ON CERTAIN UNIFIED FRACTIONAL INTEGRALS PERTAINING TO PRODUCT OF SRIVASTAVA'S POLYNOMIALS AND ℵ-FUNCTION

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Abstract: This paper deals with the evaluate of the fractional integrals involving Saigo operators of the product of the Srivastava's polynomials and the ℵ-function containing the factor $x^{\lambda}(x^k + c^k)^{-\rho}$ in its argument. Some interesting special cases are derived. The results given by Chaurasia and Gupta [1] and Saigo and Raina [11] follow as special cases of the results proved in this paper.

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1. Introduction and Preliminaries

Let α, β and η be complex numbers, and further let $x \in \mathbb{R}_+ = (0, \infty)$. Following Saigo [10], the fractional integral $(\Re(\alpha) > 0)$ and the fractional derivative $(\Re(\alpha) < 0)$ of the function $f(x)$ on $\mathbb{R}_+$ are defined by

$$\left(I_{0+}^{\alpha,\beta,\eta} f\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} {}_2F_1(\alpha+\beta, -\eta; \alpha; 1-t/x) f(t) dt, (\Re(\alpha) > 0)\,; \quad (1.1)$$

$$= \frac{d^k}{dx^k} I_{0+}^{\alpha+k,\beta-k,\eta-k} f, \qquad \Re(\alpha) \leq 0, k = [\Re(-\alpha)] + 1, \quad (1.2)$$

$$\left(I_{-}^{\alpha,\beta,\eta} f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta} {}_2F_1(\alpha+\beta, -\eta; \alpha; 1-x/t) f(t) dt, (\Re(\alpha) > 0)\,; \quad (1.3)$$

$$= (-1)^k \frac{d^k}{dx^k} I_{-}^{\alpha+k,\beta-k,\eta} f, \qquad \Re(\alpha) \leq 0, k = [\Re(-\alpha)] + 1. \quad (1.4)$$

It can be easily seen that the Riemann-Liouville and Erdélyi-Kober fractional calculus operators are special cases of Saigo operators. In what follows the symbol $\Gamma\left[\begin{smallmatrix} a\, b\, c \dots \\ d\, e\, f \dots \end{smallmatrix}\right]$ will represent the fraction of the product of gamma functions $\frac{\Gamma(a)\,\Gamma(b)\,\Gamma(c)\dots}{\Gamma(d)\,\Gamma(e)\,\Gamma(f)\dots}$.

The general class of polynomials is defined by Srivastava [18, p.1, Eq.(1)] in the following manner:

$$S_w^u[x] = \sum_{s=0}^{[w/u]} \frac{(-w)_{us}}{s!} A_{w,s} x^s, \qquad w = 0, 1, 2, \dots \quad (1.5)$$

where u is an arbitrary positive integer and the coefficients $A_{w,s}\,(w, s \geq 0)$ are arbitrary constants, real or complex.

The series representation of ℵ-function is introduced by Chaurasia *et al.* [2] as follow:

$$\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[z\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},...,[\tau_i(a_{ji},\alpha_{ji})]_{n+1,p_i;r}\\(b_j,\beta_j)_{1,m},...,[\tau_i(b_{ji},\beta_{ji})]_{m+1,q_i;r}\end{array}\right.\right]=\sum_{k=0}^{\infty}\sum_{h=1}^{m}\frac{(-1)^k\Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)}{\beta_h k!}(z)^{-\zeta}, \tag{1.6}$$

with $\zeta=\frac{b_h+k}{\beta_h}, p_i<q_i, |z|<1$ and

$$\Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)=\frac{\Pi_{j=1}^{m}\Gamma(b_j+\beta_j\zeta)\,\Pi_{j=1}^{n}\Gamma(1-a_j-\alpha_j\zeta)}{\sum_{i=1}^{r}\tau_i\Pi_{j=m+1}^{q_i}\Gamma(1-b_{ji}-\beta_{ji}\zeta)\,\Pi_{j=n+1}^{p_i}\Gamma(a_{ji}+\alpha_{ji}\zeta)}, \tag{1.7}$$

The existence of the ℵ-function defined on (1.6) depends on the following conditions.

$$\varphi_l>0,\quad |\arg(z)|<\frac{\pi}{2}\varphi_l,\quad l=1,...,r, \tag{1.8}$$

and

$$\varphi_l\geq 0,\quad |\arg(z)|<\frac{\pi}{2}\varphi_l,\quad l=1,...,r\quad \Re(\zeta)+1<0, \tag{1.9}$$

where

$$\varphi_l=\sum_{j=1}^{n}\alpha_j+\sum_{j=1}^{m}\beta_j-\tau_l\left(\sum_{j=n+1}^{p_l}\alpha_{jl}+\sum_{j=m+1}^{q_l}\beta_{jl}\right), \tag{1.10}$$

and

$$\zeta=\sum_{j=1}^{m}b_j-\sum_{j=1}^{n}a_j+\tau_l\left(\sum_{j=m+1}^{q_l}b_{jl}-\sum_{j=n+1}^{p_l}a_{jl}\right)+\frac{1}{2}(p_l-q_l),\quad l=1,...,r. \tag{1.11}$$

For the convergence conditions and other details of Aleph-function, (see: Südland *et al.* [20], [21]) and is defined in terms of the Mellin- Barnes type integrals as following manner (see, e.g., [14], [15])

Remark 1.1. On setting $\tau_i=1(i=1,...,r)$ in (1.6), yields the I-function due to Saxena [13], defined in following manner:

$$I_{p_i,q_i;r}^{m,n}[z]=\aleph_{p_i,q_i,1;r}^{m,n}[z]=\aleph_{p_i,q_i,1;r}^{m,n}\left[z\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},...,[(a_{ji},\alpha_{ji})]_{n+1,p_i}\\(b_j,\beta_j)_{1,m},...,[(b_{ji},\beta_{ji})]_{m+1,q_i}\end{array}\right.\right]$$
$$=\frac{1}{2\pi i}\int_L\Omega_{p_i,q_i,1;r}^{m,n}(\zeta)\,z^{-\zeta}d\zeta. \tag{1.12}$$

Remark 1.2. If we set $\tau_i=1(i=1,...,r)$ and $r=1$, then (1.6) reduces to the familiar Fox H-function [3]:

$$H_{p,q}^{m,n}[z]=\aleph_{p_i,q_i,1;1}^{m,n}[z]=\aleph_{p_i,q_i,1;1}^{m,n}\left[z\left|\begin{array}{l}(a_j,\alpha_j)\\(b_j,\beta_j)\end{array}\right.\right]=\frac{1}{2\pi i}\int_L\Omega_{p_i,q_i,1;1}^{m,n}(\zeta)\,z^{-\zeta}d\zeta. \tag{1.13}$$

are the kernel $\Omega_{p_i,q_i,1;1}^{m,n}(\zeta)$ can be obtained from (1.7).

A thorough and wide-ranging account of the H-function is obtainable from the monographs written by Kilbas and Saigo [4], Mathai *et al.* [6], Prudnikov *et al.* [7] and Srivastava *et al.* [19].

For our purpose, we recall the generaliged hypergeometric series defined by (see: [5, 8]):

$$ {}_pF_q\left[\begin{array}{l}\alpha_1,\ldots,\alpha_p;\\ \beta_1,\ldots,\beta_q;\end{array} z\right] = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n\cdots(\alpha_p)_n z^n}{(\beta_1)_n\cdots(\beta_q)_n n!} = {}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z), \tag{1.14}$$

where $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda\in\mathbb{C}$) by

$$(\lambda)_n = \left\{\begin{array}{ll} 1 & (n=0)\\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (n\in N=\{1,2,3,\ldots\})\end{array}\right. = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}, \quad (\lambda\in\mathbb{C}/\mathbb{Z}_0). \tag{1.15}$$

and $\mathbb{Z}_0$ denotes the set of nonpositive integers.

2. Fractional Integral Formulas

We now establish, if

$$f(t) = t^{\lambda}(t^k+c^k)^{-\rho}S_w^u[yt^{\mu}(t^k+c^k)^{-\upsilon}]\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[zt^h(t^k+c^k)^{-\delta}\,\middle|\,\begin{array}{c}(s_1)\\(s_2)\end{array}\right] \tag{2.1}$$

where

$$s_1 = (a_j,\alpha_j)_{1,n},\ldots,(\tau_i(a_{ji},\alpha_{ji}))_{n+1,p_i;r}$$

and

$$s_2 = (b_j,\beta_j)_{1,m},\ldots,(\tau_i(b_{ji},\beta_{ji}))_{m+1,q_i;r}$$

then, we have following relation:

Theorem 2.1. *Let $\Re(\alpha)>0$, $\lambda>0$, $k=1,2,3,\ldots,c$ is a positive number and ρ is a complex number, then there holds the relation*

$$\left(I_{0+}^{\alpha,\beta,\gamma}f\right)(x) = \sum_{s=0}^{[w/u]}\sum_{p=0}^{\infty}\sum_{h=1}^{m}\frac{(-1)^p(-w)_{us}A_{w,s}y^s z^{-\zeta}}{s!\,p!\,\beta_h\,c^R}x^{T-\beta}\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\zeta)$$
$$\times\frac{\Gamma(T+1)\Gamma(T+\eta-\beta+1)}{\Gamma(T-\beta+1)\Gamma(T+\alpha+\eta+1)}$$
$$\times{}_{2k+1}F_{2k}\left[\begin{array}{c}\rho+\upsilon s-\delta\zeta,\Delta(k,T+1),\Delta(k,T-\beta+\eta+1);\\ \\ \Delta(k,T+1-\beta),\Delta(k,T+\alpha+\eta+1);\end{array} -\frac{x^k}{c^k}\right], \tag{2.2}$$

where $T=\lambda+\mu s-h\zeta$ and $R=k\rho+k\upsilon s-k\delta\zeta$.

The result (2.2) is valid for $\Re(\alpha)>0$, $\Re\left(\lambda+\mu s+h\frac{b_j}{\beta_j}\right)>0$, $|\arg z|<T'(\pi/2)$, $T'>0$, $T'=\sum_{j=1}^{n}\alpha_j+\sum_{j=1}^{m}\beta_j-\tau_l\left(\sum_{j=n+1}^{p_l}\alpha_{jl}+\sum_{j=m+1}^{q_l}\beta_{jl}\right)>0$. Also c is a positive number and $\rho,\mu,\upsilon,h,\delta$ are complex numbers, $k=1,2,3,\ldots,u$ is an arbitrary positive integer and the coefficients $A_{w,s}(\mathrm{w,s}\geq 0)$ are arbitrary constants, real or complex. Here, $\Delta(k,\alpha)$ represent the sequence of parameters

$$\frac{\alpha}{k},\frac{\alpha+1}{k},\ldots,\frac{\alpha+k-1}{k},$$

and ${}_{2k+1}F_{2k}(\cdot)$ is the generalized hypergeometric function, defined in [5].

Proof. Let ℓ be the left-hand side of (2.2). using (1.5) and (1.6), applying (2.1) to (1.1), we find as:

$$\ell = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p(-w)_{us}A_{w,s}y^s z^{-\zeta}}{s!\, p!\, \beta_h} \Omega^{m,n}_{p_i,q_i,\tau_i;r}(\zeta)$$

$$\times \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} {}_2F_1(\alpha+\beta,-\eta;\alpha;1-t/x)t^{\lambda+\mu s-h\zeta}(t^k+c^k)^{-(\rho+\upsilon s-\delta\zeta)}dt, \quad (2.3)$$

using the Gauss function and series formula $(t^k+c^k)^{-\rho} = c^{-k\rho}\sum_{q=0}^{\infty}\frac{(\rho)_q}{q!}\left(-\frac{t^k}{c^k}\right)^q$ in (2.3), we get

$$= \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p(-w)_{us}A_{w,s}y^s z^{-\zeta}}{s!\, p!\, \beta_h} \Omega^{m,n}_{p_i,q_i,\tau_i;r}(\zeta)$$

$$\times \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n(-\eta)_n}{(\alpha)_n\, n!} x^{-n} \sum_{q=0}^{\infty} \frac{(\rho+\upsilon s-\delta\zeta)_q}{q!} c^{-k(\rho+\upsilon s-\delta\zeta)}\left(\frac{1}{c^k}\right)^q$$

$$\times \int_0^x t^{\lambda+\mu s-h\zeta+kq}(x-t)^{\alpha+n-1}dt,$$

Interchanging the order of integration and summation, this is valid under the given conditions, and evaluates the inner integral by means of the formula

$$\int_0^x t^{\lambda+kq}(x-t)^{\alpha+p-1} = x^{\alpha+\lambda+p+kq}\frac{\Gamma(\alpha+p)\Gamma(\lambda+kq+1)}{\Gamma(\alpha+p+\lambda+kq+1)}, \quad (2.4)$$

with the little simplification, we obtain

$$= \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p(-w)_{us}A_{w,s}y^s z^{-\zeta}}{s!p!\beta_h c^R} x^{T-\beta}\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\zeta)$$

$$\times \sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n(-\eta)_n}{(T+\alpha+kq+1)_n\, n!} \sum_{q=0}^{\infty} \frac{(\rho+\upsilon s-\delta\zeta)_q}{q!} \frac{\Gamma(T+1+kq)}{\Gamma(T+\alpha+kq+1)}\left(\frac{x^k}{c^k}\right)^q, \quad (2.5)$$

with a employing Gauss Theorem and multiplication formula, Rainville [9, p.49, 24-29] in equation (2.5). we obtain the right-hand side of (2.2). □

Theorem 2.2. *Let $\Re(\alpha) > 0, \lambda > 0$, $k = 1,2,3,\ldots, c$ is a positive number and ρ is a complex number, then there holds the relation*

$$\left(I_-^{\alpha,\beta,\eta}f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p(-w)_{us}A_{w,s}y^s z^{-\zeta}x^{T-\beta}}{s!p!\beta_h c^R}\Omega^{m,n}_{p_i,q_i,\tau_i;r}(\zeta)$$

$$\times \frac{\Gamma(\beta-T)\Gamma(\eta-T)}{\Gamma(-T)\Gamma(\alpha+\beta+\eta-T)}$$

$$\times {}_{2k+1}F_{2k}\left[\begin{matrix}\rho+\upsilon s-\delta\zeta, \Delta(k,T+1), \Delta(k,T-\alpha-\beta-\eta+1); \\ \Delta(k,T-\eta+1), \Delta(k,T-\beta+1);\end{matrix} -\frac{x^k}{c^k}\right], \quad (2.6)$$

Here c is a positive number and $\rho, \mu, \upsilon, h, \delta$ are complex numbers, $k = 1,2,3,\ldots$, The result (2.6) is valid for $\Re(\alpha) > 0$, $\Re\left(\lambda+\mu s+h\frac{b_j}{B_j}\right) > 0$, $|\arg z| < T'(\pi/2), T' > 0$, u

is an arbitrary positive integer and the coefficients $A_{w,s}(\mathrm{w}, \mathrm{s} \geq 0)$ *are arbitrary constants, real or complex.*

Proof. Let ℓ be the left-hand side of (2.6). using (1.5) and (1.6), applying (2.1) to (1.3), we obtain

$$\ell = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p(-w)_{us} A_{w,s} y^s z^{-\zeta}}{s!p!B_h} \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)$$

$$\times \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta} {}_2F_1(\alpha+\beta, -\eta; \alpha; 1-x/t) t^{\lambda+\mu s-h\zeta}(t^k+c^k)^{-(\rho+\upsilon s-\delta\zeta)} dt, \quad (2.7)$$

and express series expansion for the Gauss function and $\left(t^k+c^k\right)^{-\rho}$, then interchanging the order of integration and summation, this is valid under the given conditions, and evaluates the inner integral by means of the formula

$$\int_x^\infty t^{\lambda-\alpha-\beta-n+kq} (t-x)^{\alpha+n-1} dt = x^{\lambda-\beta+kq} \frac{\Gamma(\alpha+n)\Gamma(\beta-\lambda-kq)}{\Gamma(\alpha+\beta+n-\lambda-kq)}, \quad (2.8)$$

and, also using the relation $(\alpha)_n = \frac{(-1)^n}{(1-\alpha)_n}$, further employing Gauss Theorem and multiplication formula, we obtain the right-hand side of (2.6). □

If we set $h = \delta = 0$ in equation (2.2) and (2.6), then we obtain following new results:

Corollary 2.3.

$$\left(I_{0+}^{\alpha,\beta,\gamma}\left(x^\lambda(x^k+c^k)^{-\rho} S_w^u[yx^\mu(x^k+c^k)^{-\upsilon}]\right)\right)$$

$$= \frac{x^{\lambda-\beta}}{c^{k\rho}} \sum_{s=0}^{[w/u]} \frac{(-w)_{us} A_{w,s} y^s}{s!} \left(\frac{x^\mu}{c^\upsilon}\right)^s \frac{\Gamma(\lambda+\mu s+1)\Gamma(\lambda+\mu s+\eta-\beta+1)}{\Gamma(\lambda+\mu s-\beta+1)\Gamma(\lambda+\mu s+\alpha+\eta+1)}$$

$$\times {}_{2k+1}F_{2k}\left[\begin{array}{c} \rho+\upsilon s, \Delta(k,\lambda+\mu s+1), \Delta(k,\lambda+\mu s-\beta+\eta+1); \\ \Delta(k,\lambda+\mu s-\beta+1), \Delta(k,\lambda+\mu s+\alpha+\eta+1); \end{array} -\frac{x^k}{c^k}\right], \quad (2.9)$$

Corollary 2.4.

$$\left(I_{-}^{\alpha,\beta,\gamma}\left(x^\lambda(x^k+c^k)^{-\rho} S_w^u[yx^\mu(x^k+c^k)^{-\upsilon}]\right)\right)$$

$$= \frac{x^{\lambda-\beta}}{c^{k\rho}} \sum_{s=0}^{[w/u]} \frac{(-w)_{us} A_{w,s} y^s}{s!} \left(\frac{x^\mu}{c^\upsilon}\right)^s \frac{\Gamma(\beta-\lambda-\mu s)\Gamma(\eta-\lambda-\mu s)}{\Gamma(-\lambda-\mu s)\Gamma(\alpha+\beta+\eta-\lambda-\mu s)}$$

$$\times {}_{2k+1}F_{2k}\left[\begin{array}{c} \rho+\upsilon s, \Delta(k,\lambda+\mu s+1), \Delta(k,\lambda+\mu s-\alpha-\beta-\eta+1); \\ \Delta(k,\lambda+\mu s-\eta+1), \Delta(k,\lambda+\mu s-\beta+1); \end{array} -\frac{x^k}{c^k}\right]. \quad (2.10)$$

3. Interesting Special Cases

(I) When $\beta = -\alpha$, the operators (1.1) and (1.2) coincide with the classical Riemann-Liouville fractional integrals of order $\alpha \in \mathbb{C}$ with and $x > 0$ (see, e.g., [12]):

$$\left(I_{0+}^{\alpha,-\alpha,\eta} f\right)(x) = \left(I_{0+}^{\alpha} f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \tag{3.1}$$

$$\left(I_{0+}^{\alpha} f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p (-w)_{us} A_{w,s} y^s z^{-\zeta}}{s! p! \beta_h c^R} x^{T+\alpha} \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)$$
$$\times \frac{\Gamma(T+1)}{\Gamma(T+\alpha+1)} {}_{k+1}F_k \left[\begin{matrix} \rho + \upsilon s - \delta\zeta, \Delta(k, T+1); \\ \Delta(k, T+1-\beta); \end{matrix} -\frac{x^k}{c^k} \right], \tag{3.2}$$

and

$$\left(I_{-}^{\alpha,-\alpha,\eta} f\right)(x) = \left(I_{-}^{\alpha} f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} f(t)dt, \tag{3.3}$$

$$\left(I_{-}^{\alpha} f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p (-w)_{us} A_{w,s} y^s z^{-\zeta} x^{T+\alpha}}{s! p! \beta_h c^R} \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)$$
$$\times \frac{\Gamma(-T-\alpha)}{\Gamma(-T)} {}_{k+1}F_k \left[\begin{matrix} \rho + \upsilon s - \delta\zeta, \Delta(k, T+1); \\ (k, T-\beta+1); \end{matrix} -\frac{x^k}{c^k} \right]. \tag{3.4}$$

(II) When $\beta = 0$, in (1.1) - (1.2) yields the so-called Erdélyi-Kober integrals of order $\alpha \in \mathbb{C}$ with and $x > 0$ (see, e.g., [12]):

$$\left(I_{0+}^{\alpha,0,\eta} f\right)(x) = \left(I_{\eta,\alpha}^{+} f\right)(x) = \frac{x^{-\alpha-\eta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} t^\eta f(t)dt, \tag{3.5}$$

$$\left(I_{\eta,\alpha}^{+} f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p (-w)_{us} A_{w,s} y^s z^{-\zeta}}{s! p! \beta_h c^R} x^{T} \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)$$
$$\times \frac{\Gamma(T+\eta+1)}{\Gamma(T+\alpha+\eta+1)} {}_{k+1}F_k \left[\begin{matrix} \rho + \upsilon s - \delta\zeta, \Delta(k, T+\eta+1); \\ \Delta(k, T+\alpha+\eta+1); \end{matrix} -\frac{x^k}{c^k} \right], \tag{3.6}$$

and

$$\left(I_{-}^{\alpha,0,\eta} f\right)(x) = \left(K_{\eta,\alpha}^{-} f\right)(x) = \frac{x^{\eta}}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\eta} f(t)dt, \tag{3.7}$$

$$\left(K_{\eta,\alpha}^{-} f\right)(x) = \sum_{s=0}^{[w/u]} \sum_{p=0}^{\infty} \sum_{h=1}^{m} \frac{(-1)^p (-w)_{us} A_{w,s} y^s z^{-\zeta} x^{T}}{s! p! \beta_h c^R} \Omega_{p_i,q_i,\tau_i;r}^{m,n}(\zeta)$$
$$\times \frac{\Gamma(\eta-T)}{\Gamma(\alpha+\beta+\eta-T)} {}_{k+1}F_k \left[\begin{matrix} \rho + \upsilon s - \delta\zeta, \Delta(k, T-\alpha-\eta+1); \\ \Delta(k, T-\eta+1); \end{matrix} -\frac{x^k}{c^k} \right]. \tag{3.8}$$

4. Conclusion

We have established two new integral relations involving the product of the Srivastava's polynomials and the ℵ-function. We can also derived analogous result in the form of Riemann-Liouville and Erdélyi-Kober fractional integral operators, which have been depicted in corollaries. In another direction, using remark (1.1) and (1.2), we can also find the numerous result in the form of I-function and H-function. Therefore, the results presented in this article are easily converted in terms of a similar type (1, 11, 16, 17) of new interesting integrals with different arguments after some suitable parametric replacements.

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