MHD boundary layer flow of nanofluid and heat transfer over a porous exponentially stretching sheet in presence of thermal radiation and chemical reaction with suction

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ABSTRACT:
A mathematical model of the steady boundary layer flow of nanofluid due to anexponentially stretching sheet with magnetic field i n presence of chemical reaction, thermal radiation and viscous dissipation is presented. In this paper the effects of Brownian motion and thermophoresis on he at transfer and nano-particle volume fraction on mass transfer are studied. Using similarity transformations, the governing equations are transformed into coupled, nonlinear ordinary differential equations, which are solved by using Keller Box method. Effects of various physical parameters on the velocity, temperature and concentration profiles are presented graphically. Numerical values of different involved parameters on skin friction coefficient, local Nusselt and Sherwood numbers are obtained. Chemical reaction parameter increases, the nanoparticle volume fraction decreases significantly. The temperature profile increases with the increasing values of Eckert number and radiation parameter.

Key Words: *Nanofluid, Exponentially Stretching Sheet, Heat Transfer, Keller-box method, MHD, Chemical Reaction, Thermal Radiation.*

INTRODUCTION:

The study of magnetohydrodynamic (MHD) flow is very important because the influence of a magnetic field on the viscous flow of electrically conducting fluid is essential in many industrial processes, such as in magnetic materials processing, purification of crude oil, MHD electrical power generation, glass manufacturing, geophysics and paper production. The magnetohydrodynamic is one of the important factors by which the cooling rate can be controlled and the product of the desired quality can be achieved. The study of Magnetohydrodynamics (MHD) boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes. Crane [1] was first to study the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. K.Jafar et al. [2] studied the MHD flow and heat transfer over stretching/shrinking sheets with external magnetic field, viscous dissipation and Joule Effects. K.V.Prasad et al. [3] discussed the effect of variable viscosity on MHD visco-elastic fluid flow and heat transfer over a stretching sheet. P.R. Sharma and G. Singh [4] analyzed the effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet.

A new kind of fluid known as Nanofluid is first time proposed in Argonne National Laboratory in US by Choi at the time of investigations on coolants techniques and cooling processes. The Nanofluids are of nanometer sized particles that are metals, oxides, carbides or carbon nanotubes. The properties of Nanofluids are of special importance over the base fluid since thermal conductivity and convective properties of the Nanofluid predominant over the properties of the base fluid. The common base fluids include water, ethylene glycol, toluene and oil. Thermal conductivity is observed to be more effectively enhanced in the range of 15% - 40% over the base fluid. A new engineering medium, called nanofluid attracted a wide range of researchers on many cooling processes in engineering applications, for instance in transportation, nuclear reactor, electronics as well as biomedicine and food. S. U. S. Choi [5] studied the Enhancing thermal conductivity of fluids with nanoparticles. Khan and Pop [6] discussed the boundary layer flow of nanofluids over a stretching surface. SandeepNaramgari and C. Sulochana [7] discussed the effect of MHD flow over a permeable stretching/shrinking sheet of a nanofluid with suction/injection. Sohail Nadeem and Changhoon Lee [8] analyzed Boundary layer flow of nanofluid over an exponentially stretching surface. NorfifahBachok et al. [9] presented the Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid. Wubshet Ibrahim and Bandari Shankar [10] discussed MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. M.M. Rashidi et al. [11] studied Homotopy simulation of nanofluid dynamics from a non-linearly stretching isothermal permeable sheet with transpiration.

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Various devices are designed in streambeds to reduce the kinetic energy of flowing waters, reducing their erosive potential on banks and river bottoms. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flows such as injection modeling or extrusion at high rates. Mohsen Sheikholeslami et. al [12] presented the Numerical simulation of MHD nanofluid flow and heat transfer by considering viscous dissipation. Sin Wei Wong et al. [13] discussed the Boundary layer flow and heat transfer over an exponentially stretching/shrinking permeable sheet with viscous dissipation. Yohannes Yirga and Daniel Tesfay [14] analyzed Heat and mass transfer in MHD flow of nanofluids through a porous media due to a permeable stretching sheet with viscous dissipation and chemical reaction effects.

The effect of thermal radiation on flow and heat transfer processes is of major importance in the design of many advanced energy conversion systems operating at high temperature. Thermal radiations within such systems occur because of the emission by the hot walls and working fluid. Boundary layer flow over a stretching surface in connection with thermal radiation and transverse magnetic field within nanofluid medium flow is important due to its application in engineering and industries, especially in the design of reliable equipment's, polymer and processing engineering, metallurgy, petrochemical industry, nuclear plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles.Mutuku-Njane and Makinde [15] were discussed the heat transfer analysis of MHD nanofluid flow over a permeable plate. Fekry M Hady et al. [16] discussed radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet. Liancun Zhenga et al. [17] analyzed flow and radiation heat transfer of a nanofluid over a stretching sheet with velocity slip and temperature jump in porous medium. Abdul Sattar Dogonchi and Davood Domiri Ganji [18] investigated thermal radiation effect on the nanofluid buoyancy flow and heat transfer over a stretching sheet. Radiation and MHD boundary layer stagnation-point of nanofluid flow towards a stretching sheet embedded in a porous medium studied by Emad H. Aly [19]. Maria Imtiaz et al. [20] discussed the flow of magneto nanofluid by a radiative exponentially stretching surface with dissipation effect. Dulal Pal and Gopinath Mandal [21] examine the effect of Mixed convection–radiation on stagnation-point flow of nanofluids over a stretching/shrinking sheet in a porous medium with heat generation and viscous dissipation.

The study of heat transfer with chemical reaction in the presence nanofluids of large realistic importance to engineers and scientists, because of its almost universal incidence in many branches of engineering and science. This frequently occurs in petro-chemical industry, cooling systems and power, chemical vapour deposition on surfaces, heat exchanger design, cooling of nuclear reactors, geophysics and forest fire dynamics as well as in magneto hydrodynamic power generation system. N. G. Rudraswamy and B. J. Gireesha [22] presented the influence of chemical reaction and thermal radiation on MHD boundary layer flow and heat transfer of a nanofluid over an exponentially stretching sheet. Mohamed R. Eid [23] discussed the influence of Chemical reaction on MHD boundary-layer flow of two-phase nanofluid model over an exponentially stretching sheet with a heat generation. M.Das et al. [24] studied the mixed convection and nonlinear radiation in the stagnation point nanofluid flow towards a stretching sheet with homogenous-heterogeneous Reactions effects. P. Sreedevi et al. [25] examine the Heat and mass transfer analysis of nanofluid over linear and non-linear stretching surfaces with thermal radiation and chemical reaction.

Motivated by the above reference work and the numerous possible industrial applications of the problem, it is of paramount interest in this study to investigate the MHD boundary layer nanofluid flow over an porous exponentially stretching sheet by considering the effect of viscous dissipation, thermal radiation and chemical reaction and solved numerically by adopting the well-known implicit finite difference scheme known as Keller-Box method. The aim of the present study is to extend the work of Bhattacharyya and Layek [26].The numerical solutions are analyzed graphically for different flow parameters involved in the problem by using MATLAB. The numerical values of local skin friction coefficient, local Nusselt number and local

Sherwood number for different values of the flow parameters are also tabulated.

MATHEMATICAL FORMULATION:

Consider a steady two-dimensional flow of an incompressible viscous and electrically conducting nanofluid caused by a stretching sheet, which is placed in a quiescent ambient fluid of uniform temperature of the plate and species concentration are raised to *Tw (> T∞)* and *Cw (> C∞)*, respectively, which are thereafter maintained constant, where *Tw*, *Cw* are temperature and species concentration at the wall and *T∞*, *C∞* are temperature and species concentration far away from the plate, respectively. The *x*-axis is taken along the stretching sheet in the direction of the motion and *y*-axis is perpendicular to it. Consider that a variable magnetic field *B(x)*is applied normal to the sheet and that the induced magnetic field is neglected, which is justified for MHD flow at small magnetic Reynolds number. A physical model with the coordinate system of the problem shown in Figure (a).

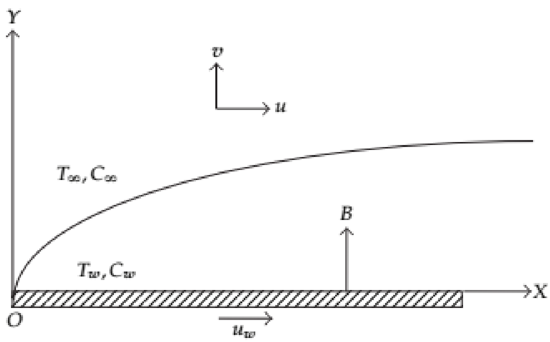

Figure a: Physical model and coordinate system

Under the above assumptions and usual boundary layer approximation, the steady MHD boundary layer flow of nanofluid flow over an exponentially stretching sheet in presence of chemical reaction and thermal radiation are governed by the following equations of momentum, energy and species concentration are written in usual notation as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho_f}u-\frac{\upsilon}{K}u, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\tau\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}-\frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y} \qquad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)-K_0(C-C_\infty) \qquad (4)$$

where u and v are the velocity components in$x$-and$y$-directions, respectively,$\upsilon$is the kinematic viscosity, ρ_f is the density of the base fluid, T is the temperature, T_∞ is constant temperature of the fluid in the inviscid free stream, $\alpha$is the thermal conductivity, $(\rho c)_p$ is the effective heat capacity of nanoparticles, $(\rho c)_f$ is heat capacity of the base fluid, Cis nanoparticle volume fraction, K is the permeability of the porous medium, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, c_p is the specific heat at constant pressure. Here, the variable magnetic field B(x)is taken in the form

$$B(x)=B_0\exp\left(\frac{x}{2L}\right), \qquad (5)$$

where B_0 is a constant.

It is assumed that the permeability K of the porous medium takes the following form

$K(x)=2k_0\exp\left(\frac{-x}{L}\right)$ where k_0 is reference permeability.

The boundary conditions are given by

$$u=U_w(x),\quad v=v_w\,, T=T_w=T_\infty+T_0\exp\left(\frac{x}{2l}\right),$$

$$C=C_w=C_\infty+C_0\exp\left(\frac{x}{2l}\right) \quad \text{at } y=0, \qquad (6)$$

$$u\to 0,\; T\to T_\infty,\;\; C\to C_\infty \text{ at } y\to\infty,$$

Where T_w is the variable temperature at the sheet with T_0 being a constant which measures the rate of temperature increase along the sheet, C_w is the variable wall nanoparticle volume fraction with C_0 being a constant, and C_∞ is constant nano-particle volume fraction in free stream. The stretching velocity U_w is given by

$$U_w(x) = c\exp\left(\frac{x}{L}\right) \quad (7)$$

where $c > 0$ is stretching constant.

Here v_w is the variable wall mass transfer velocity and is given by $v_w(x) = v_0 \exp\left(\frac{x}{2L}\right)$ (8)

Where v_0 is a constant with $v_0 < 0$ for mass suction and $v_0 > 0$ for mass injection.

The radiative heat flux in the x-direction is considered negligible as compared to y-direction. Hence, by Using Rosseland approximation for radiation, the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$$

where k^* is the mean absorption coefficient, σ^* is the Stefan-Boltzman constant, we Assume that the temperature difference with in the flow sufficiently small such that the term T^4 may be expanded as a linear function of temperature. This is done by expanding T^4 in a Taylor series about a free stream temperature T_∞ and neglecting higher order terms we get $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.

Hence $q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y}$

then $\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3(\rho c_p)_f k^*}\frac{\partial^2 T}{\partial y^2}$

Now we introduce the similarity transformations:

$$\psi = \sqrt{2\upsilon Lc}\, f(\eta)\exp\left(\frac{x}{2L}\right), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \eta = y\sqrt{\frac{c}{2\upsilon L}}\exp\left(\frac{x}{2L}\right)$$

(9)

Where ψ is the stream function with $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ and η is the similarity variable.

Using (9), the equations (1)-(3) are transformed to the following ordinary differential equations:

$$f''' - 2f'^2 + ff'' - (M + K1)f' = 0 \quad (10)$$

$$\left(1 + \frac{4R}{3}\right)\theta'' + \Pr\left[\begin{array}{l} f\theta' - f'\theta + Ecf''^2 + \\ Nb\theta'\phi' + Nt\theta'^2 \end{array}\right] = 0$$

(11)

$$\phi'' + Le(f\phi' - f'\phi) + \frac{Nt}{Nb}\theta'' - Le\gamma \operatorname{Re}_x \phi = 0$$

(12)

Where $M = \frac{2\sigma B_0^2}{c\rho}$ is the Magnetic parameter,

$\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number,

$Ec = \frac{c^2 \exp\left(\frac{2x}{L}\right)}{(T_w - T_\infty)(C_p)_f}$ is the Eckert number, $K1 = \frac{ck0}{L\upsilon}$ is the permeability parameter, $R = \frac{4\sigma^* T_\infty^3}{\kappa k^*}$ is the radiation parameter,

$\gamma = \frac{\nu K_0}{U_w^2}$ is the Chemical Reaction parameter and

$\operatorname{Re}_x = \frac{xU_w}{\nu}$ is the Local Reynolds number. The dimensionless parameters Nb (Brownian motion parameter) and

Nt (Thermophoresis parameter) are defined as

$$Nb = D_B \frac{(\rho c)_p}{(\rho c)_f} \frac{(C_w - C_\infty)}{\upsilon},$$
$$Nt = \frac{D_T}{T_\infty} \frac{(\rho c)_p}{(\rho c)_f} \frac{(T_w - T_\infty)}{\upsilon}$$

The boundary conditions (6) reduce to the following form:

$$f(\eta) = S,\ f'(\eta) = 1, \theta(\eta) = 1, \varphi(\eta) = 1 \text{ at } \eta = 0, \tag{13}$$

$$f'(\eta) \to 0, \theta(\eta) \to 0, \varphi(\eta \to 0) \text{ as } \eta \to \infty,$$

Where $S = -v_0 \Big/ \sqrt{\frac{\upsilon c}{2L}}$ is the wall mass transfer parameter, $S > 0(v_0 < 0)$ corresponds to mass suction and $S < 0(v_0 > 0)$ corresponds to mass injection.

The quantities of physical interest for this problem are the local skin friction coefficient C_f, the local Nusseltnumber Nu_x, and the local Sherwood number Sh_x, which are respectively, defined as

$$C_f = \frac{\upsilon}{U_w^2 e^{\frac{x}{2L}}} \frac{\partial u}{\partial y}\Big|_{y=0}, \quad Nu_x = -\frac{x}{(T_w - T_\infty)} \frac{\partial T}{\partial y}\Big|_{y=0}, \quad Sh_x = -\frac{x}{(C_w - C_\infty)} \frac{\partial C}{\partial y}\Big|_{y=0}, \tag{14}$$

that is,

$$f''(0) = \sqrt{2\mathrm{Re}_x}\, C_f, \quad \frac{Nu_x}{\sqrt{2\mathrm{Re}_x}} = -\sqrt{\frac{x}{2L}}\,\theta'(0), \quad \frac{Sh_x}{\sqrt{2\mathrm{Re}_x}} = -\sqrt{\frac{x}{2L}}\,\varphi'(0), \tag{15}$$

Where $\mathrm{Re}_x = \frac{U_w x}{\upsilon}$ is the local Reynolds number.

METHOD OF SOLUTION:

Equations (10)-(12) are nonlinear, it is impossible to get the analytical solutions. Consequently, the equations using the boundary conditions (13) are solved numerically by means of a finite-difference scheme known as the Keller-Box method. This method has four fundamental steps. First step is converting the Eq. (10)-(12) into a system of first order ordinary differential equations. The second step is approximating the derivatives in system of first order equations with central difference approximations. The third step is linearizing the nonlinear algebraic equations with Newton's method and then casting as the matrix vector form. Finally, solving the system of linear equations using block tri-diagonal elimination scheme with the suitable initial solution. This method has a second order accuracy and is unconditionally stable. For this iterative scheme to solve the system of equations, a convergent criterion i.e., when difference between two successive approximations is sufficiently small ($\leq 10^{-5}$) is used. In this study a uniform grid of size $\Delta\eta$ = 0.001 is taken and the solutions are obtained with an error of tolerance 10^{-5} in all cases, which gives four decimal places accurate for most of the prescribed quantities as shown in the table. One of the factors modifying the accuracy of the method depends on the initial guesses. The following initial guesses dependent on the convergence criteria and the boundary conditions (13). We assume the initial guesses as

$$f(\eta) = (1 + S) - e^{-\eta}, \theta(\eta) = e^{-\eta}, \phi(\eta) = e^{-\eta}$$

RESULTS AND DISCUSSIONS:

In this study, we analyzed the effect of thermal radiation, viscous dissipation, permeability parameter and chemical reaction on the MHD boundary layer flow of a nanofluid over an exponentially stretching permeable sheet. The transformed nonlinear ordinary differential equations (10)–(12) with boundary conditions (13) are solved numerically using Keller box method. Dimensionless velocity, temperature and concentration profiles as well as the local skin friction coefficient, Nusselt number and Sherwood number were analyzed for different emerging flow parameters involved in the problem. The numerical results were discussed for the various values of the parameters graphically and in the tabular form.

To validate the numerical accuracy, the values of skin friction coefficient are compared with the results of Magyari and Keller [27] and Krishnendu Bhattacharyya and G.C.Layek [26]in Table 1 without magnetic field, chemical reaction, viscous dissipation, radiation and nonporous stretching sheet and the results are found in good agreement. Thus we are very much confident that the present results are accurate.

Table 1: The comparison of values of $f''(0)$ for M = 0, R = K1 = S = 0, Ec =0 and γ =0

	Magyari &Keller [27]	Bhattacharyya &Layek [26]	Present
- $f''(0)$	1.281808	1.28180838	1.281807536

Figure 1 exhibits the effect of magnetic parameter (M) on the dimensionless velocity. It is observed that the velocity profile of the nanofluid is reduced with increasing values of M. As expected, the velocity decreases with an increase in the magnetic parameter. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. We also see that velocity profiles decrease with the increase of magnetic effect indicating that magnetic field tends to retard the motion of the fluid. Magnetic field may control the flow characteristics. Figures 2 and 3 are shows the effect of magnetic parameter M on the dimensionless temperature and nanoparticle concentration. The dimensionless temperature and concentration profiles are accelerated with the increasing values of magnetic parameter (M). As the Lorentz force is a resistive force which opposes the fluid motion, so heat is produced and as a result, the thermal boundary layer thickness and nanoparticle volume fraction boundary layer thickness become thicker for stronger magnetic field.

Figures 4, 5 and 6 illustrate the effect of suction/bowling parameter S on the velocity, temperature and concentration profiles in the stretching sheet boundary layer flow. The flow is observed to be strongly decreased with suction ($S>0$) whereas it is increased significantly with blowing (injection $S<0$). Suction causes the boundary layer to adhere more closely to the wall and this destroys momentum leading to fall in velocity. Momentum boundary layer thickness is therefore decreased with suction. Conversely injection adds nanofluid via lateral mass flux through the sheet and this assists increase in momentum, improving velocity and causing a simultaneous increase in momentum boundary layer thickness. Admirable flow control is achieved in the nanofluid sheet regime with suction. Temperature and concentration are also found to be strongly decreased with the presence of suction (Figs. 5 and 6). Suction achieves a strong suppression of nano-particle species diffusion and also regulates the diffusion of thermal energy in the boundary layer. This shows that suction has significant effects on the establishment of engineered nanofluids and shows that suction is an exceptional mechanism for achieving flow control, cooling and nano-particle distribution in nanofluid fabrication.

The influence of the Prandtl number on the temperature distribution is shown in Figure 7. As the Prandtl number increases the thermal boundary layer thickness is decreases and thermal diffusion in the regime is inhibited leading to fall in temperature.

Figure 8 shows the effect of the Brownian motion parameter (Nb) and thermophoresis parameter (Nt) on the temperature profile. Increasing the values of Nb & Nt accelerate the temperature profile throughout the region. Geometrically, smaller nano-particles possess higher Nb values which assist in thermal diffusion in the boundary layer via enhanced thermal conduction. Conversely larger nano-particles show lower Nb values and this suppresses thermal conduction. The regime is heated and better distribution of nano-particles is achieved with increasing thermophoresis parameter Nt. Figure 9display the Brownian motion parameter (Nb) on the concentration profile, as Brownian motion parameter increases the nanoparticle volume boundary layer thickness decreases. Figure 10 shows the influence of thermophoresis parameter (Nt) on the concentration distribution. The concentration boundary layer thickness increases with the increasing of Nt.

Figure11 reveals that the influence of Eckert number (Ec) on temperature profile. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. The wall temperature of the sheet increases as the values of Ec increase. Moreover, when the values of Ec increase, the thermal boundary layer thickness increases. This is due to the fact that the heat transfer rate at the surface decreases with increasing of Ec as shown in Table 3.

Figure 12 depicts the effect of Lewis number (Le) on concentration profile. Lewis number Le is the ratio of thermal diffusion rate to species diffusion rate in the boundary layer regime. Larger Le values will suppress concentration profile.

Effect of chemical reaction parameter (γ) on nanoparticle volume fraction profile is shown in Fig. 13. It is observed that the concentration profile of the fluid decreases with increasing of chemical reaction parameter. This is due to the fact that chemical reaction in this system results in consumption of the chemical and hence results in decrease of concentration profile. The most important effect is that the first order chemical reaction has a tendency to shrink the over shoot in the profiles of the concentration in the boundary layer.

Figure 14 shows effect of permeability parameter K1 on the velocity profile. It is obvious that the presence of a porous medium causes higher restriction to the fluid flow which, in turn, slows its motion. As a result of this the shear stress at the surface increases. Therefore with increasing permeability parameter the resistance to the fluid motion also increases. This causes the fluid velocity to decrease.

Influence of radiation parameter on the temperature profile is depicted in Fig. 15. We observed that an increase in radiative parameter leads to an enhancement in the temperature profile. Thus larger values of R show a dominance of the thermal radiation over conduction. Consequently larger values of R are indicative of larger amount of radiative heat energy being poured into the system, causing a rise in temperature.

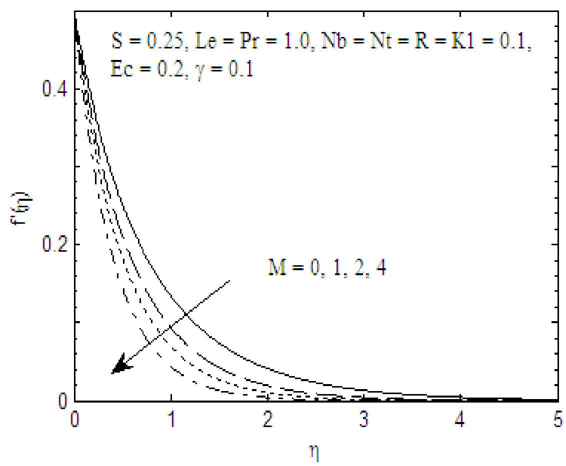

Figure 1: Effect Magnetic parameter (M) on Velocity Profile

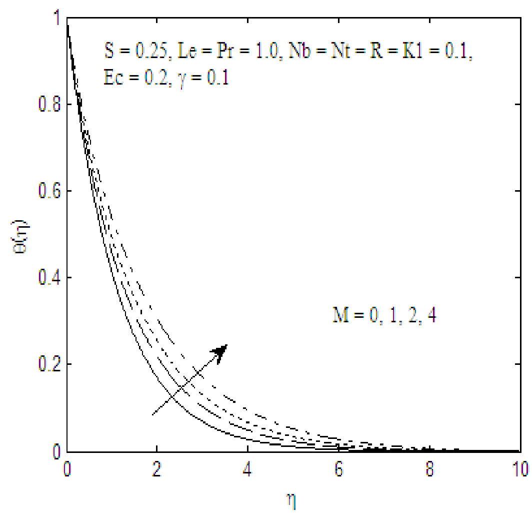

Figure 2: Effect Magnetic parameter (M) on Temperature Profile

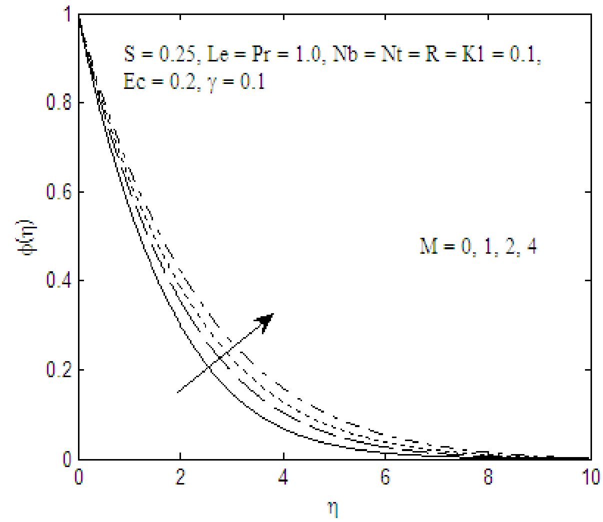

Figure 3: Effect Magnetic parameter (M) on Concentration Profile

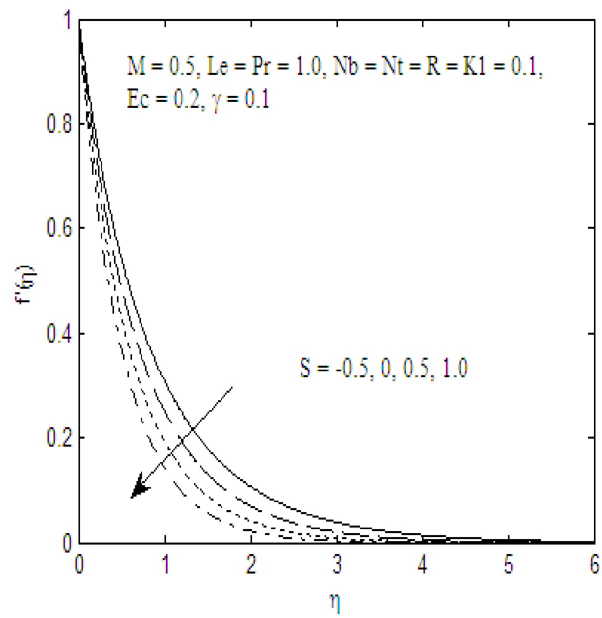

Figure 4: Effect Suction parameter (S) on Velocity Profile

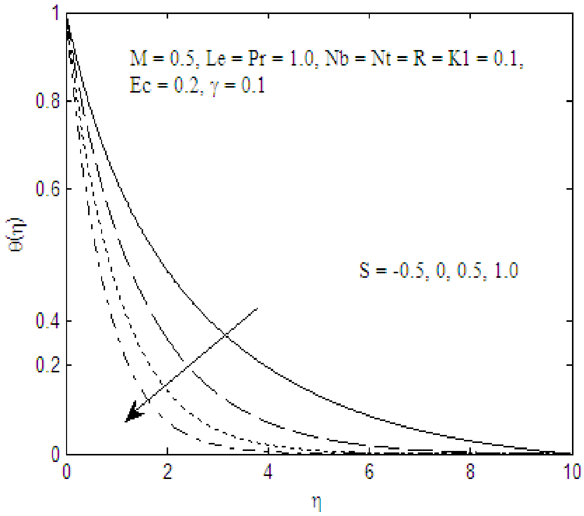

Figure 5: Effect Suction parameter (S) on Temperature Profile

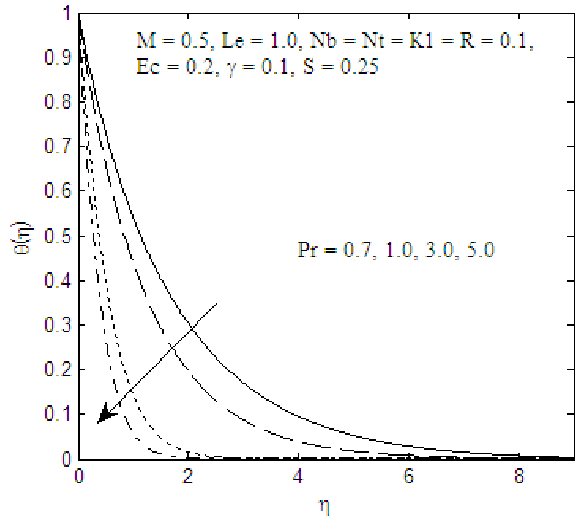

Figure 7: Effect of prandtl number Pr on Temperature profile

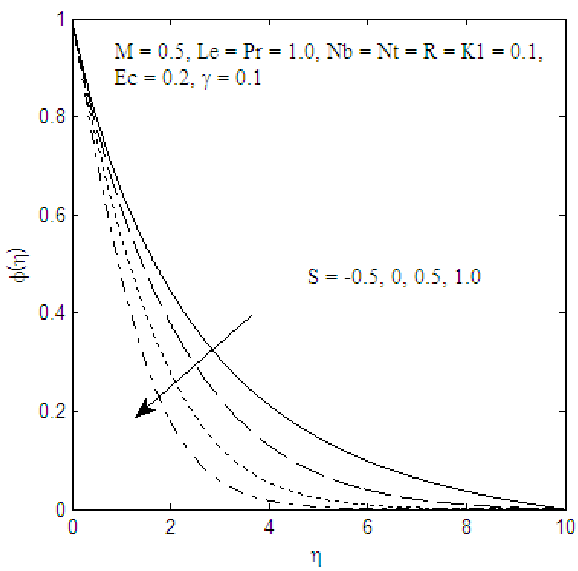

Figure 6: Effect Suction parameter (S) on Concentration Profile

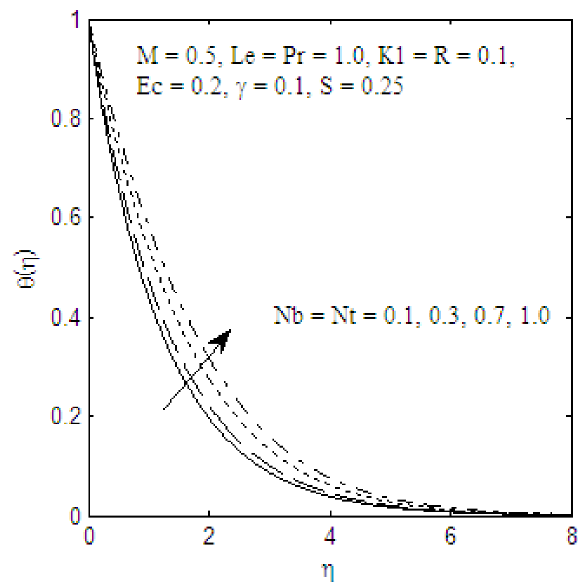

Figure 8: Effect of Nb & Nt on Temperature profile

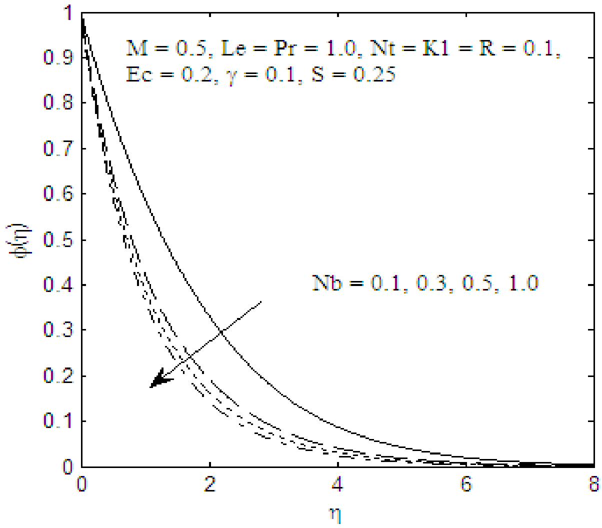

Figure 9: Effect of Brownian motion parameter on concentration profile

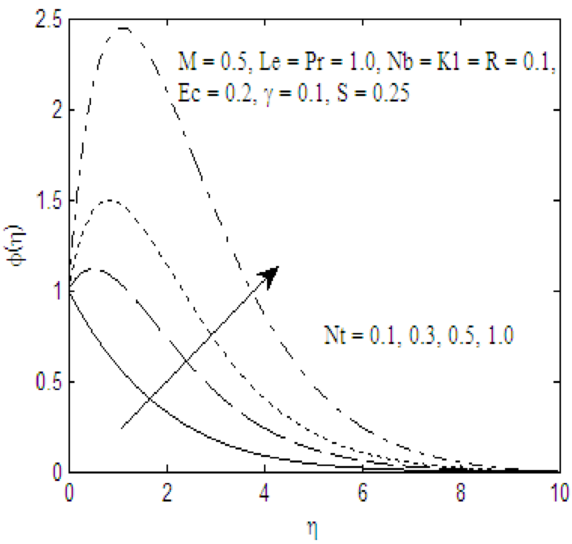

Figure 10: Effect of Thermophoresis parameter on concentration profile

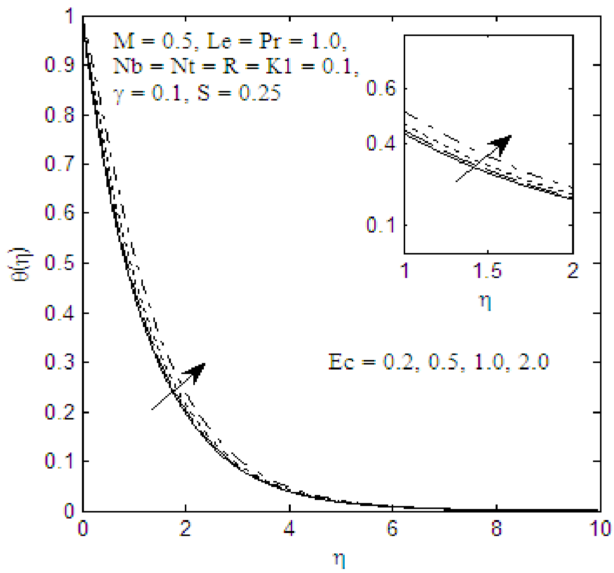

Figure 11: Effect of Eckert number on Temperature profile

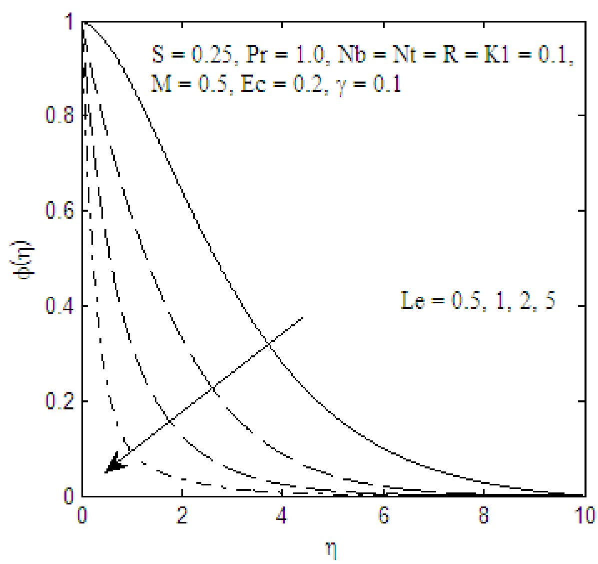

Figure 12: Effect of Lewis number on concentration profile

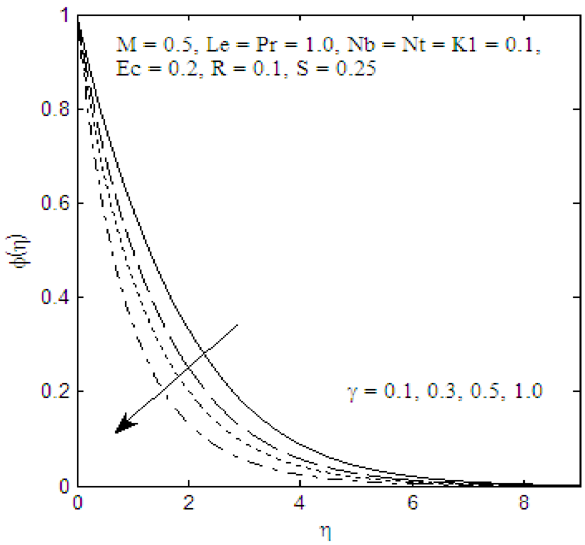

Figure 13: Effect of Chemical Reaction parameter on concentration profile

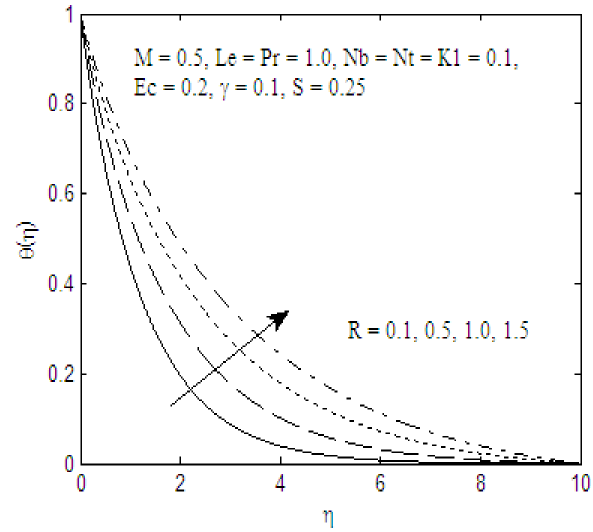

Figure 15: Effect of Thermal Radiation parameter R on Temperature profile

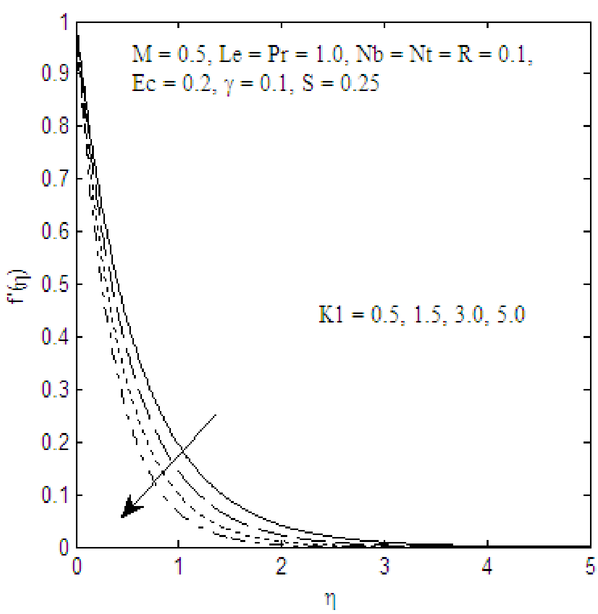

Figure 14: Effect of permeability parameter K1 on Velocity profile

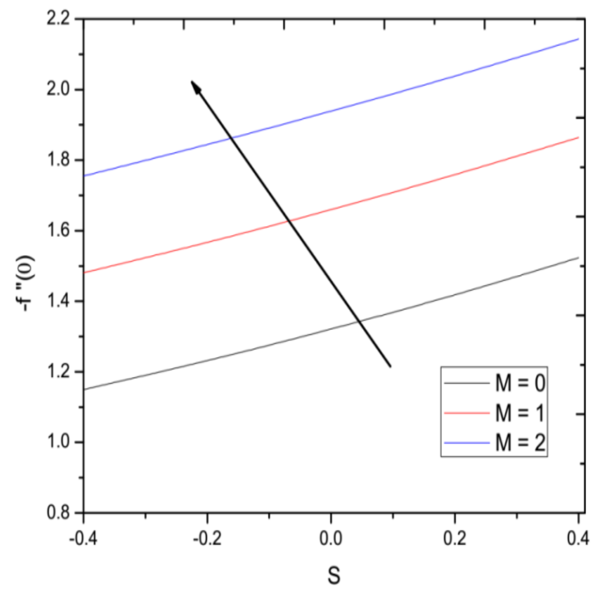

Figure 16: Effect of Magnetic field M and suction S on skin friction coefficient

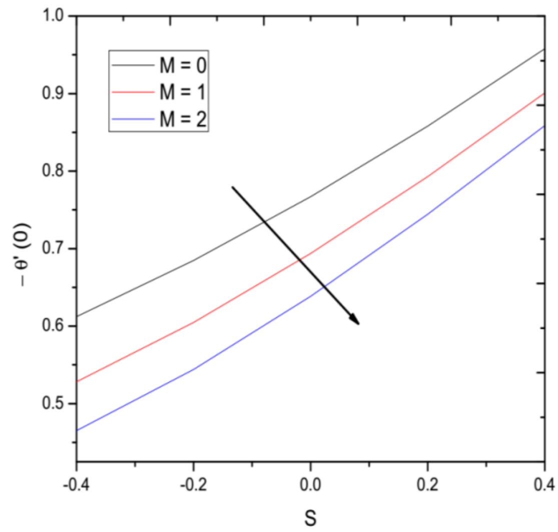

Figure 17: Effect of Magnetic field M and suction S on local nusselt number

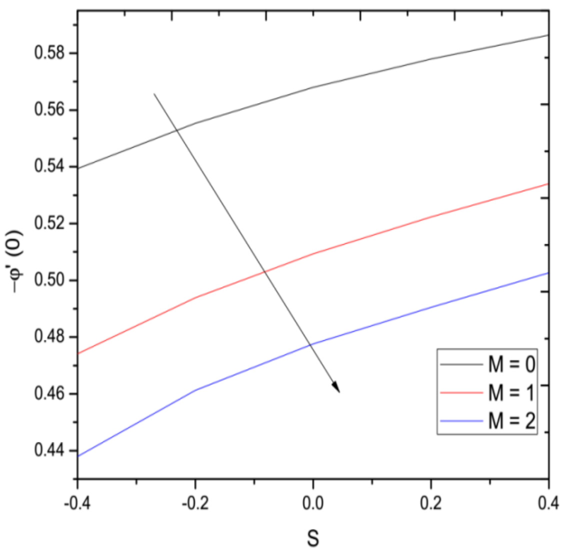

Figure 18: Effect of Magnetic field M and suction S on local Sherwood number

The effects of Magnetic field M and suction parameter S on the skin friction coefficient, the local Nusselt and Sherwood numbers are plotted in Figures 16, 17 and 18. It is observed that an increase in the Magnetic parameter M leads to increase in the skin friction coefficient, where as the values of reduced Nusselt and Sherwood numbers are decreases.

Influence of both the Brownian motion parameter Nb and thermophoresis parameter Nt on local Nusselt number is shown in figure 19. As both parameters increases, the heat transfer rate on the surface of a sheet decreases. This indicates that an increment in thermophoresis parameter induces resistance to the diffusion of mass, this results in the reduction of heat transfer rate on the surface. Fig. 20reveals the variation of local Sherwood number in response to a change in Brownian motion parameter Nb. The graph shows that an increase in both Nb and Nt leads to increase in the local Sherwood number.

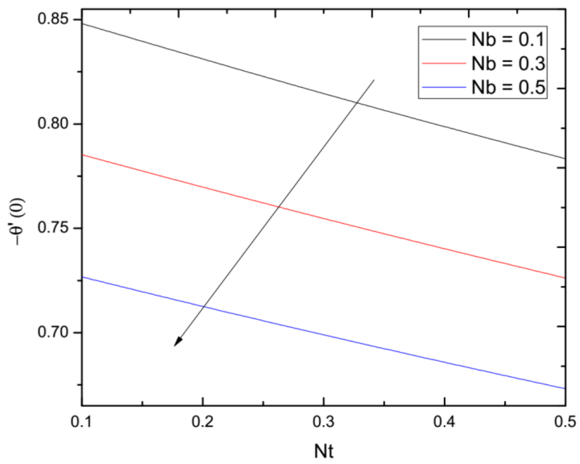

Figure 19: Effect of Nb&Nton local Nusselt Number

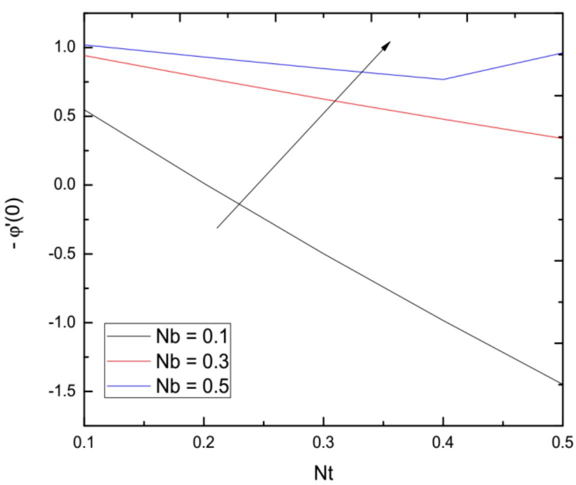

Figure 20: Effect of Nb&Nton local Sherwood number

The numerical values of the skin-friction coefficient, Nusselt number and the Sherwood number for various physical parameters are presented in Table 2. It is observed that as *M*, K1increases, both the Nusselt number and the Sherwood number decrease whereas skin-friction coefficient increases. An increase in suction parameter S leads to increase in the Nusselt number, Sherwood number and skin-friction coefficient.

Table 3 reveals the computation of the local Nusselt number and Sherwood number for different flow parameters. It is identified that an increase in the radiation parameter R, chemical reaction parameter γ , Lewis number Le and viscous dissipation parameter Ec leads to a fall in the Nusselt number and a rise in the Sherwood number. As Pr increases, the Sherwood number decreases while the Nusselt number increases. As *Nt* increases, both the Nusselt number and the Sherwood number decreases rapidly. With an increase in the values of *Nb*, the Nusselt number decreases whereas the Sherwood number increases.

Table 2: Effect of M, K1 and Son skin-friction coefficient ($-f''(0)$), Rate of heat and mass transfer coefficients ($-\theta'(0)$ and $-\varphi'(0)$) when Pr = 1.0, Nb = Nt = 0.1, R = γ = 0.1, Le = 1.0.

K1	M	S	$-f''(0)$	$-\theta'(0)$	$-\varphi'(0)$
0.2	0.5	0.25	1.6579	0.8420	0.5432
0.5			1.7538	0.8247	0.5294
0.7			1.8145	0.8140	0.5213
0.1	0.1		1.4822	0.8749	0.5729
	0.5		1.6245	0.8481	0.5484
	1		1.7844	0.8193	0.5252
	0.5	-0.2	1.4090	0.6418	0.5192
		0.2	1.5988	0.8229	0.5457
		0.4	1.7042	0.9269	0.5563

Table 3: Effect of R, Ec, Pr, Le, Nt, Nband γ on Rate of heat and mass transfer coefficients ($-\theta'(0)$ and $-\varphi'(0)$) when M = 0.5, K1 = 0.1, S = 0.25.

R	Ec	γ	Pr	Le	Nt	Nb	$-\theta'(0$	$-\varphi'(0)$
0.1	0.2	0.1	1.0	1.0	0.1	0.1	0.8481	0.5484
0.2							0.7837	0.5986
0.5							0.6419	0.7070
0.1	0.1						0.8721	0.5272
	0.3						0.8241	0.5697
	0.5						0.7762	0.6122
	0.2	0.3					0.8442	0.7319
		0.5					0.8416	0.8463
		1.0					0.8371	1.1564
		0.1	1				0.8481	0.5484
			2				1.3503	0.1390
			5				2.3631	-0.7556
			1.0	1			0.8481	0.5484
				3			0.8291	1.9390
				5			0.8219	2.9710
				1.0	0.1		0.8481	0.5484
					0.3		0.8146	-0.4984
					0.5		0.7834	-1.4496
					0.1	0.1	0.8481	0.5484
						0.3	0.7852	0.9416
						0.5	0.7267	1.0194

CONCLUSIONS:

The effect of chemical reaction, thermal radiation and viscous dissipation on steady MHD boundary layer flow of Nanofluid over an exponentially permeable stretching sheet has been investigated. The velocity, temperature, and concentration profiles alongthe exponential permeable sheet are studied and the results are shown graphically. The skin-friction coefficient, the rate of heat transfer and the rate of mass transferare presented in the tabular form. The study concludes the following results:

1. The Magnetic field reduces the velocity profile, whereas temperature and concentration profiles are enhanced with increasing of Magnetic parameter (M).

2. An increase of suction parameter S leads to fall in the velocity, temperature and concentration distributions.

3. It is observed that the velocity of the fluid decreases with the increase of permeability parameter K1.

4. The temperature profile increases with the increasing of Brownian motion parameter (Nb) and Thermophoresis parameter (Nt).

5. The concentration profile decreases with the increase of Brownian motion parameter (Nb), whereas reverse trend is observed with the increase of Thermophoresis parameter (Nt).

6. The fluid temperature increases with the increase of Eckert number (Ec), Radiation parameter (R) and decreases with the increase of Pr.

7. Concentration profile is reduced by enhancing both chemical reaction parameter γ and Lewis number (Le).

8. The skin friction coefficient increases with the increasing values of Magnetic parameter.

9. The local Nusselt number decreases with the increasing of Magnetic parameter, chemical reaction parameter and Eckert number.

10. The local Sherwood number decreases with the increasing of Magnetic parameter, whereas it is increases with the increasing of Eckert number and chemical reaction parameter.

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Nomenclature:

- u, v: Velocity components in the x- and y-axis, respectively (*m/s*)
- U_w: Velocity of the wall along the x-axis (*m/s*)
- $B(x)$: Magnetic field strength ($A\ m^{-1}$)
- C: Nano particle concentration ($mol\ m^{-3}$)
- C_{fx}: Skin-friction coefficient (*Pascal*)
- Nu_x: Nusselt number
- Sh_x: Sherwood number
- C_w: Nano particles concentration at the stretching surface ($mol\ m^{-3}$)
- C_∞: Nano particle concentration far from the sheet ($mol\ m^{-3}$)
- C_p: Specific heat capacity at constant pressure ($J\ Kg^{-1}\ K$)
- D_T: Brownian diffusion coefficient
- D_B: Thermophoresis diffusion coefficient
- k^*: Rosseland mean absorption coefficient
- k: Thermal conductivity ($W\ m^{-1}\ K^{-1}$)
- Le: Lewis number
- M: Magnetic parameter
- Nb: Brownian motion parameter
- Nt: Thermophoresis parameter
- S: Suction / Injection parameter
- Ec: Eckert Number
- K: Variable thermal conductivity
- Pr: Prandtl number
- R: Radiation parameter
- Re_x: Reynolds number
- T: Fluid temperature (K)
- T_W: Temperature at the surface (K)
- T_∞: Temperature of the fluid far away from the stretching sheet (K)
- q_w: Surface heat flux (W/m^2)
- q_m: Surface mass flux
- q_r: Radiative heat flux
- K1: Permeability parameter
- v_w: variable wall mass transfer Velocity

Greek Symbols:

- α: Thermal diffusivity (m^2/s)
- σ: Electrical conductivity, *mho/m, Henry/meter*
- σ^*: Stefan-Boltzmann constant
- ψ: Stream function
- η: Dimensionless similarity variable
- μ: Dynamic viscosity of the base fluid(*kg/m.s*)
- υ: Kinematic viscosity ($m^2\ s^{-1}$)
- ρ_f: Density of the fluid ($Kg\ m^{-3}$)
- ρ_p: Density of the nanoparticle ($Kg\ m^{-3}$)
- τ: The ratio of the nanoparticle heat capacity and the base fluid heat capacity
- $(\rho c)_f$: Heat capacity of the base fluid ($kg/m.s^2$)
- $(\rho c)_p$: Heat capacity of the nano particle ($kg/m.s^2$)
- ϕ_W: Nanoparticle volume fraction at wall temperature
- ϕ_∞: Ambient nanoparticle volume fraction
- γ: Chemical Reaction Parameter

Sub Scripts:

- W: Condition at the surface
- ∞: Ambient Conditions