hyper connected. For example:

Let X = {a,b,c,d} τ_1 = {∅, X, {a,b}, {b,c}, {a,b,c}, {b,d},{a,b,d}, {b,c,d}} τ_2= {∅, X, {c,d}, {a,b,c}}The $\tau_1\tau_2$* open sets are {∅, X, {a,b}, {b,c}, {a,b,c}, {b,d}{a,b,d}, {b,c,d}, {c,d}}

Clearly (X, τ_1) is hyper connected and (X, τ_2) is hyper connected. But $\tau_1\tau_2$* closure of {a,b} is {a,b} . Hence (X, τ_1,τ_2) is not generalized hyper connected.

Now to find a situation where the converse is true.

Theorem: 3.8

If (X, τ_1) is hyper connected, (X,τ_2) is hyper connected and the intersection of any non empty τ_1 open set and any non empty τ_2 open set is non empty then (X, τ_1, τ_2) is generalized hyper connected space.

Proof:

Let A and B be two non empty τ_1 τ_2* open sets. Then A = τ_1 int A U τ_2 int A and

B = τ_1 int B U τ_2 int B Now, A∩B = (τ_1 int A U τ_2 int A) ∩ (τ_1 int BU$\tau_2$int B) Hence, A∩B = (τ_1 int A ∩ τ_1 int B) U (τ_1 int A ∩ τ_2 int B) U (τ_2 intA∩τ_1 int B) U (τ_2 int A ∩ τ_2 int B) Since A is non-empty, at least one of the set τ_1 int A or τ_2 int A is non empty. Also, since B is non empty, atleast one of the set τ_1 int B or τ_2 int B is non-empty.

Case (i)

Suppose τ_1 int A and τ_1 int B are non-empty. Since (X, τ_1) is hyper connected, τ_1 int A ∩ τ_1 int B ≠ ∅

Case (ii)

Suppose τ_1 int A and τ_2 int B are non empty. By hypothesis the intersection of any non empty τ_1 open set and any non empty τ_2 open set is non empty. Therefore, τ_1 int A ∩ τ_2 int B ≠ ∅

Case (iii)

Suppose τ_2 int A and τ_1 int B are non empty. By above case (ii) τ_2 intA ∩ τ_1 int B ≠ ∅

Case (iv)

Suppose τ_2 int A and τ_2 int B are non empty. Since (X, τ_2) is hyper connected, τ_2 int A ∩ τ_2 int B ≠ ∅ In each case, A ∩ B ≠ ∅. Therefore, (X, $\tau_1\tau_2$) is generalized hyper connected space.

Theorem: 3.9

If (X, τ_1, τ_2) is generalized hyper connected space then (X, τ_1) is hyper connected, (X, τ_2) is hyper connected and the intersection of any non empty τ_1 open set and any non empty τ_2 open set is non empty.

Proof:

Let (X, τ_1,τ_2) be generalized hyper connected . By theorem 2.11, (X, τ_1) is hyper connected and (X, τ_2) is hyper connected. Let A be non empty τ_1 open and B be non empty τ_2 open set.

Then A and B are non empty $\tau_1\tau_2$* open sets. Since X is generalized hyper connected, $A \cap B \neq \emptyset$

Theorem: 3.10

A bigeneralized topological space (X,τ_1,τ_2) is generalized hyper connected space if and only if (X,τ_1) is hyper connected, (X,τ_2) is hyper connected and the intersection of any non empty τ_1 open set and any non empty τ_2 open set is non empty.

Proof:

It follows from previous theorem 3.8 and 3.9.

Definition: 3.11

Let (X, τ_1,τ_2) be a bigeneralized topological space. Let x∈X is said to be $\tau_1\tau_2$* limit point if $U \cap (A - \{x\}) \neq \phi$ for every $\tau_1\tau_2$* open set U containing x.

Theorem: 3.12

Let X be a generalized hyper connected space and A be a proper $\tau_1\tau_2$* open set. Then every point $x \notin A$ is a $\tau_1\tau_2$* limit point of A.

Proof:

Let $x \notin A$ and U be a $\tau_1\tau_2$* open set containing x. Since X is generalized hyper connected and A is $\tau_1\tau_2$* open, $U \cap A \neq \emptyset$. Hence x is a $\tau_1\tau_2$* limit point of A.

Theorem:3.13

In a generalized hyper connected space, there is no proper set which is both $\tau_1\tau_2$* open and $\tau_1\tau_2$* closed.

Proof:

Let (X, τ_1,τ_2) be a generalized hyper connected space. Suppose A be both $\tau_1\tau_2$* open and $\tau_1\tau_2$* closed set. Then A^c is $\tau_1\tau_2$* open. Also $A \cap A^c = \emptyset$. Which is contradiction to X is generalized hyper connected space.

Definition: 3.14

Let (X, τ_1,τ_2) be a generalized hyper connected space. A point x∈X is called a point of attraction if the intersection of all the non empty $\tau_1\tau_2$* open set is contains x. The set of all points of attraction is called the set of attraction. X is called on attractive space if X has a point of attraction.

Theorem: 3.15

A $(X, \tau_1\tau_2)$ bigeneralized topological space. If X is attractive then it is generalized hyper connected.

Proof:

Since $(X, \tau_1\tau_2)$ is attractive space, then X has a point of attraction. Let x∈X be the point of attraction. Then the intersection of all the non-empty $\tau_1\tau_2$* open set is contains x. Therefore, every $\tau_1\tau_2$* open sets are intersects. Hence X is generalized hyper connected space.

Result: 3.16

The converse is not true. If X is a generalized hyper connected space then X is not a attractive space.

For example,

Let X = {a,b,c,d} τ_1 = {∅, X, {a,b}, {b,d}, {a,b,d}, {b,c}, {a,b,c}, {b,c,d}} τ_2 = {∅, X, {a,c,d}, {b,c,d}}$\tau_1\tau_2$* open sets are ϕ, X, {a, b}, {b, d}, {a, b, d}, {b, c}, {a, b, c},{b, c, d}, {a, c, d}. Clearly X is generalized hyper connected space and, X is not a attractive space.

Definition: 3.17

Let $(X, \tau_1\tau_2)$ be a Bigeneralized topological space. Let A be a subset of X and x∈X . Then x is called a $\tau_1\tau_2$* boundary point of A if $x \in \tau_1\tau_2$* cl A ∩ $\tau_1\tau_2$*cl (X – A). We denote the set of all $\tau_1\tau_2$* boundary point of A by $\tau_1\tau_2$* Bd(A).

Theorem: 3.18

Let X be a generalized hyper connected space. If A is $\tau_1\tau_2$* open then $\tau_1\tau_2$* Bd (A) = X – A

Proof:

Let A be $\tau_1\tau_2$* open set. Then X –A is $\tau_1\tau_2$* closed set. That is, τ_1 cl (X – A) ∩ τ_2 cl (X – A) = X – A Since X is generalized hyper connected, $\tau_1\tau_2$* cl (A) = τ_1 cl A ∩ τ_2 cl A = X. Now, $\tau_1\tau_2$* Bd (A) = (τ_1 cl A ∩ τ_2 cl A) ∩ (τ_1 cl (X – A)∩ τ_2 cl (X – A)) = X ∩ (X – A) = X – A

Definition: 3.19

Let (X, τ_1, τ_2) and (Y, μ_1, μ_2) be bigeneralized topological spaces. Then $f : (X, \tau_1, \tau_2) \to (Y, \mu_1, \mu_2)$ is said to be star continuous if inverse image of $\mu_1\mu_2$*closed set is $\tau_1\tau_2$* closed.

Theorem: 3.20

Let f : (X, τ_1, τ_2) → (Y, μ_1, μ_2) be onto and star continuous. If X is generalized hyper connected then Y is generalized hyper connected.

Proof:

Let B_1, B_2 be two non-empty $\mu_1\mu_2$ * open sets of Y. Since f is onto, f^{-1} (B_1) and f^{-1} (B_2) are non empty, Since f is continuous, f^{-1} (B_1) and f^{-1} (B_2) are $\tau_1\tau_2$ * open sets of X. Since X is generalized hyper connected, $f^{-1}(B_1) \cap f^{-1}(B_2) \neq \phi$ This implies $f^{-1}(B_1 \cap B_2) \neq \phi$ Hence $B_1 \cap B_2 \neq \phi$ Therefore, Y is generalized hyper connected.

Remark: 3.21

In the above theorem, f : X → Y is onto is necessary. Suppose f: (X, τ_1, τ_2) → (Y, μ_1, μ_2) is star continuous but not onto. If X is generalized hyper connected then Y is not generalized hyper connected. For example, Let X = {a, b, c} and Y={a, b,c} Take $\tau_1 = \tau_2$ = {ϕ, X, {a, b}, {b, c}} and $\mu_1 = \mu_2$ = {ϕ, X, {a, b}, {c}} Define f : (X, τ_1, τ_2) → (Y, μ_1, μ_2) by f (a) = a ; f(b)= b and f (c) = a clearly f is star continuous but not onto, Also X is generalized hyper connected and Y is not generalized hyperconnected.

Theorem: 3.22

Let (X, τ_1, τ_2) be a bigeneralized topological space and τ_2 = {ϕ, X}. Then X is generalized hyper connected if and only if (X, τ_1) is hyper connected.

Proof:

If X is generalized hyper connected then by theorem 3.2 (X, τ_1) is hyper connected. Suppose (X, τ_1) is hyper connected. Since τ_2 open sets are ϕ and X, the τ_1 τ_2 * open sets are only the τ_1 open sets. Since any two non empty τ_1 open sets are intersect, any two non empty τ_1 τ_2* open sets are intersect. Hence (X, τ_1, τ_2) is generalized hyper connected.

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