Some Vertex Prime Graphs And A New Type Of Graph Labeling.

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Abstract

In this paper we investigate some new families of vertex prime graphs.We show that the graphs kayak paddle KP(k,m,l) , book graph $\theta(Cm)^n$,*Irregular book graph $\theta(C)^n$,C_3 snake $S(C_3,m)$, *m-fold triangular snake $S(C_3,m,n)$, sunflower graph SF(1,n),*m-fold-petal sunflower graph SF(m,n),$(C_n)^k$ and *One point union of k cycles not of equal length $(C)^k$ has Vertex Prime label. The * indicates new families are discussed and shown tobe vertex prime. We introduce **a new type of graph labeling called as L-cordial labeling** and show that $K_{1,n}$,path P_n, C_n, $S(C_3,m)$ are families of L-cordial graphs .

Key words: Vertex Prime labeling,L-cordial labeling,path,cycle.

AMS subject classification number (2010) :05C78

1.Introduction

All graphs considered here are simple ,finite,connected ,undirected.A graph G(V,E) has **vertex prime labeling** if it's edges can be labeled with distinct integers 1,2,3..|E| ie a function f:E→{1,2,..|E|} defined such that for each vertex with degree at least 2 the the greatest common divisor of the labels on it's incident edges is 1 [4].The edge labels are the actual images under function f:E(G)→{1,2,….|E|} Deretsky,Lee and Mitchen [4] shows that the forests,all connected graphs,$C_{2k}UC_n$,$5C_{2m}$ have vertex prime labeling.The graph with exactly two components one of which is not an odd cycle has a vertex prime labeling.They also show that a 2-regular graph with at least two odd cycles has no vertex prime labeling.The graph admitting vertex prime labeling is called as verterx prime graph.

We introduce a new type of graph labeling called as L-cordial labeling and show that $K_{1,n}$,path P_n, C_n,$S(C_3,n)$ are families of L-cordial graphs .

We refer for terminology and symbols J.F.Harary [7] and Dynamic survey of graph labeling by Galian J.A.[5]

In this paper main results follows after some definations.

1.1 Definition: A book graph $\theta(C_m)^n$ is made from m copies of cycle C_m that share an edge in common. It is n page book with each page as a m-gone.

1.2 Definition : Irregular book graph $\theta(C)^n$ is a book on n pages such that not all cycles are identical polygons.

1.3 Definition: A kayak padle G = KP(k,m,t) is a graph obtained by joining cycle C_k and cycle C_m by a path of length t

1.4 Definition :A graph $S(C_3,m)$ is a snake of length m on C_3 . It is obtained from a path p_m=($v_1,v_2,..v_{m+1}$) by joining vertices v_i and v_{i+1} to new vertex w_i (i= 1,2,…m) giving edges q_i =(w_iv_i) and edge q_i'=(w_iv_{i+1}).

1.5 Defination: A m-fold triangular snake $S(C_3,m,n)$ of length n is obtained from a path $v_1,v_2,\ldots v_n,v_{n+1}$ by joining v_i and v_{i+1} to new m vertices $w^i_1, wi_2, ..w^i_m$, i = 1,2…n giving edges $(v_i w^i_j)$ and e^i_j'$= (w^i_j v_{i+1})$,j=1...m. i= 1,2,..n

1.6 Definition :A unifold-petal sunflower graph SF(1,n) is a sunflower graph SF(n) obtained by a n-cycle $(v_1,v_2,..v_n,v_1)$ and creating new vertices $w_1,w_2,..w_n$ with new edges (w_iv_i) and (w_iv_{i+1}).It also can be obtained from a C_3 snake $S(C_3,n)$ by identifying end points v_1 and v_{n+1} of path P_{n+1}.

1.7 Definition :A m-fold-petel sunflowere graph SF(m,n) is obtained from a cycle $v_1,v_2,\ldots v_n,v_1$ by joining v_i and v_{i+1} to new m vertices $w^i_1, wi_2, ..w^i_m$, i = 1,2…n giving edges $e^i_j=(v_iw^i_j)$ and e^i_j'$=(w^i_jv_{i+1})$,i=1...n. ,taken modulo n. j= 1,2,..m.

1.8 Definition :$(C_n)^k$ is a graph obtained by taking one point union of k copies of cycle C_n.

1.9 Definition :$(C)^k = (C_{r1r2\ldots rk})^k$ is a graph obtained by taking one point union of k cycles of lengths $r_1,r_2,\ldots r_k$ not all same.

2.Main Results proved :

2.1 Theorem :A book graph $\theta(Cm)^n$ is a vertex prime .

Proof: A book graph G = $\theta(C_m)^n$ has n(m-1)+1 edges.The n copies of cycle C_m are denoted by $C^1,C^2..C^n$ The consecutive edges on cycle C^k are $e^k_1 =(v^1_1v^1_2)$. (it is common to all cycles and is denoted by e_1) , $e^k_{2,..}e^k_m$ Where k = 1,2,..n.and vertex v^i_j is the j^{th} vertex on i^{th} cycle .The edge $e^i_j =(v^i_jv^i_{j+1})$ is the j^{th} edge on i^{th} cycle , i= 1,2..n and j = 1,2,..m. The edge e_1 is adjacent to e^k_2 and e^k_m ,k = 1,2,..n. Define $f :E(G)\rightarrow \{1,2,\ldots|E|\}$ such that

$f(e^1_x)= x$, x= 1,2…m.

$f(e^2_x)=m+x-1$, x= 2,3,4…m.

$f(e^k_x) = m+(m-1)(k-2)+x-1$; k=3,4,..n and x = 2,3,..m.

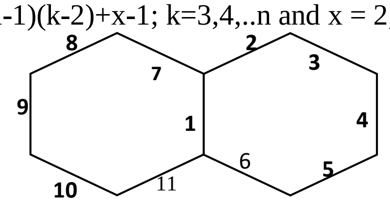
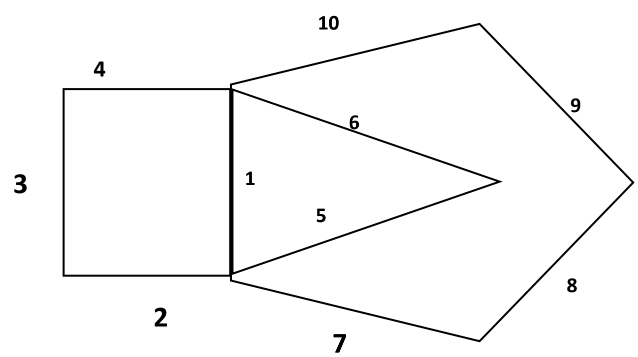


fig.2.1 $\theta(C_6)^2$

For any vertex the incident edges have label numbers that are consecutive positive integers or one of the incident edge is of label 1. Thus graph G is vertex prime.

2.2 Theorem: Irregular book graph G= $\theta(C)^n$ or G = $\Theta(C_{r1,r2,..rn})^n$ has vertex prime labeling.

Fig2.2 $\Theta(C_{3,4,5})^3$



2

Proof: The different n cycles are $C_{r1},C_{r2}..C_{rn}$.These cycles are also refered as $1^{st},2^{nd},\ldots n^{th}$ cycle on edges $r_1,r_2,..r_n$ respectively. The edge common to all cycles is e_1 . e^k_x be the x^{th} edge on k^{th} cycle where $x = 2,3,..r_k$ and $k = 1,2,3,..n$.

Define $f : E(G) \rightarrow \{1,2,\ldots|E|\}$ as

$f(e^k_x) \quad = x \qquad$ for $k = 1$ and $x = 1,2,\ldots r_1$

$\qquad = \sum_{t=1}^{k-1} r_t - k + 1 + x \quad$ for $k=2,3..n$ and $x=2,3\ldots r_k$.

It follows that for any vertex on any cycle $C_{r1},C_{r2},..C_{rn}$ the incident edges have label numbers that are consecutive positive integers or one of the incident edge is of label 1. As such the graph $\theta(C)^n = \Theta(C_{r1,r2,..rn})^n$ has vertex prime labeling.

2.3 Theorem :A kayak paddle G= KP(k,m,t) is vertex prime.

Proof: A Kayak graph G= KP(k,m,t) has vertex set $V(G)= \{v_1,v_2..v_k,u_1,u_2,\ldots u_{t-1},w_1,w_2,\ldots w_m\}$where the cycle $C_k = (v_1,v_2..v_k,v_1)$and path of length $t = P_{t+1} = (v_1,u_1,u_2,\ldots u_{t-1},w_1)$and $C_m=(w_1,w_2,\ldots w_m,w_1)$The edge set $E(G) = \{e_i=(v_iv_{i+1}),\ i =1,2..k,$where $k+1$ is taken as $1\}$U $\{p_1=(v_1u_1),\ p_t=(u_lw_1)$ U$\{p_{i+1}=(u_iu_{i+1}),\ i=1,2..,t-2.\}$ U$\{e_i'=(w_iw_{i+1}),\ i=1,2,,m,v_{m+1}$ is taken as $v_1\}$

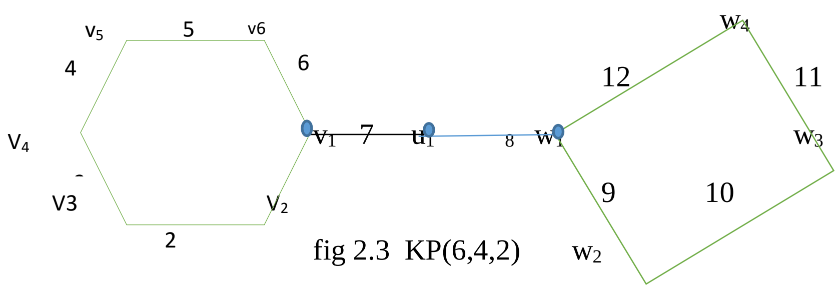


fig 2.3 KP(6,4,2)

Define a function $f:E(G)\rightarrow\{1,2,..|E|\}$ as

$f(e_i)=i$ for $i = 1..k$,$f(p_1)= k+1,f(p_i) =k+i;\ i = 1\ldots(t-1);\quad i=1,2..,t-1$.

$f(p_t) = k+t$;

$f(e_i') = k+t+i$,$i=1..m\}$This gives every vertex is incident with at least two edges whose labels are consecutive positive integers. Therefore G is vertex prime.

2.4 Theorem: A C_3 snake G = $S(C_3,m)$ is vertex prime.

Proof: Define $f:E(G)\rightarrow\{1,2,..|E|\}$ as

$f(e_i)=i$ for $e_i =(v_iv_{i+1})$ $i= 1,2,\ldots,m$

$f(e_i')=m+1+2(m-i),\ \ i= 1,2,..m,$

$f(e_i)= f(e_i')+1\ \ , i= 1,2..m$

As each vertex is incident with edges whose label are consecutive intigers ,the resultant labeling is vertex prime .

2.5 Theorem: A sunflower graph SF(n) $n\geq 3$ has vertex prime labeling.

Proof. We first obtain the vertex prime labeling of S(C3,n) as stated in theorem 1.4.By identifying the vertex v_1 with vertex v_{n+1} will give the required labeling of SF(n)

2.6 Theortem : A m-fold triangular snake $S(C_3,m,n)$ is vertex prime.

Proof : Define $f:E(G)\rightarrow\{1,2,..|E|\}$ as follows,

$f(v_iv_{i+1})=i, i=1,2..n. f(e^i_j{}') = n+2n(j-1)+2(n-i)+1; j = 1,2..m$

$f(e^i_j{}') = f(e^i_j{}')+1$

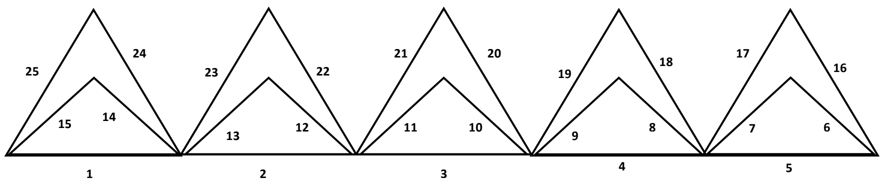


diagram 2.4 $S(C_3,2,5)$

It follows that $S(C_3,m,n)$ is vertex prime.

2.7 Theorem: A m-fold-petel sunflowere graph SF(m,n) is vertex prime.

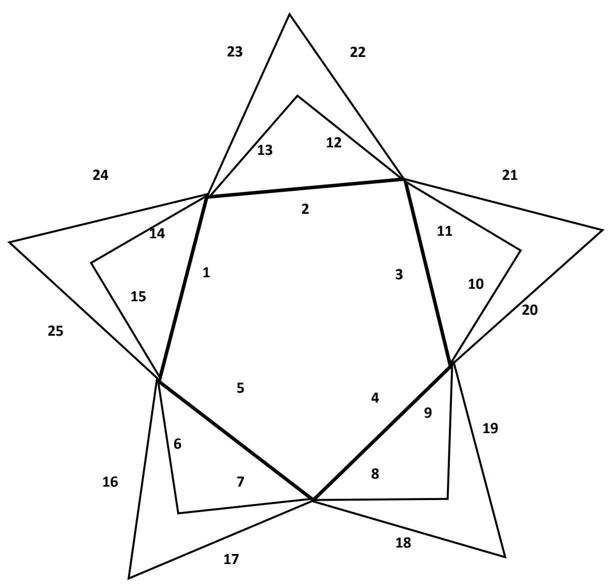


Fig 2.5 2-fold sunflower graph SF(2,5)

Proof :A m-fold-petel sunflowere graph SF(m,n) is obtained from $S(C_3,m,n)$ by identifying vertex v_1 and v_n of S(C3,m,n).Identifying the these two vertices do not have any effect on vertex prime labeling.The resultant graph is vertex prime.

2.8 Theorem: The graph $G = (C_{r1r2\ldots rk})^k$ is vertex prime.

Proof: Let the k cycles be C^j, j=1,2..k. with length $r_1, r_2, .. r_k$ respectively.The vertex common to all cycles be v_1 and $e^j_i = (v^j_i \, v^j_{i+1})$ i = 1,2..n.

Define f:E(G)→{1,2,..|E|} as follows,

$f(e^j_i) = r_1 + r_2 + \, . \, .. r_{(j-1)} + i$ for j = 1..k

This labeling produces at least two edges on any vertx with label as consecutive natural numbers .Resultant graph is vertex prime.

If we take $r_1 = r_2 = r_3 = \ldots.. = r_k$ we get all cycles of same length say,m,The resultant graph is $(C_m)^k$

3. Future challenges in vertyex prime labelings.:

3.1 Definition A block - cutpoint graph of a graph G is a bipartite graph in which one partite set consists of the cut vertices of G and the other has a vertex bi for each block Bi of G.

3.2 Definition A triangular cactus is a connected graph all of whose blocks are triangles.On similar lines one can define n-gonal cactus as a graph in which all of it's blocks are n-gone.

We define n-gonal snake on the same lines as the triangular snakes.

3.3 We observe that n-gonal snake is vertex prime .Obtain the particular labeling to this effect.

3.4 A n-gonal cactus is vertex prime. Obtain the particular labeling to this effect.

4. We define a new type of labeling : L-cordial labeling of a graph.

A graph G(V,E) has a L-cordial labeling if there is a bijective function f:E(G)→{1,2,…|E|} . This induces the vertex label as 0 if among all the labels on the incident edges the biggest label is an even number and 1 otherwise . Further the condition is satisfied that $v_f(0)$ the number of vertices labeled with 0 and $v_f(1)$ the number of vertices labeled with1 follows the condition that $|v_f(1) - v_f(0)| \leq 1$. Here isolated vertices are not considered for labeling. A graph which admits L-cordial labeling is called as L-cordial graph.

4.1 Theorem: $K_{1,n}$ is L-cordial iff n is even.

Proof : Label the pendent edges in $K_{1,n}$ as 1,2,3..n

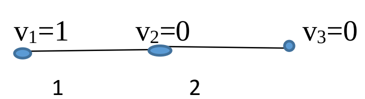
Case : n =2m m= 0,1,2... we have m edges with odd label and m edges with even label producing m vertices with label 0 and m vertices with label 1.The vertex with n degree will receive label 0. Thus $|v(0) - v(1)| \leq 1$.

Case : n = 2m + 1.(m = 0,1,2..)There will be m+1 edges with odd label number and m edges with even number as label. This will giving m+1 vertices with label 1 and m vertices with label 0. Since the biggest edge label at the n degree vertex is odd number the vertex label will be 1.The resultant labeling gives v(0) =m and v(1) = m+2 . which is not L- cordial labeling.

3.2 Theorem : A path G = P_n is L-cordial.n≥3

Proof: case n=2 there is no L-cordial labeling of P_2.

Case n=3



case $n = 2m + 1$, $m = 1,2,3...$

Define $f:E(G)\rightarrow\{1,2,..|E|\}$ as follows:

$f(e_1)=1, f(e_2)=3, f(e_3)=2, f(e_i)=i$ for $i\geq 4 | v_f(0)= m$, $v_f(1)=m+1$

Case $n = 2m, m = 1,2,\ldots$: Define $f:E(G)\rightarrow\{1,2,..|E|\}$ as follows:

$f(e_1)=2, f(e_2)=1, f(e_3)=3,$

$f(e_i)=i$ for $i\geq 4 | v_f(0)= m$, $v_f(1)=m$

The resultant labeling is L-cordial labeling.

4.3 Theorem: Cycle C_n is L-cordial.

Proof: Let the cycle be $C_n=(v_1,v_2,\ldots v_n,v_1)$ and any edge $e_i = (v_i v_{i+1})$, $i= 1,2..n$ (n+1 taken modulo n)

case 1 : n is even.

Define $f:E(G)\rightarrow\{1,2,\ldots E\}$ as follows,

$f(e_1) =1, f(e_2)=3, f(e_3) = 2, f(e_i)=i$ for $i = 4,5..n$.

The edge with label number n is adjacent to the edge with label 1. $v_f(0)= v_f(1) = n/2$. For odd cycle ($n=2m+1$) Redefine the function as $f(e_i)=i$ for $i = 1,2,3....n$. gives $v_f(0) + 1= v_f(1)$#

4.4 Theorem: $S(C_3,m)$ is L-cordial.

Proof: Define $f:E(G)\rightarrow\{1,2,\ldots|E|\}$ as

$f(v_i v_{i+1})= 3(i-1)+1$ $i=1,2..m$.

$f(w_j v_j)= 2 +(j-1)3$; $j=1,2,..m$

$f(w_j v_{j+1})= 3j$ $j= 1,2\ldots m$ When n is odd $v_f(1) = v_f(0) +1$ and when n is even $v(0)= v(1) + 1$. Thus $S(C_3,n)$ is L-cordial.

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