Infra Generalized b-Closed Sets in Infra Topological Space

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Abstract: *In this paper we introduce and investigate a new class of generalized closed sets in infra topological spaces are called infra generalized b-closed set. Furthermore we introduce and study the characterization of infra generalized-closed set, infra generalized semi- closed set, infra semi generalized-closed set, infra generalized pre- closed set, infra generalized α - closed set, infra α generalized-closed set, infra generalized - b closed set, infra weakly generalized- closed set, infra generalized β - closed set, infra generalized α b-closed set, infra semi generalized b - closed set, infra semi weakly generalized- closed set.*

Keywords: *IG-closed set, IGS-closed set, ISG-closed set, IGP-closed set, IGα-closed set, IαG-closed set, IGB-closed set, IWG-closed set, IG β-closed set, IG α B-closed set, ISGB-closed set, ISWG-closed set.*

1. INTRODUCTION

Adel.M.AL.Odhari[1] derived the concept of infra topological space. In 1970, Levine [12] introduced the concept of generalized closed sets which formed a strong tool in the characterization of topological spaces. In 1996, D.Andrijevic'[2] introduced and studied the class of generalized open sets in a topological space called b-open sets .This class of sets contained in the class of β -open sets [3], and contains all semi-open sets [13], and all pre-open sets[20]. During recent years' different types of generalized closed sets were introduced and studied by many topologist [8,9,10,12,13,15,16,17]. Nagaveni[15] introduced weakly generalized closed sets and investigated the properties of these sets. Bhattacharyya & Lahiri [7] introduced semi generalized closed set and Arya &Nour [6] (1990) introduced generalized semi closed set in Topological space.

In this paper we introduce many new concepts of generalized infra topological sets and its relationship with other existing sets are discussed in section 3. The section 4 deals with the properties if infra generalized closed sets are dealt with in that section.

2. PRELIMINERIES

Definition 2.1: Let X be any arbitrary set. An Infra – topological space on X is a collection τ_{iX} subsets of X such that the following axioms are satisfying:

Ax-1: ϕ , X $\epsilon\tau_{iX}$.

Ax-2:The intersection of the elements of any sub collection of τ_{iX} in X

i.e) If $O_i \epsilon\tau_{iX}$,$1 \leq i \leq n \rightarrow \cap O_i \epsilon\tau_{iX}$.

Terminology, the order pair (X,τ_{iX}) is called infra-topological space. We simply say X is an infra space.

Definition 2.2: Let (X,τ_{iX}) be an infra-topological space and A$\subset X$. A is called an infra open set (IOS) if A $\epsilon\tau_{iX}$.

Definition 2.3: Let (X,τ_{iX}) be an infra topological space. A subset C $\subset X$ is called infra-closed set (ICS) in X if X \C is infra-open set in X.

(i.e) C is infra-closed set (ICS) iff X \C $\epsilon\tau_{iX}$.

Definition 2.4: Let (X,τ_{iX}) be an infra topological space and A$\subset X$. The **Infra Closure Point (ICP)** of A is a set denoted by icp(A) and

given by : icp(A) = $\cap \{C_i : A \subseteq C_i, X - C_i \epsilon\tau_{iX}\}$

(i.e) icp(A) is the intersection of all infra closed set containing the set A.

Definition 2.5: Let (X,τ_{iX}) be an infra topological space and A$\subset X$. The **Infra Interior Point (IIP)** of A is a set denoted by iip(A) and given by: iip(A) = $\cup \{O_i : O_i \subseteq A, O_i \epsilon\tau_{iX}\}$

(i.e) iip(A) is the union of all infra open set contained in the set A.

Definition 2.6: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi-open if A

$\subseteq icp\ (iip\ (A))$and infra semi- closed set if $iip\ (icp\ (A)) \subseteq$ A.

Definition 2.7: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra pre-open if A $\subseteq iip\ (icp\ (A))$ and infra pre-closed set if $icp\ (iip\ (A)) \subseteq A$.

Definition 2.8: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra α-open if

A $\subseteq\ iip\ (icp(iip(A)))$and infra α - closed set if $icp\ (iip\ (icp(A))) \subseteq A$.

Definition 2.9: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra b-open if

A $\subseteq iip(icp\ (A)) \cup icp\ (iip\ (A))$ *and* infra b-closed set if $iip(icp\ (A)) \cup icp\ (iip\ (A)) \subseteq A$.

Definition 2.10: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra β -open if A $\subseteq icp\ (iip\ (icp(A)))$ *and* infra β - closed set if $iip(icp\ (iip(A))) \subseteq A$.

3. INFRA GENERALIZED b-CLOSED SETS IN INFRA TOPOLOGICAL SPACE

In this section, we form the new concept of infra generalized closed sets such as, infra generalized semi- closed set, infra semi generalized-closed set, infra generalized pre- closed set, infra generalized α - closed set, infra α generalized-closed set, infra generalized - b closed set, infra weakly generalized-closed set, infra generalized α b- closed set, infra semi generalized b - closed set, infra semi weakly generalized- closed set, infra generalized semi-pre closed set (or) infra generalized β – closed in an infra topological space and study some of its properties.

Definition 3.1: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized- closed set if (IG-Closed set) if icp(A) ⊆ U whenever A⊆U and U is infra open.

The complement of an infra generalized- closed set is an infra generalized –open set.

Definition 3.2: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized semi-closed set if (IGS-Closed set) if sicp (A) ⊆ U whenever A⊆U and U is infra open.

The complement of an infra generalized semi-closed set is an infra generalized semi–open set.

Definition 3.3: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi generalized closed if (ISG -Closed set) if sicp (A) ⊆ U whenever A⊆U and U is infra semi-open.

The complement of an infra semi generalized closed set is an infra semi generalized open set.

Definition 3.4: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized pre-closed set if (IGP-Closed set) if picp (A) ⊆ U whenever A⊆U and U is infra open.

The complement of an infra generalized pre- closed set is an infra generalized pre–open set.

Definition 3.5: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized α- closed if (IGα -Closed set) if αicp (A) ⊆ U whenever A⊆U and U is infra α-open.

The complement of an infra generalized α - closed set is an infra generalized α –open set.

Definition 3.6: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra α generalized closed if (IαG-Closed set) if αicp (A) ⊆ U whenever A⊆U and U is infra -open.

The complement of an infra α generalized closed set is an infra α generalized open set.

Definition 3.7: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized b- closed set (IGB- closed set) if $b$icp (A) ⊆ U whenever A⊆U and U is infra open.

The complement of an infra generalized b - closed set is an infra generalized b –open set.

Definition 3.8: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra weakly generalized closed if (IWG-Closed set) if icp (iip (A)) ⊆ U whenever A⊆U and U is infra -open.

The complement of an infra weakly generalized closed set is an infra weakly generalized open set.

Definition 3.9: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized β - closed set if (IG β- closed set) if βicp (A) ⊆ U whenever A⊆U and U is infra open.

The complement of an infra generalized β- closed set is an infra generalized β–open set.

Definition 3.10: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra generalized α b-closed if (IGαB-Closed set) if bicp (A) ⊆ U whenever A⊆U and U is infra α-open.

The complement of an infra generalized α b - closed set is an infra generalized αb –open set.

Definition 3.11: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi generalized b-closed if (ISGB-Closed set) if bicp (A) ⊆ U whenever A⊆U and U is infra semi-open.

The complement of an infra semi generalized b - closed set is an infra semi generalized b-open set.

Definition 3.12: Let (X,τ_{iX}) be an infra topological space. A set 'A' is called infra semi weakly generalized closed if (ISWG-Closed set) if icp (iip (A)) ⊆ U whenever A⊆U and U is infra semi-open.

The complement of an infra semi weakly generalized closed set is an infra semi weakly generalized open set.

The collection of all infra generalized b-closed (resp. infra generalized closed, infra generalized semi-closed, infra semi generalized closed, infra generalized pre-closed, infra generalized α -closed, infra generalized β–closed, infra weakly generalized closed, infra generalized α b-closed, infra semi generalized b-closed,infra αgeneralized closed, infra semi generalized-closed, infra semi weakly generalized-closed) subset of (X,τ_{iX}) is denoted by IGBCP(X)(resp.IGCP(X),IGSCP(X),ISGCP(X),IGP CP(X),IGαCP(X),IGSPCP(X),IWGCP(X),IG αBCP (X),ISGBCP(X),IαGCP(X), ISWGCP(X)).

And the collection of all infra generalized b-open (resp. infra generalized open, infra generalized semi-open,infra semi generalized open, infra generalized pre-open, infra generalized α -open, infra generalized β –open, infra weakly generalized open,infra generalized α b-open,infra semi generalized b-open,infra α generalized open, infra semi weakly generalized open)subset of (X,τ_{iX}) is denoted by IGBOP(X) (resp. IGSOP(X),ISGOP(X),IGPOP(X),IG α OP(X), IGSPOP(X),IWGOP(X),IG αBOP(X),ISGBOP(X),I α GOP(X), ISWGOP(X)).

Theorem 3.13:

(a)Every Infra closed set is infra generalized semi-closed set (IGS closed set).

(b)Every Infra closed set is infra generalized pre-closed set (IGP closed set).

(c)Every Infra closed set is infra generalized α-closed set (IGα closed set).

(d)Every Infra closed set is infra generalized b-closed set (IGB closed set).

(e)Every Infra closed set is infra generalized β -closed set (IG β closed set).

(f)Every Infra closed set is infra weakly generalized -closed set (IWG- closed set).

(g)Every Infra closed set is infra generalized closed set (IG-closed set).

Proof:

(a)Let A be any infra closed set in (X,τ_{iX}) and A⊆ U, where U is infra open.
Since $sicp\ (A)$ ⊆ icp(A)= A ⊆ U.
Therefore $sicp\ (A)$ ⊆ U. Hence A is infra generalized semi-closed set.

Proof of (b), (c), (d), (e) and (f) are obvious by definition.

But the converse of above theorem need not be true as seen from the following example.

Example 3.14: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi , X , \{b\} , \{c\}\}$.The set $\{a\}$ is infra generalized semi-closed set (resp. infra generalized pre-closed set, infra generalized α -closed set, infra generalized b-closed set, infra generalized β -closed set, infra weakly generalized -closed set, infra generalized closed set) but is not infra closed in (X,τ_{iX}) .

Theorem 3.15:

(a)Every Infra semi-closed set is infra generalized semi-closed set (IGS closed set).

(b)Every Infra pre-closed set is infra generalized pre-closed set (IGP closed set).

(c)Every Infra α-closed set is infra generalized α-closed set (IGα closed set).

(d)Every Infra β-closed set is infra generalized β-closed set (IGβ closed set).

Proof:

(a)Let A be any infra semi-closed set in (X,τ_{iX}) and U be any infra open set containing A. Since A is infra semi-closed, Therefore sicp(A) ⊆ U. Hence A is infra generalized semi-closed set.

Proof of (b), (c) and (d) are obvious by definition.

But the converse of above theorem need not be true as seen from the following example.

Example 3.16: (i) Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{b\}, \{c\}\}$ .The set $\{a\}$ is infra generalized semi-closed set (resp.infra generalized pre-closed set, infra generalized α -closed set) but is not infra semi- closed (Infra pre-closed, Infra α-closed) in (X,τ_{iX}).

(ii) Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{c\}\{a, b\}\}$ .The set $\{a, c\}$ is infra generalized β -closed set but it is not infra β-closed set.

Theorem 3.17: Every infra b-closed set is infra generalized b-closed set.

Proof:

Let A be any infra b-closed set in (X,τ_{iX}) such that U be any infra open set containing A.
Since A is infra b closed, bicp (A)=A. Therefore, bicp (A) ⊆U. Hence A is infra generalized b-closed set.
But the converse of above theorem need not be true as seen from the following example.

Example 3.18: Let X=$\{a, b, c\}$, $\tau_{iX} = \{\phi, X, \{b\}, \{c\}\}$ .Then the family of all infra b-closed set of X is
IBCP(X) = $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}\}$. But the family of all infra generalized b-closed set of X is
IGBCP(X)
= $\{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$. Then it is clear that $\{b, c\}$is infra generalized b-closed set but it is not infra b-closed set.

Theorem 3.19:

Every infra α-closed set is infra generalized β -closed set (IG β closed set).

Proof:

Let A be any infra α - closed set in (X,τ_{iX}) and U be any infra open set containing
A. Since A is infra α –closed, αicp (A) ⊆U.
Therefore $\beta icp(A) \subseteq \alpha$icp(A)⊆U.

Hence A is infra generalized β -closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.20: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{b\}, \{c\}\}$ .The set $\{a\}$ is infra generalized β -closed set but is not infra α -closed in (X,τ_{iX}).

Theorem 3.21:

Every infra b-closed set is infra generalized β -closed set (IG β closed set).

Proof:

Let A be any infra b- closed set in (X,τ_{iX}) and U be any infra open set containing
A. Since A is infra b –closed, bicp (A) ⊆U.
Therefore $\beta icp(A) \subseteq b$icp (A) ⊆U.

Hence A is infra generalized β -closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.22: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{b\}, \{c\}\}$ .The set $\{b, c\}$ is infra generalized β -closed set but is not infra b -closed in (X,τ_{iX}).

Theorem 3.23: Every infra b-closed set is infra semi generalized b-closed set (ISGB-closed set).
Proof: Let A be infra b-closed set in (X,τ_{iX}) and let U be a semi-open set contains A in(X,τ_{iX}). Now A ⊆U and A=bicp (A). Hence, every infra b- closed set is ISGB-closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.24: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{c\}\}$ .The set $\{a, c\}$ is infra semi generalized b-closed set but is not infra b-closed in (X,τ_{iX}).

Theorem 3.25:

(a) Every Infra generalized closed set is infra generalized b-closed set (IGB closed set).

(b)Every Infra generalized closed set is infra generalized semi-closed set (IGS closed set).

(c)Every Infra generalized closed set is infra generalized pre-closed set (IGP closed set).

(d)Every Infra generalized closed set is infraα generalized-closed set (IGα closed set).

(e)Every Infra generalized closed set is infra generalized β -closed set (IG β closed set).

Proof:(a)Let U be infra open in (X,τ_{iX}) such that A⊆ U. By hypothesis, A is an infra generalized closed set then icp(A) ⊆ U. This implies that $b$icp(A) ⊆ U. Therefore, A is infra generalized b-closed set.

Proof of (b), (c), (d) and (e) are obvious by definition.

But the converse of above theorem need not be true as seen from the following example.

Example 3.26: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}\}$ .The set $\{a\}$ is infra generalized b-closed set (resp. infra generalized semi-closed set, infra αgeneralized -closed set, infra generalized β -closed set) but is not infra generalized closed in (X,τ_{iX}).

Example 3.27: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{a, c\}\}$ .The set $\{c\}$ is infra generalized per-closed set but is not infra generalized closed in (X,τ_{iX}).

Theorem 3.28:

(a) Every Infra generalized semi-closed set is infra generalized b-closed set (IGB closed set).

(b)Every Infra α generalized closed set is infra generalized semi-closed set (IGS closed set).

(c)Every Infra α generalized closed set is infra generalized b-closed set (IGB closed set).

(d)Every Infra semi generalized closed set is infra generalized b-closed set (IGB closed set).

(e)Every Infra semi generalized closed set is infra generalized semi-closed set (IGS closed set).

(f)Every Infra generalized α-closed set is infra generalized b-closed set (IGB closed set).

Proof:

(a)Let U be infra open in (X,τ_{iX}) such that A⊆ U. By assumption, A is an infra generalized semi closed set then sicp (A) ⊆ U. This implies that $b$icp (A) ⊆ U. Therefore, A is infra generalized b-closed set.

Proof of (b), (c), (d), (e) and (f), are obvious by definition.

But the converse of above theorem need not be true as seen from the following example.

Example 3.29 :(i)Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{c\}\}$.

The set $\{a\}$ is in infra generalized semi -closed set but is not infra α generalized -closed set and the set $\{b\}$ is in infra generalized semi -closed set but is not infra semi generalized -closed set in (X,τ_{iX}).

(ii) Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$.

The set $\{a\}$ is in infra generalized b-closed set but is not in infra α generalized -closed set, infra generalized semi -closed set, infra semi generalized -closed set, infra generalized α -closed set in (X,τ_{iX}).

Theorem 3.30:

(i)Every Infra generalized α-closed set (IGα closed set) is infra generalized β-closed set (IGβ closed set).

(ii)Every Infra generalized semi-closed set (IGS closed set) is infra generalized β -closed set (IG β closed set).

(iii)Every Infra generalized b-closed set (IGB closed set) is infra generalized β-closed set (IGβ closed set).

Proof:

(i)Suppose A⊆ U and U is infra open in (X,τ_{iX}). By assumption, αicp(A) ⊆ U⟹ βicp(A) ⊆ αicp(A) ⊆ U.

Hence A is infra generalized β -closed set (IG β closed set).

(ii)Let A be Infra generalized semi-closed set (IGS closed set) of (X,τ_{iX}). Suppose A⊆ U and U is infra open in (X,τ_{iX}). sicp(A) ⊆ U⟹ βicp(A)⊆ U. Hence A is infra generalized β -closed set (IG β closed set).

(iii)Suppose A⊆ U and U is infra open in (X,τ_{iX}). By assumption, bicp(A) ⊆ U⟹ βicp(A) ⊆bicp(A) ⊆ U.

Hence A is infra generalized β -closed set (IG β closed set).

But the converse of above theorem need not be true as seen from the following example.

Example 3.31: Let X=$\{a, b, c\}$ with $\tau_{iX} = \{\phi, X, \{b\}, \{c\}, \{a, c\}\}$ .The set $\{a\}$ is infra generalized β -closed set but is not infra generalized α-closed set and infra generalized semi- closed in (X,τ_{iX}).

Remark 3.32:

Finite intersection of infra generalized b-closed set (IGB closed set) need not be infra generalized b-closed set (IGB closed set).

Example 3.33: Let X=$\{a,b,c\}$ and the corresponding infra topological space be

$\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\}$. Let A= $\{a,b\}$,B=$\{b,c\}$then A and B are infra generalized b-closed (IGB closed set) subsets of (X,τ_{iX}). But A⋂B = $\{b\}$ is not a infra generalized b-closed (IGB closed set) subset of (X,τ_{iX}).

Theorem 3.34: Every infra generalized α-closed set is infra semi generalized b (ISGB-closed set) closed set.

Proof: Let A be infra generalized α-closed set then, αicp(A)⊆U whenever A⊆U and U is infra α- open. Since, every $infra\ \alpha$- closed sets are infra b-closed sets

bicp(A)⊆ αicp(A)⊆U and U is infra semi – open. Therefore, A is ISGB-closed.

Hence, every infra generalized α-closed set is infra semi generalized b (ISGB-closed set) closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.35: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{c\}\}$ .The set $\{a\}$ is infra semi generalized b-closed set but is not infra generalized α-closed in (X,τ_{iX}).

Theorem 3.36: If a subset A of infra topological space (X,τ_{iX}) is IG α B-closed set, then it is IGB-closed set.

Proof:

Suppose A is IG α B-closed set in (X,τ_{iX}). Since every infra open set is infra α-open sets, U is infra open set. Therefore, bicp(A) ⊆U. And U is infra open. Thus A is IGB-closed.

But the converse of above theorem need not be true as seen from the following example.

Example 3.37: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\}$. The set $\{a\}$ $is\ IGB$-closed but it is not IG α B-closed.

Theorem 3.38 :(i)If a subset A of infra topological space (X,τ_{iX}) is IG α-closed Set (respectively, IαG) then it is IG α B -closed set.

(ii) If a subset A of infra topological space (X,τ_{iX}) is ISG-closed set, then it is IG α B-closed set.

Proof:

(i) Let A be infra generalized α-closed set. Now bicp(A) ⊆ $icp_{\alpha}(A)$ ⊆U, implies that bicp(A) ⊆U and U is infra α open. Thus, A is IG α B-closed set.

(ii) Let A be infra semi generalized-closed set, sicp(A)⊆U whenever A⊆ U and U is infra semi-open. Since every infra semi-closed set is infra b-closed set, bicp(A) ⊆U, A⊆ U and U is infra α open. Thus, A is IG α B-closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.39: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\}$. The set $\{a\}$ $is\ IG\alpha$-closed, I$\alpha G - closed$ set, ISG-closed set but it is not IG α B-closed.

Remark 3.40: The following example shows that the infra weakly generalized closed sets and infra generalized α b closed sets are independent.

Example 3.41: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\}$ .The set $\{b\}$ is infra generalized α b -closed set but is not infra weakly generalized -closed in (X,τ_{iX}).

Example 3.42: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{a,c\}\}$ .The set $\{a\}$ is infra weakly generalized -closed set but is not infra generalized α b -closed in (X,τ_{iX}).

Theorem 3.43: Every infra semi generalized-closed set is infra semi generalized b (ISGB-closed set) closed set.

Proof: Let A be infra semi generalized-closed set, then sicp (A)⊆U whenever A⊆ U and U is infra semiopen. Since every infra semi-closed set is infra b-closed sets,

bicp(A)⊆sicp(A)⊆U and U is infra semi open. Therefore, A is infra semi generalized b (ISGB-closed set) closed set. Hence, every infra semi generalized-closed set is infra semi generalized b (ISGB-closed set) closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.44: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{c\}\}$ .The set $\{b\}$ is infra semi generalized b-closed set but is not infra semigeneralized -closed in (X,τ_{iX}).

Theorem 3.45: Every infra semi weakly generalized closed set (ISWG-closed set) is infra semi generalized b (ISGB-closed set) closed set.
Proof: Let A be infra semi weakly generalized closed set then, icp(iip(A))⊆U whenever A⊆U and U is infra semi-open. Since, every *infra semi*-closed sets are infra b-closed sets, bicp (A)⊆icp(iip(A)) ⊆U and U is infra semi – open. Therefore, A is ISGB-closed.

Hence, every infra semi weakly generalized -closed set is infra semi generalized b (ISGB-closed set) closed set.

But the converse of above theorem need not be true as seen from the following example.

Example 3.46: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{c\}\}$ .The set $\{b\}$ is infra semi generalized b-closed set but is not infra semi weakly generalized –closed set in (X,τ_{iX}).

Theorem 3.47:
Every infra generalized pre-closed set (IGP-closed set) is infra semi generalized b (ISGB-closed set) closed set.
Proof: Let A be infra generalized pre-closed set, then picp (A)⊆U whenever A⊆ U and U is infra open. Since every infra pre-closed set is infra b-closed sets, bicp(A)⊆picp(A)⊆U and U is infra open. Therefore, A is infra semi generalized b (ISGB-closed set) closed set.
Hence, every infra generalized pre-closed set is infra semi generalized b (ISGB-closed set) closed set.
But the converse of above theorem need not be true as seen from the following example.

Example 3.48: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\}$ .The set $\{a\}$ is infra semi generalized b-closed set but is not infrageneralized pre -closed in (X,τ_{iX}).

Remark 3.49: The following example shows that the infra weakly generalized closed sets and infra semi generalized closed sets are independent.

Example 3.50: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}\}$ .The set $\{a\}$ is infra semi generalized b-closed set but is not infra weakly generalized -closed in (X,τ_{iX}).

Example 3.51: Let X=$\{a,b,c\}$ with $\tau_{iX} = \{\phi, X, \{a\}, \{b\}, \{a,c\}\}$ .The set $\{a\}$ is infra weakly generalized -closed set but is not infra semi generalized b-closed in (X,τ_{iX}).

4. THE CHARACTERISTICS OF INFRA GENERALIZED b-CLOSED SETS IN INFRA TOPOLOGICAL SPACE

Theorem 4.1: Let A be a IGB-closed subset of (X,τ_{iX}).Then bicp(A)-A does not contain any non-empty infra closed sets.

Proof: Let F be an infra closed set in (X,τ_{iX}) such that F ⊆ bicp(A)-A. Since X – F is infra open , A ⊆X – F and A is IGB-closed set, it follows that bicp(A) ⊆ X – F . Hence F ⊆X – bicp(A).

This implies that F ⊆ (X – bicp(A))∩ (bicp(A)-A)=∅ and hence F = ∅.

Corollary 4.2: Let A be IGB-closed set. Then A is infra b-closed set if and only if bicp(A) – A is infra closed.

Proof: Let A be IGB-closed set. If A is infra b-closed, then we have bicp(A) – A = ∅ which is infra closed set.

Conversely, let bicp(A) – A be closed. Then by theorem(4.1),bicp(A) – A does not contain any non-empty infra closed subset and since bicp(A) – A is closed subset of itself, then bicp(A) – A= ∅. This implies that A=bicp(A) and so A is infra b-closed set.

Corollary 4.3: Let A be infra open and IGB-closed set. Then A ∩ F is IGB-closed whenever F ⊆bicp(A).

Proof: Since A is IGB-closed and infra open, then bicp(A) ⊆ A and thus A is infra b-closed set . Hence A∩ F is infra b-closed in X which implies that A ∩ F is IGB-closed in X.

Theorem 4.4: If A is a IGB-closed set and B is any set such that A ⊆ B ⊆bicp(A) then B is a IGB-closed set.

Proof: Let B ⊆ U where U is infra open set. Since A is IGB-closed set and A⊆ U, then bicp(A) ⊆ U and bicp(A) = bicp(B) . Therefore bicp(B) ⊆ U and hence B is a IGB-closed set.

Theorem 4.5: A subset A ⊆ X is IGB-open if and only if F ⊆bicp(A) whenever F is infra closed set and F ⊆ A.

Proof: Necessity: Let A be a IGB –open set and Suppose F ⊆ A where F is infra closed. Then X – A is a IGB –closed set contained in the infra open set X – F.

Hence bicp(X – A) ⊆ X – F and X – biip(A) ⊆ X – F. Thus F ⊆biip(A).

Sufficiency: Suppose F is an infra closed set with F ⊆biip(A) and F ⊆ A , then X – biip(A) ⊆ X – F. Thus bicp(X - A) ⊆ X – F. Hence X – A is a IGB-closed set and A is a IGB-open set.

Theorem 4.6:

If A is infra open and IGB-closed set, then A is infra b-closed set.

Proof:

Let A be infra open and IGB-closed. Then bicp(A) ⊆ A. But always A⊆ bicp(A). Therefore, A = bicp(A). Hence A is infra b-closed set.

Theorem 4.7:

If biip(A) ⊆ B ⊆ A and A is IGB-open then B is IGB-open.

Proof:

Given biip(A) ⊆ B ⊆ A implies X – A ⊆ X – B ⊆ X – biip(A). Then X – A ⊆ X – B ⊆ X – bicp(X - A). Since X – A is IGB-closed, by theorem (4.4), X – B is IGB-closed and hence B is IGB-open.

Theorem 4.8:

Let B ⊆ A ⊆ X, where A is a IGB-closed set and infra open set. Then B is IGB-closed relative to Aif and only if B is IGB-closed set in (X,τ_{iX}).

Proof:

We first note that since B ⊆ A and A is both a IGB-closed and infra open set, then bicp(A) ⊆ A and thus bicp(B) ⊆ bicp((A) ⊆ A. Now from the fact that A ∩bicp(B) = bicp $_A$ (B), we have bicp(B) = bicp $_A$ (B) ⊆ A. If B is IGB-closed set relative to A and U is infra open subset of (X,τ_{iX}) such that B ⊆ U, then B=B ∩ A ⊆ U ∩ A, where U ∩ A is infra open in A. Hence as B is IGB-closed relative to A, bicp (A) = bicp $_A$ (B) ⊆ U ∩ A ⊆U .Therefore B is IGB-closed set in (X,τ_{iX}).

Conversely, if B is IGB-closed in (X,τ_{iX}) and U is infra open subset of A such that B ⊆ U, then U = V ∩ A for some infra open subset V of (X,τ_{iX}). As B ⊆ V and B is IGB-closed in (X,τ_{iX}), bicp (B) ⊆ V. Thus bicp $_A$ (B) =bicp (B) ∩A ⊆ V ∩ A = U. Therefore, B is IGB-closed relative to A.

Theorem 4.9: Let A be IG αB-closed subset of (X,τ_{iX}), then biip(A)-A does not contain any non-empty infra α-closed sets.

Proof: Necessary part: Suppose F is a non-empty infra α-closed subset of (X,τ_{iX}) such that F ⊆ bicp(A)-A. Now F⊆bicp(A)-A ⟹F ⊆ bicp(A) ∩ A^c ⟹ F ⊆ bicp (A) and F ⊆ A^c ⟹A ⊆ F^c.

Since F^c is infra α-open and A is IG α B-closed set, bicp(A) ⊆ F^c ⟹F ⊆ $(bicp(A))^c$. Thus, F⊆ $(bicp(A) \cap (bicp(A))^c$ = ∅. That is F=∅. Implies that bicp(A)-A=∅ contains no non-empty $infra\ \alpha - closed\ set.$

Sufficient part: Let A ⊆ U and U is infra $\alpha - open,$ then bicp (A) ⊆ U. Suppose that bicp(A) does not contain in U, then bicp(A) ∩ U^c is a non-empty infra closed set of bicp(A)-A, which is a contradiction. Therefore, bicp(A) ⊆ U. Hence, A is IG α B-closed.

Theorem 4.10: Let A be IG αB-closed set, then A is IGB -closed sets if and only if bicp(A)-A=∅.

Proof: Necessary part: Assume that A is IG αB-closed set. Since bicp(A)=A, bicp (A)-A=∅ is IGB-closed and hence infra closed.

Sufficient part: Assume that bicp (A)-A be infra closed. By the previous theorem, bicp (A)-A does not contain any non-empty infra α-closed set. That is bicp (A)-A=∅, so A=bicp (A). Therefore, A is IGB – closed.

Theorem 4.11: Let A be IG α B-closed and suppose that F is an infra α-open, then A∩ F is IG α B-closed.

Proof: To show that A∩ F is IG α B-closed one has to show that bicp (A∩ F) ⊆ $U,$ where U is infra α-open and A ∩ F⊆ U.

Now icp(iip(icp(A ∩ F))) ⊆ $A \cap F$, icp(iip(icp(A ∩ F))) ⊆ $A \cap F \subseteq U$.Implies that icp(iip(icp(A ∩ F))) ⊆ U . Thus, icp(iip(A ∩ F)) ⊆ U and iip(icp(A ∩ F)) ⊆ U, as U is infra α-open. Now, icp(iip(A ∩ F)) ∪ iip(icp(A ∩ F)) ⊆ U, that is bicp(A ∩ F) ⊆ U. Hence proved.

Result 4.12: Suppose that B ⊆ A ⊆ X, B is IG α B-closed set relative to A and A is IG α B-closed set in (X,τ_{iX}), then B is IG α B-closed set relative to X.

Result 4.13: If a subset A is IG α B-closed set and A ⊆ B⊆ bicp(A) , then B is IG α B-closed set.

Result 4.14: Let A ⊆Y ⊆X and suppose that A is IG α B-closed set in X, A is then IG α B-closed set relative to Y.

Proof: Given that A⊆Y⊆ X and A is IG α B-closed set in (X,τ_{iX}), to show that A is IG α B-closed set relative to Y. Let A⊆ Y∩U, where U is infra $\alpha - open$

in (X,τ_{iX}), then bicp(A) ⊆ U and bicp(A) ∩Y is the infra b-closure if A in Y. Thus A is IG α B-closed set relative to Y.

Theorem 4.15: Let A be a ISGB-closed subset of (X,τ_{iX}).Then bicp(A)-A does not contain any non-empty infra closed sets.

Proof: Let F be an infra closed set in (X,τ_{iX}) such that F ⊆ bicp(A)-A.Since X – F is infra semi- open , A ⊆X – F and A is IGB-closed set, it follows that bicp(A) ⊆ X – F .

Hence F ⊆X – bicp(A).

This implies that F ⊆ (X – bicp(A))∩ (bicp(A)-A)=∅ and hence F = ∅.

Corollary 4.16: Let A be ISGB-closed set. Then A is infra b-closed set if and only if bicp(A) – A is infra closed.

Proof: Necessary part: Let A be ISGB-closed set. If A is infra b-closed, then we have

bicp(A) – A = ∅ which is infra closed set.

Conversely, let bicp(A) – A be infra closed. Then by theorem (4.15) bicp (A) – A does not contain any non-empty infra closed subset and since bicp (A) – A is closed subset of itself, then bicp(A) – A= ∅. This implies that A=bicp(A) and so A is infra b-closed set.

Theorem 4.17:

Let subset A be infra semi open and ISGB-closed set, A∩F is then ISGB-closed whenever F ϵ bicp (A)

Proof: Since A is ISGB-closed set and infra semi open set, then bicp(A) ⊆ A and thus A is IB-closed. Hence, A ∩F is IB-closed which implies that A∩F is ISGB-closed in (X,τ_{iX}).

Theorem 4.18: If A is a ISGB-closed set and B is any set such that A ⊆ B ⊆bicp(A) then B is a ISGB-closed set.

Proof: Let B ⊆ G where G is infra semi open set. Since A is ISGB-closed set and A⊆G, then bicp(A) ⊆ G and bicp(A) = bicp(B) . Therefore bicp (B) ⊆ G and hence B is an IGB-closed set.

Result 4.19:

Let B ⊆ A⊆ X, where A is an ISGB-closed set and infra semi open set. Then B is ISGB-closed relative to A if and only if B is ISGB-closed set in (X,τ_{iX}).

Theorem 4.20:

Intersection of any two ISGB-closed set is ISGB-closed set.

Proof: Let A and B be two ISGB-closed set. That is bicp(A) ⊆ G whenever A ⊆ G and G is infra semi open and bicp(B) ⊆ G whenever B⊆ G and G is infra semi open.

Now, bicp(A) (A∩B) = bicp(A) ∩ bicp(B) ⊆ G ,where A∩B ⊆ G and G is infra semi open. Thus, intersection of any two ISGB-closed set is ISGB-closed set.

Theorem 4.21: A subset A ⊆ (X,τ_{iX}) is ISGB-closed set if F⊆biip(A) whenever F is infra closed set and F⊆ A.

Proof: Let A be ISGB-open set and suppose F⊆ A, where F is infra closed, X-A is then an ISGB-closed set contained in the infra semi open set X-F. Hence, biip(X-A) ⊆ X-F and X-biip(A) ⊆ X-F. Thus F ⊆ biip(A).

Conversely, if F is infra closed set with F⊆ biip(A) and F ⊆ A then X-biip(A) ⊆ X-F, then biip(X-A) ⊆ X-F. Hence, X-A is ISGB-closed set and A is ISGB-open set.

CONCLUSION

The Infra closed sets and infra generalized closed sets can be used to derive infra continuity, infra closed map, infra open map, infra closure and infra interior and new separation axioms.

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