Analysis of the M/M/1 queue with single working vacation and vacation interruption

Shakir Majid*, P.Manoharan

Department of Mathematics Annamalai University, Annamalainagar-608002, Tamil Nadu, India.

Abstract

In this work, a vacation interruption in M/M/1 queue with single working vacation is considered. Using the matrix analytic method, we obtain the distributions for the mean queue length and the mean sojourn time and their stochastic decomposition structures. Finally, we demonstrate the effects of system parameters on the performance measures and present some special cases.

Key words: M/M/1 queue, Vacation, Working vacation, Stochastic decomposition, Matrix-geometric solution.
AMS classification: 60K25, 68M20

1. Introduction

Queuing systems with vacations have been studied extensively by various researchers for their profound applications in many real life situations such as telecommunication, computer networks, production systems, and so on. The details have been reported in the survey of Doshi (1986), the monographs of Takagi (1991) and Tian and Zhang (2006). In these studies, it is assumed that the customers service is completely stopped during a vacation. However, there are numerous examples, where the server serves the customers at a lower rate during a vacation, rather than being completely inactive. This type of semi vacation policy was first introduced by Servi and Finn (2002). This type of vacation policy is called a working vacation (WV). Servi and Finn (2002) analyzed M/M/1 queue with multiple working vacation policy and derived the PGF of the queue length and LST of the waiting time, and utilized their results to analyze the system performance of gateway router in fiber communication networks. Later M/M/1 with multiple working vacation model was also studied by Liu, Xu and Tian (2007) to obtain explicit expressions of the performance measures and their stochastic decomposition by using the matrix-geometric method. Subsequently, by applying the same method, M/M/1 queue with single working vacation was analyzed by Tian and Zhao (2008) and obtained various steady state indicators. Kim, Choi and Chae (2003), Wu and Takagi (2006) and Li et al. (2011) extended the study in Servi and Finn (2002) to an M/G/1/WV queue. Baba (2005) applied the matrix-geometric solution method to generalize the work of Servi and Finn (2002) to a GI/M/1 queue with general arrival process and multiple working vacations. Later, Li and Tian (2011) investigated the GI/M/1 queue with single working vacation.

Situations often arise where the server must stop the vacation and come back to normal working level, once some system indices, such as the number of customers, attain a certain value during a vacation. In many real life scenarios, important events happen during a vacation and the server rather than continuing to take residual vacation must return to normal work. For example, when the number of waiting customers in the system exceeds a certain value during a vacation and server continues to be in vacation, it leads to a large cost of waiting customers. Hence the server vacation interruption is more effective and reasonable to vacation queues. Li and Tian (2007) first studied the vacation interruptions in an M/M/1 queuing model with working vacation and derived the steady state distributions of the expected queue length and expected waiting time along with stochastic decomposition structures in terms of matrix-

---

(Shakir Majid)

geometric method. Later, Li and Tian (2007) and Li et al. (2008) generalized this work to GI/Geo/1 and GI/M/1 queue by using matrix-analytic method. Zhang and Hou (2010) analyzed the vacation interruptions in M/G/1 queue with working vacation by utilizing the method of supplementary variable. They obtained the queue length distribution and LST of the stationary waiting time. Zhang and Hou (2011) extended the work of Zhang and Hou (2010) to an MAP/G/1/WV queue and computed the system size distribution and the Laplace-Stieltjes transform (LST) of waiting time. Later, Gao et al. (2014) analyzed the M/G/1/MWV queue with retrials. Laxmi and Jyothsna (2015) studied the impatient customer in M/M/1 queue under Bernoulli schedule vacation interruption and obtained various performance measures by adopting the PGF method.

In this paper, we consider an M/M/1 queuing system with two vacation policies. The working vacation and vacation interruption are connected and the server takes a working vacation as soon as the system becomes empty and customers are served at the lower rate during the working vacation period. If the system is non empty at the service completion instant during the vacation period, the server will return to the normal busy period no matter whether or not the vacation ends. Otherwise, he continues the vacation until there are customers after a service or a vacation ends. The purpose of our work is to develop a set of stationary performance measures such as the mean queue length and waiting time. With these performance measures, we illustrate the effects of system parameters on the performance indices of the system.

The rest of the paper is organized as follows. In section 2, we describe the model as a quasi-birth death process. In section 3, we obtain the steady state distribution of the queue length and the state probabilities of the server. Section 4 discusses the stochastic decomposition structures of the queue length and sojourn time, and obtains the distributions of the additional queue length and additional delay. Finally the numerical examples are presented in section 5.

2. Model description

Consider a classical M/M/1 queuing system with Poisson arrival rate λ and a service rate μ. The server begins a working vacation as soon as the system becomes empty and the vacation period V follows an exponential distribution with the parameter ϕ. During a vacation period, the arriving customers are served at an exponential rate η, where $\eta < \mu$. The server is assumed to interrupt the vacation if there are customers in the system at a service completion instant during a working vacation period and changes its service rate from η to μ, thereby giving rise to a normal working level. Otherwise, the server continues the working vacation until the system is non empty after the service or a vacation ends. Meanwhile, if the server finds no customer in the system after returning from the working vacation, he does not take another vacation but stays idle until the next arrival. In this service discipline, the server may return from the vacation without completing it and can stay in the vacation period if the system is empty at the instant of a service completion. Meanwhile, the vacation service rate is applicable only to the first customer arrived during a vacation period. The inter-arrival times, the service times, and the working vacation times all are taken to be mutually independent. In addition, the service order is first in first out (FIFO).

Let L(t) be the number of customers in the system at time t and J(t) be the status of the server, which is defined as follows:

$$J(t) = \begin{cases} 0 & \text{When the server is in WV period at time t, ,} \\ 1 & \text{When the server is in normal working period at time t,} \end{cases}$$

Then $\{(L(t), J(t)), t \geq 0\}$ defines a continuous time Markov chain with state space

$$S = \{(k, j) : k = 0, 1, 2, ..., j = 0, 1\}$$

Using the lexicographical order for the states, the infinitesimal generator is given by

$$Q = \begin{pmatrix} A_0 & C & & & \\ B_1 & A & C & & \\ & B & A & C & \\ & & B & A & C \\ & & & \vdots & \vdots & \vdots \end{pmatrix}$$

2

Where $A_0 = \begin{pmatrix} -(\lambda + \phi) & \phi \\ 0 & -\lambda \end{pmatrix}$, $C = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$, $B_1 = \begin{pmatrix} \eta & 0 \\ \mu & 0 \end{pmatrix}$,

$A = \begin{pmatrix} -(\lambda + \phi + \eta) & \phi \\ 0 & -(\lambda + \mu) \end{pmatrix}$, $B = \begin{pmatrix} 0 & \eta \\ 0 & \mu \end{pmatrix}$,

The matrix structure of Q indicates that the Markov Chain $\{(L(t), J(t)), t \geq 0\}$ is a quasi birth and death process. To analysis the QBD process, we first need to get the rate matrix, denoted by R, which is the minimal non-negative solution of the matrix quadratic equation

$$R^2 B + RA + C = 0 \tag{1}$$

The following lemma presents the explicit solution of R.

Lemma 1. If $\frac{\lambda}{\eta} < 1$, the minimal non-negative solution of matrix quadratic equation (1) has the following expression:

$$R = \begin{pmatrix} r & \rho \frac{\lambda+\phi}{\lambda+\phi+\eta} \\ 0 & \rho \end{pmatrix} \tag{2}$$

$$r = \frac{\lambda}{\lambda + \phi + \eta} \tag{3}$$

Proof. Since A, B and C in (1) are all upper triangular matrices, therefore we can consider that the solution matrix R has the same structure as

$$R = \begin{pmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{pmatrix}$$

Substituting the matrix value R into (1) leads to

$$-(\lambda + \phi + \eta)r_{11} + \lambda = 0$$

$$\mu r_{22}^2 - (\lambda + \mu)r_{22} + \lambda = 0 \tag{4}$$

$$\eta r_{11}^2 + \mu r_{12}(r_{11} + r_{22}) + \phi r_{11} - (\lambda + \mu)r_{12} = 0$$

We can obtain the minimal non-negative solution of (1) from the above set of equations, by using the fact that $\mu z^2 - (\lambda + \mu)z + \lambda = 0$ has a unique root $r_{22} = \rho$ in interval (0,1). Substituting $r_{11} = r$ and $r_{22} = \rho$ into the last equation of (4), we get $r_{12} = \rho \frac{\lambda+\phi}{\lambda+\phi+\eta}$, then the proof is complete.

According to the expression of R in Neuts (1981, Theorem 3.1.1) and lemma 1, it is easy to prove that the QBD process $\{(L(t), J(t)), t \geq 0\}$ is positive recurrent if and only if $\rho < 1$.

3. Stationary distribution of queue length

If $\rho < 1$, let (L,J) be the stationary limit of the QBD process $\{(L(t), J(t)), t \geq 0\}$ and define

$$\pi_k = (\pi_{k0}, \pi_{k1}), \quad k \geq 0$$

$$\pi_{kj} = P\{L = k, J = j, t \geq 0\} = \lim_{t \to \infty} \{(L(t) = k, J(t) = j\}, \quad (k, j) \in S,$$

Theoem 1. If $\rho < 1$, the Stationary probability distribution of (L,J) is

$$\pi_{k0} = Kr^k, \quad k \geq 0,$$

$$\pi_{01} = K\frac{\phi}{\lambda} \tag{5}$$

$$\pi_{k1} = K\left[\frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta} \sum_{j=0}^{k-1} \rho^j r^{k-1-j} + \frac{\phi}{\mu}\rho^{K-1}\right], \quad k \geq 1$$

Where

$$K = (1-r)(1-\rho)\left[(1-\rho) + \frac{\phi}{\lambda}(1-r)(1-\rho) + \frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta} + \frac{\phi}{\mu}(1-r)\right]^{-1} \tag{6}$$

3

Proof. Using the matrix geometric solution method by Neuts (1981), we get

$$\pi_k = (\pi_{k0}, \pi_{k1}) = (\pi_{10}, \pi_{11})R^{k-1}, \quad k \geq 1 \tag{7}$$

and $(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11})$ satisfies the following set of equations

$$(\pi_{00}, \pi_{01}, \pi_{10}, \pi_{11})B[R] = 0 \tag{8}$$

Where

$$B[R] = \begin{pmatrix} A_0 & C \\ B_1 & RB + A \end{pmatrix} = \begin{bmatrix} -(\lambda + \phi) & \phi & \lambda & 0 \\ 0 & -\lambda & 0 & \lambda \\ \eta & 0 & -(\lambda + \phi + \mu) & (\lambda + \phi) \\ \mu & 0 & 0 & -\mu \end{bmatrix}$$

Substituting B[R] into (8), we obtain the set of equations as follows

$$-(\lambda + \phi)\pi_{00} + \eta\pi_{10} + \mu\pi_{11} = 0$$

$$\phi\pi_{00} - \lambda\pi_{01} = 0$$

$$\lambda\pi_{00} - (\lambda + \phi + \eta)\pi_{10} = 0 \tag{9}$$

$$\lambda\pi_{01} - (\lambda + \phi)\pi_{10} - \mu\pi_{11} = 0$$

Taking $\pi_{00} = K$ and solving the above equations in terms of K, we obtain

$$\pi_{10} = Kr, \qquad \pi_{01} = k\frac{\phi}{\lambda}, \qquad \pi_{11} = K\left[\frac{\rho(\lambda + \phi)}{\lambda + \phi + \eta} + \frac{\phi}{\mu}\right]$$

From (2), we obtain

$$R^k = \begin{bmatrix} r^k & \frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta}\rho \sum_{j=0}^{k-1} \rho^j r^{k-1-j} \\ 0 & \rho^k \end{bmatrix}, \quad k \geq 1 \tag{10}$$

Substituting (π_{10}, π_{11}) and the matrix expression R^{k-1} into (7), we obtain (5) and noting that constant factor K can be determined by utilizing the normalization condition.

From theorem 1, the probabilities that the system is in working vacation period, idle period and regular busy period are obtained as follows, respectively:

$$P\{J=0\} = \sum_{k=0}^{\infty} \pi_{k0} = K_0(1 - \rho)$$

$$P\{\text{the server is in idle period}\} = K\frac{\phi}{\lambda} \tag{11}$$

$$P\{J=1\} = \sum_{k=1}^{\infty} \pi_{k1} = K_0(1 - r)\left[\frac{\rho(\lambda + \phi)}{(\lambda + \phi + \eta)(1 - r)} + \frac{\phi}{\mu}\right]$$

Where

$$K_0 = \left[(1 - \rho) + \frac{\phi}{\lambda}(1 - r)(1 - \rho) + \frac{\rho(\lambda + \phi)}{\lambda + \phi + \eta} + \frac{\phi}{\mu}(1 - r)\right]^{-1} \tag{12}$$

4. Stochastic decompositions

To achieve a better comparison with the queuing models already existing, we often attempt to decompose the quantities of interest into various factors. In vacation queuing models, the stochastic decomposition structures plays a significant role and underlines the influence of system vacation on system performance indices such as mean queue length and mean sojourn times. For the system under investigation, we strive to do a similar decomposition.

Theoem 2. If $\rho < 1$ and $\eta > \mu$, the stationary queue length L in system can be decomposed into sum of two independent random variables: $L = L_0 + L_d$, where L_0 is the stationary queue length of the classical M/M/1 queue

4

without vacation and follows a geometric distribution with parameter 1-ρ and the additional queue length L_d has a modified geometric distribution

$$P\{L_d=\text{K}\} = \begin{cases} K^*\frac{\lambda+\phi}{\lambda}(1-r), & \text{K}=0 \\ K^*\left(r-\rho+r\frac{\lambda+\phi}{\mu}\right)(1-r)r^K, & K \geq 1 \end{cases} \tag{13}$$

$$\text{Where} \quad K^* = \frac{K}{(1-r)(1-\rho)} = \left[\frac{\lambda+\phi}{\lambda}(1-r)+r-\rho+r\frac{\lambda+\phi}{\mu}\right]^{-1} \tag{14}$$

Proof. By theorem 1, the PGF of L can be expressed as follows

$$L(z) = \sum_{k=0}^{\infty}\pi_{k0}z^k + \sum_{k=0}^{\infty}\pi_{k1}z^k = K\left[\frac{\phi}{\lambda}+\frac{1}{1-rz}+\frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta}\frac{z}{(1-rz)(1-\rho z)}+\frac{\phi}{\mu}\frac{z}{1-\rho z}\right]$$

$$= \frac{1-\rho}{1-\rho z}K^*\left[\frac{\phi}{\lambda}(1-r)(1-\rho z)+\frac{1-r}{1-rz}(1-\rho z)+\frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta}\frac{z(1-r)}{(1-rz)}+\frac{\phi}{\mu}(1-r)z\right]$$

Note that

$$\frac{\phi}{\lambda}\rho = \frac{\phi}{\mu}, \qquad \frac{1-r}{1-rz}(1-\rho z) = (1-r)+(r-\rho)\frac{z(1-r)}{1-rz}$$

Hence $L(z)$ can be rewritten as

$$L(z) = \frac{1-\rho}{1-\rho z}K^*\left[\frac{\lambda+\phi}{\lambda}(1-r)+\left(r-\rho+\frac{r(\lambda+\phi)}{\mu}\right)\frac{z(1-r)}{1-rz}\right] = L_0(z)L_d(z) \tag{15}$$

where

$$K^* = \left[(1-\rho)+\frac{\phi}{\lambda}(1-r)(1-\rho)+\frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta}+\frac{\phi}{\mu}(1-r)\right]^{-1}$$

$$= \left[1-r+\frac{\phi}{\lambda}(1-r)(1-\rho)+r-\rho+\frac{\rho(\lambda+\phi)}{\lambda+\phi+\eta}+\frac{\phi}{\mu}(1-r)\right]^{-1}$$

$$= \left[\frac{\lambda+\phi}{\lambda}(1-r)+r-\rho+r\frac{\lambda+\phi}{\mu}\right]^{-1}$$

Expanding $L_d(z)$ in power series of z, we get (13).

Theorem 2 shows that the additional queue length $L_d(z)$ is a mixture of two random variables

$$i.e \qquad L_d = K^*\frac{(\lambda+\phi)(1-r)}{\lambda}X_0 + K^*(r-\rho+\frac{r(\lambda+\phi)}{\mu})X_1 \tag{16}$$

Where $X_0 = 0$ and X_1 follows the geometric distribution with parameter 1-r.
Based on stochastic decomposition in theorem 2, we obtain

$$E(L_d) = K^*\left[r-\rho+r\frac{\lambda+\phi}{\mu}\right]\frac{1}{1-r} \tag{17}$$

$$E(L) = \frac{\rho}{1-\rho}+K^*\left[r-\rho+r\frac{\lambda+\phi}{\mu}\right]\frac{1}{1-r} \tag{18}$$

Theoem 3. If $\rho < 1$ and $\mu > \eta$, the stationary sojourn time S of an arrival can be decomposed into the sum of two independent variables: $S = S_0 + S_d$, where S_0 is the sojourn time of an arrival in a corresponding classical M/M/1 queue and is exponentially distributed with parameter $\mu - \lambda$ and S_d is the additional delay with the LST given by

$$S_d^*(s) = K^*\left\{\left[(\frac{\lambda+\phi}{\lambda})(1-\rho)-\frac{r-\rho}{r}\right](1-r)+\left[\frac{r-\rho}{r}+\frac{\lambda+\phi}{\mu}\right]\frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s}\right\} \tag{19}$$

5

Where

$$K^* = \left[\left((\frac{\lambda+\phi}{\lambda})(1-\rho) - \frac{r-\rho}{r}\right)(1-r) + \frac{r-\rho}{r} + \frac{\lambda+\phi}{\mu}\right]^{-1}$$

Proof. The classical relation between the PGF of a queue length L and the LST of sojourn time S by Keilson and Sevi (1988) is given as follows

$$L(z) = S^*(\lambda(1-z))$$

From theorem 2, we get

$$L(z) = \frac{1-\rho}{1-\rho z}K^*\left[(\frac{\lambda+\phi}{\lambda})(1-r) + \left(r-\rho + \frac{r(\lambda+\phi)}{\mu}\right)\frac{z(1-r)}{1-rz}\right] \tag{20}$$

Taking $z = 1 - \frac{s}{\lambda}$ in (20) and noting that

$$\frac{1-\rho}{1-\rho z} = \frac{\mu(1-\rho)}{\mu(1-\rho)+s}, \qquad \frac{1-r}{1-rz} = \frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s}$$

We obtain

$$S^*(s) = \frac{\mu(1-\rho)}{\mu(1-\rho)+s}K^*\left[(\frac{\lambda+\phi}{\lambda})(1-r) + (r-\rho+\frac{r(\lambda+\phi)}{\mu})(1-\frac{1}{r}+\frac{1}{r}\frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s})\right]$$

$$= \frac{\mu(1-\rho)}{\mu(1-\rho)+s}K^*\left[(\frac{\lambda+\phi}{\lambda}-\frac{\lambda+\phi}{\mu}-\frac{r-\rho}{r})(1-r) + (\frac{r-\rho}{r}+\frac{(\lambda+\phi)}{\mu})(\frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s})\right]$$

$$= \frac{\mu(1-\rho)}{\mu(1-\rho)+s}K^*\left\{\left[(\frac{\lambda+\phi}{\lambda})(1-\rho) - \frac{r-\rho}{r}\right](1-r) + \left[\frac{r-\rho}{r}+\frac{\lambda+\phi}{\mu}\right]\frac{\frac{\lambda}{r}-\lambda}{\frac{\lambda}{r}-\lambda+s}\right\}$$

$$= \frac{\mu(1-\rho)}{\mu(1-\rho)+s}S_d^*(s) \tag{21}$$

From (14), we have

$$K^* = \left[\frac{\lambda+\phi}{\lambda}(1-r) + r - \rho + r\frac{\lambda+\phi}{\mu}\right]^{-1}$$

$$= \left[\frac{\lambda+\phi}{\lambda}(1-r) + r - \rho + (r+1-1)\frac{\lambda+\phi}{\mu}\right]^{-1} \tag{22}$$

$$= \left[\left((\frac{\lambda+\phi}{\lambda})(1-\rho) - \frac{r-\rho}{r}\right)(1-r) + \frac{r-\rho}{r} + \frac{\lambda+\phi}{\mu}\right]^{-1} \tag{23}$$

Hence, this verifies that $S_d^*(s)$ is a LST.

Similarly, theorem 2 indicates that the additional sojourn time $S_d^*(s)$ is a mixture of two random variables

$$S_d^*(s) = K^*\left\{\left[(\frac{\lambda+\phi}{\lambda})(1-\rho) - \frac{r-\rho}{r}\right]Y_0 + \left[\frac{r-\rho}{r}+\frac{\lambda+\phi}{\mu}\right]Y_1\right\} \tag{24}$$

Where $Y_0 = 0$ and Y_1 follows the exponential distribution with parameter $\frac{\lambda}{r} - \lambda$.
Based on the above stochastic decomposition structure, we get easily obtain the means as follows

$$E(S_d) = K^*\left[r - \rho + r\frac{\lambda+\phi}{\mu}\right]\frac{1}{\lambda(1-r)} = \frac{1}{\lambda}E(L_d) \tag{25}$$

$$E(S) = \frac{1}{\mu(1-\rho)} + K^*\left[r-\rho+r\frac{\lambda+\phi}{\mu}\right]\frac{1}{\lambda(1-r)} \tag{26}$$

6

5. Numberical Results

In this section, we provide some numerical examples to demonstrate the above obtained results and discuss the influence of system parameters on system performance indices. Figs 1 and 2 shows how the mean queue length and mean sojourn time respectively changes with the vacation service rate η at different values of ϕ. Fig.3(a) illustrates the comparison of mean queue length in our model with $M/M/1/MWV + VI$ (Li and Tian, 2007). Fig.3(b) demonstrates the effect of the mean vacation time on the mean sojourn time and presents the comparison of the mean sojourn time in our model with two different vacation policies i.e the single vacation (SV) and the single working vacation (SWV). The main findings in this study are itemized as

- From fig.1(a), evidently as we increase the value of η, the mean queue length decreases. Also, we find that the mean queue length depends on the vacation rate ϕ i.e when η is fixed and if ϕ is smaller, E(L) is bigger. Meanwhile, the vacation rate ϕ has a small effect on the mean queue length, when the vacation rate ϕ increases to a certain value. Furthermore, Fig.1(b) indicates that if ρ is larger, the mean queue length E(L) is also larger and this is reasonable in practice. Hence the traffic intensity ρ also has some effects on E(L). Fig.2 illustrates that the mean sojourn time E(S) decreases along with the increase of the η and it also decreases if ϕ is bigger.

- From 3(a), our model gives significantly a better performance in terms of E(L). For example, obviously, when the vacation service rate η are the same in two situations, expected queue length in M/M/1 with SWV and VI is smaller than that in M/M/1 with MWV and VI and the later causes more jobs or customers to wait. Hence, we can achieve a better service under working vacation if we consider vacation interruption policy that utilizes the server and decreases the waiting time effectively.

- From fig.3(b), the E(S) increases with an increase of mean vacation rate ϕ^{-1} and E(S) reaches to a fixed value, When ϕ^{-1} tends to zero i.e our system is reduced to a classical M/M/1 queue. Further, the SWV + VI policy gives a better performance as compared to the SV policy and the SWV policy. This is illustrated by the following: higher is the probability that vacation interruption occurs if the mean vacation time is longer. In other words, the server will come back more frequently to a normal working level if the mean vacation time is longer. As a result, the server serves more customers at a normal service rate.
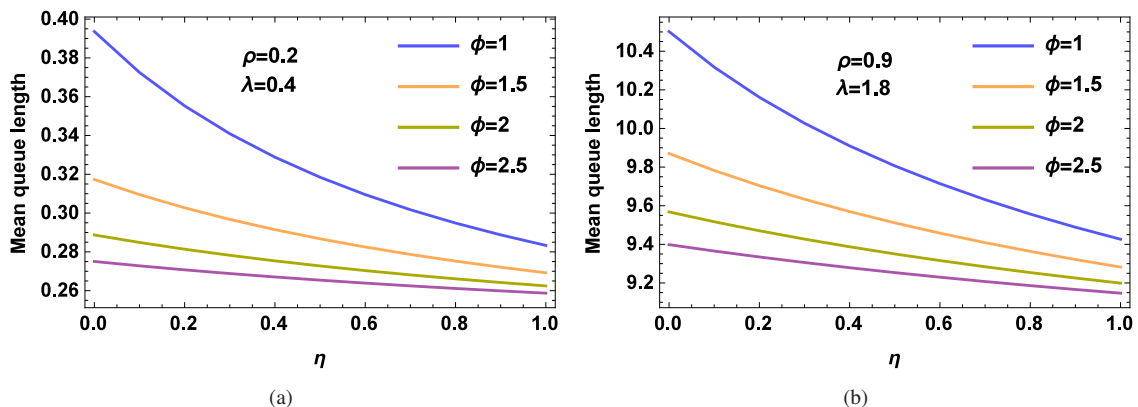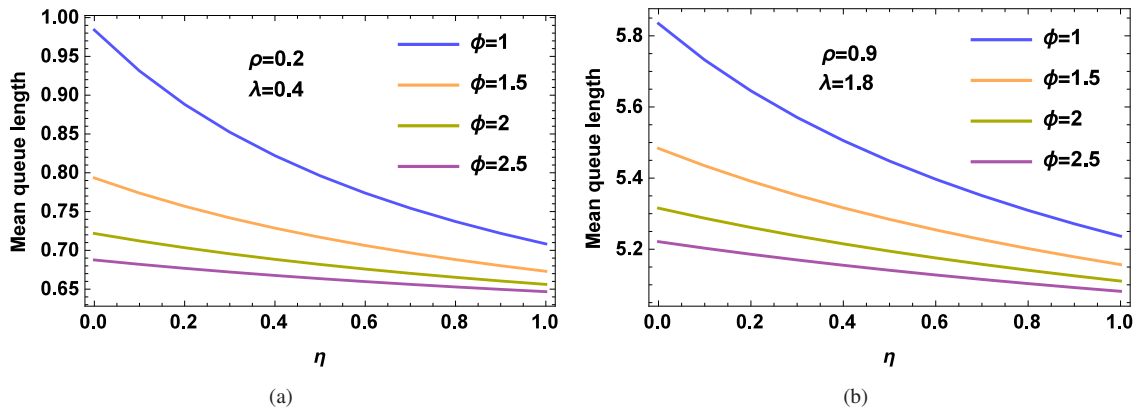

Figure 1: The relation of E(L) with η
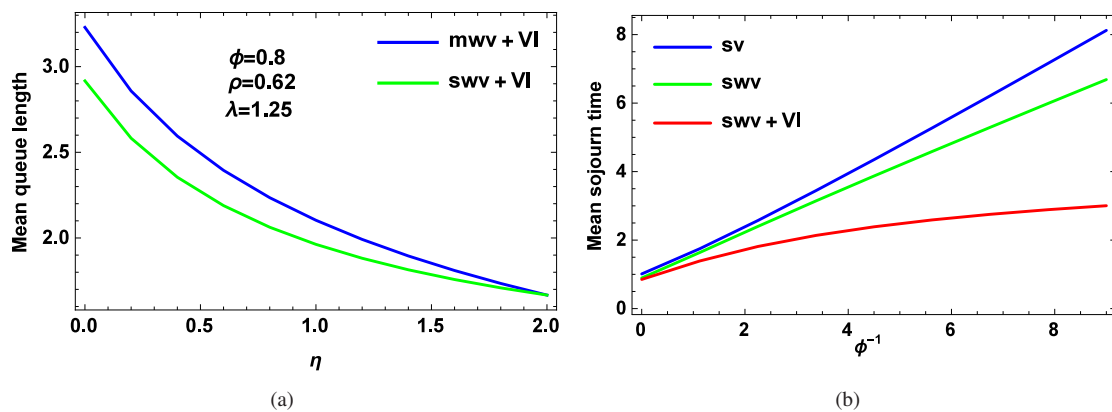
7

Figure 2: The relation of E(S) with η

Figure 3: Comparisons among different models
.

References

Doshi,B.T., 1986. Queueing systems with vacations-A survey, Queueing Syst. 1, 29-66.

Takagi,H., 1991. Queueing Analysis: A Foundation of Performance Evaluation. Vol. 1: Vacation and Priority Systems, Part1, North-Holland Elsevier, New York.

Tian,N.,Zhang,Z.G., 2006. Vacation Queueing Models-theory and Application. Springer, New York.

Servi,L.D.,Finn, S.G., 2002. M/M/1 queues with working vacation (M/M/1/WV). Perform. Eval. 50, 41-52.

Liu, W.,Xu,X.,Tian,N.,2007. Stochastic decompositions in the M/M/1 queue with working vacations. Oper. Res. Lett. 35, 595-600.

Tian,N.,Zhao,X., 2008. The M/M/1 queue with single working vacation. Int. J. Inf. Manage. Sci. 19(4), 621-634.

Kim, J.D.,Choi, D.W.,Chae,K.C., 2003. Analysis of queue-length distribution of the M/G/1 queue with working vacation. Hawaii International Conference on Statistics and Related Fields.

Wu, D.,Takagi,H., 2006. M/G/1 queue with multiple working vacations. Perform. Eval. 63, 654-681.

Li,J.,Tian, N.,Zhang,Z.G.,Luh,H.P, 2011. Analysis of the M/G/1 queue with exponentially working vacationsa matrix analytic approach. Queueing Syst. 61, 139-166.

Baba,Y., 2005. Analysis of a GI/M/1 queue with multiple working vacations. Oper. Res. Lett. 33, 201-209.

Li,J.,Tian, N., 2011. Performance analysis of an GI/M/1 queue with single working vacation. Appl. Math. Comput. 217, 4960-4971.

Li, J., Tian, N., 2007. The M/M/1 queue with working vacations and vacation interruptions. Journal of System Sciences and Systems Engineering 16(1), 121-127.

Li, J., Tian, N., 2007. The discrete-time GI/Geo/1 queue with working vacations and vacation interruption. Applied Mathematics and Computation 185(1), 1-10.

Li, J., Tian, N., Ma, Z., 2008. Performance analysis of GI/M/1 queue with working vacations and vacation interruption. Applied Mathematical Modelling 32(12), 2715-30.

Zhang, M., Hou, Z., 2010. Performance analysis of M/G/1 queue with working vacations and vacation interruption. Journal of Computational and Applied Mathematics 234(10), 2977-85.

Gao, S., Wang, J., Li, WW., 2014. An M/G/1 retrial queue with general retrial times, working vacations and vacation interruption. Asia-Pacific Journal of Operational Research 31(2), article no. 1440006.

Zhang, M., Hou, Z., 2011. Performance analysis of MAP/G/1 queue with working vacations and vacation interruption. Applied Mathematical Modelling 35(4), 1551-60.

Laxmi, PV., Jyothsna, K., 2015. Impatient customer queue with Bernoulli schedule vacation interruption. Computers and Operations Research 56, 1-7.

Neuts,M., 1981. Matrix-Geometric Solution in Stochastic Model, Hopkins University Press, Baltimore.

Keilson,J.,Sevi,L.D., 1988. A distribution form of Littles law, Oper. Res. Lett. 7(5), 223-227.

9