On $i\pi gs$- Homeomorphism in Intuitionistic Fuzzy Topological spaces

Dr. K. Ramesh
Department of Mathematics, Jayirams Arts and Science college, Karur, Tamilnadu.

Abstract *- This paper introduces intuitionistic fuzzy π generalized semi homeomorphism and intuitionistic fuzzy iπ generalized semi homeomorphism in intuitionistic fuzzy topological spaces. Also they have been related to the fundamental concepts of intuitionistic fuzzy continuous mappings and intuitionistic fuzzy open mappings.*

Keywords - *Intuitionistic fuzzy topology, intuitionistic fuzzy π generalized semi closed set, intuitionistic fuzzy π generalized semi continuous mapping, intuitionistic fuzzy π generalized semi homeomorphism, intuitionistic fuzzy i π generalized semi homeomorphism.*

1.Introduction The concept of fuzzy sets was introduced by Zadeh [13] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy π generalized semi homeomorphism and intuitionistic fuzzy i π generalized semi homeomorphism. Also we have studied the relations with the fundamental concepts of intuitionistic fuzzy continuous mappings and intuitionistic fuzzy open mappings. We have arrived at some characterizations of intuitionistic fuzzy π generalized semi homeomorphism.

2. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$,where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

Definition 2.3:[3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Definition 2.5:[8] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) ⊆ A,

(ii) intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A)) ⊆ A.

Definition 2.6:[9] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy π - generalized closed set (IFπGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ).

Result 2.7:[9] Let A be an IFS in (X, τ), then

(i) scl(A) = A ∪ int(cl(A)),

(ii) sint(A) = A ∩ cl(int(A)).

Result 2.8: [9] (i) Every IFπOS is an IFOS in (X, τ).

(ii) Every IFπCS is an IFCS in (X, τ).

Definition 2.9:[8] An IFS A in an IFTS (X, τ) is an

(i) intuitionistic fuzzy generalized closed set (IFGCS) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

(ii)intuitionistic fuzzy alpha generalized closed set (IFαGCS) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Definition 2.10:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(i) intuitionistic fuzzy continuous (IF continuous) if f^{-1}(B) ∈ IFO(X, τ) for every B ∈ σ.

(ii)intuitionistic fuzzy semi continuous (IFS continuous) if f^{-1}(B) ∈ IFSO(X, τ) for every B ∈ σ.

(iii)intuitionistic fuzzy α-continuous (IFα continuous) if f^{-1}(B) ∈ IFαO(X, τ) for every B ∈ σ.

(iv)intuitionistic fuzzy generalized continuous (IFG continuous) if f^{-1}(B)∈IFGC(X, τ) for every IFCS B in (Y, σ).

(v)intuitionistic fuzzy generalized semi continuous (IFGS continuous) if f^{-1}(B) ∈IFGSC(X, τ) for every IFCS B in (Y, σ).

(vi)intuitionistic fuzzy α-generalized continuous (IFαG continuous) if f^{-1}(B)∈ IFαGC(X) for every IFCS B in (Y, σ).

Definition 2.11:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy π generalized semi open mapping (IFπGS open mapping) if f (A) ∈ IFπGSOS (X, τ) for every IFOS A in (X, τ).

Definition 2.12:[11] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy π - generalized semi closed mapping* (IFπGS closed) if f (A) is an IFπGSCS in (Y,σ) for every IFCS A of (X, τ).

Definition 2.13:[9] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then the map $f$is said to be an intuitionistic fuzzy π- *generalized semi irresolute* (IFπGS irresolute in short) if f^{-1}(B) ∈ IFπGCS(X) for every IFπGCS B in (Y, σ).

Definition 2.14:[8] Let f be a bijection mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(i) intuitionistic fuzzy homeomorphism (IF homeomorphism) if f and f^{-1} are IF continuous mappings.

(ii) intuitionistic fuzzy semi homeomorphism (IFS homeomorphism) if f and f^{-1} are IFS continuous mappings.

(iii) intuitionistic fuzzy alpha homeomorphism (IFα homeomorphism in short) if f and f^{-1} are IFα continuous mappings.

(iv) intuitionistic fuzzy generalized homeomorphism (IFG homeomorphism in short) if f and f^{-1} are IFG continuous mappings.

(v) intuitionistic fuzzy generalized semi homeomorphism (IFGS homeomorphism in short) if f and f^{-1} are IFGS continuous mappings (vi)intuitionistic fuzzy alpha generalized homeomorphism (IFαG homeomorphism in short) if f and f^{-1} are IFαG continuous mappings.

Definition 2.15:[9] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi T_{1/2}$ (in short IF$\pi T_{1/2}$) space if every IFπGCS in (X, τ) is an IFCS in (X, τ).

Definition 2.16:[9] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi g T_{1/2}$ (in short IF$\pi g T_{1/2}$) space if every IFπGCS in (X, τ) is an IFGCS in (X, τ).

3. INTUITIONISTIC FUZZY π GENERALIZED SEMI HOMEOMORPHISMS

In this section, we introduce intuitionistic fuzzy π generalized semi homeomorphism and study some of its properties.

Definition 3.1: A bijection mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy π generalized semi homeomorphism (briefly IFπGS homeomorphism) if f and f^{-1} are IFπGS continuous mappings.

We denote the family of all IF π GS homeomorphisms of an IFTS (X, τ) onto itself by **IFπGS-*h*(X, τ).**

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.12, 0.12), (0.54, 0.63) ⟩, G_2 = ⟨ y, (0.33, 0.61), (0.31, 0.12⟩. Then, τ = {$0_\sim$, G_1, $1_\sim$} and σ = { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is an IFπGS continuous mapping and f^{-1} is also an IFπGS continuous mapping. Therefore, f is an IF π GS homeomorphism.

Proposition 3.3: Every IF homeomorphism is an IFπGS homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be an IF homeomorphism. Then f and f^{-1} are IF continuous and f is bijection. Since every IF continuous mapping is IFπ GS continuous mapping, we have f and f^{-1} are IFπGS continuous mappings. Therefore, f is IFπGS homeomorphism.

The converse of the above proposition need not be true as seen from the following example.

Example 3.4: Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.31, 0.21), (0.62, 0.71) ⟩, G_2 = ⟨ y, (0.51, 0.41), (0.4, 0.21) ⟩. Then, τ = {$0_\sim$, G_1, $1_\sim$} and σ = { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a bijection mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then, f is an IFπGS homeomorphism but not an IF homeomorphism since f and f^{-1} are not IF continuous mappings.

Proposition 3.5 : Every IFS homeomorphism is an IFπGS homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFS homeomorphism. Then, f and f^{-1} are IFS continuous mappings. Since every IFS continuous mapping is an IFπGS continuous mapping, f and f^{-1} are IFπGS continuous mappings. Therefore, f is an IFπGS homeomorphism.

Example 3.6 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.421, 0.31), (0.43, 0.421) ⟩, G_2 = ⟨ y, (0.123, 0.12), (0.61, 0.723) ⟩. Then, τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then, f is an

IFπGS homeomorphism. For an IFCS A = ⟨ y, (0.61, 0.723), (0.123, 0.12) ⟩ in (Y, σ), f^{-1}(A) = ⟨ x, (0.61, 0.723), (0.123, 0.12) ⟩ is not an IFSCS in (X, τ). This implies f is not an IFS continuous mapping. Hence f is not an IFS homeomorphism.

Proposition 3.7 : Every IFGS homeomorphism is an IFπGS homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be an IFGS homeomorphism. Then f and f^{-1} are IFGS continuous mappings. Since every IFGS continuous mapping is an IFπGS continuous mapping, f and f^{-1} are IFπGS continuous mappings. Therefore, f is an IFπGS homeomorphism.

The converse of the above proposition need not be true as seen from the following example.

Example 3.8 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0, 0.91), (0.53, 0.12) ⟩, G_2 = ⟨ y, (0.7, 0.7), (0, 0.31) ⟩. Then, τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then, f is an IFπGS homeomorphism. For an IFCS A = ⟨ y, (0, 0.31), (0.7, 0.7) ⟩ in (Y, σ), f^{-1}(A) = ⟨ x, (0, 0.31), (0.7, 0.7) ⟩ is not an IFGSCS in (X, τ). This implies that f is not an IFGS continuous mapping. Hence f is not an IFGS homeomorphism.

Proposition 3.9 : Every IFα homeomorphism is an IFπGS homeomorphism but the converse need not true.

Proof: Let f: (X, τ) → (Y, σ) be an IFα homeomorphism. Then f and f^{-1} are IFα continuous mappings. Since every IFα continuous mapping is an IFπGS continuous mapping, f and f^{-1} are IFπGS continuous mappings. Therefore, f is an IFπGS homeomorphism.

Example 3.10 : Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.543, 0.421), (0.52, 0.632) ⟩, G_2 = ⟨ y, (0.212, 0.21), (0.7, 0.842) ⟩. Then, τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then, f is an IFπGS homeomorphism. For an IFCS A = ⟨ y, (0.7, 0.842), (0.212, 0.21) ⟩ in (Y, σ). Then f^{-1}(A) = ⟨ x, (0.7, 0.842), (0.212, 0.21) ⟩ is not an IFαCS in (X, τ). This implies that f is not an IFα continuous mapping. Hence f is not an IFα homeomorphism.

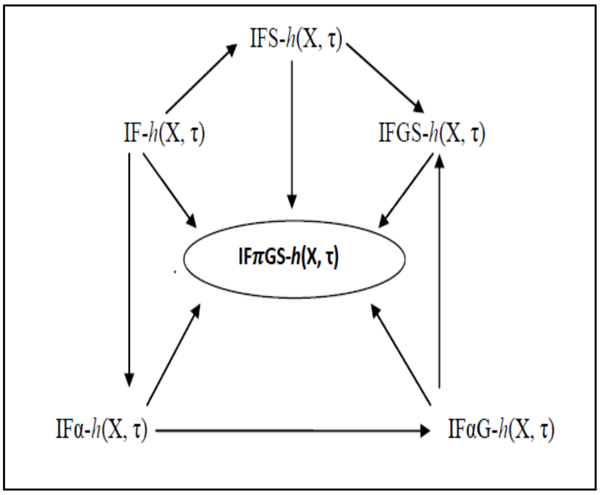
Proposition 3.11: Every IFαG homeomorphism is an IFπGS homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFαG homeomorphism. Then f and f^{-1} are IFαG continuous mappings. Since every IFαG continuous mapping is an IFπGS continuous mapping, f and f^{-1} are IFπGS continuous mappings. Therefore, f is an IFπGS homeomorphism.

Example 3.12: Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.21, 0.32), (0.41, 0.52) ⟩, G_2 = ⟨ y, (0.51, 0.612), (0.12, 0) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then f is an IFπGS homeomorphism. For an IFCS A = ⟨ y, (0.12, 0), (0.51, 0.612) ⟩ in (Y, σ). Then f^{-1}(A) = ⟨ x, (0.12, 0), (0.51, 0.612) ⟩ is not an IFαGCS in (X, τ). This implies that f

is not an IFαG continuous mapping. Hence f is not an IFαG homeomorphism.

From the above discussions and known results we have the following diagram.



Theorem 3.13: Let f: (X, τ) → (Y, σ) be an IFπGS homeomorphism. Then f is an IF homeomorphism if (X, τ) and (Y, σ) are IFπ$T_{1/2}$ space.

Proof: Let B be an IFCS in (Y, σ). Since f is an IFπGS homeomorphism, f^{-1}(B) is an IFπGSCS in (X, τ). Since (X, τ) is an IFπ$T_{1/2}$ space, f^{-1}(B) is an IFCS in (X, τ). Hence f is an IF continuous mapping. Also, f^{-1}: (Y, σ) → (X, τ) is a IFπGS continuous mapping. Let A be an IFCS in (X, τ). Then, $(f^{-1})^{-1}$(A) = f (A) is an IFπGSCS in (Y, σ). Since (Y, σ) is an IFπ$T_{1/2}$ space, f (A) is an IFCS in (Y, σ). Hence f^{-1} is an IF continuous mapping. Therefore, the mapping f is an IF homeomorphism.

Theorem 3.14: Let f: (X, τ) → (Y, σ) be an IFπGS homeomorphism. Then f is an IFG homeomorphism if (X, τ) and (Y, σ) are IFπ$T_{1/2}$ space.

Proof: Let B be an IFCS in (Y, σ). Since f is an IFπGS homeomorphism, f^{-1}(B) is an IFπGSCS in (X, τ). Since (X, τ) is an IFπ$T_{1/2}$ space, f^{-1}(B) is an IFGCS in (X, τ). Hence f is an IFG continuous mapping. Also, f^{-1}: (Y, σ) → (X, τ) is a IFπGS continuous mapping. Let A be an IFCS in (X, τ). Then, $(f^{-1})^{-1}$(A) = f (A) is an IFπGSCS in (Y, σ). Since (Y, σ) is an IFπ$T_{1/2}$ space, f(A) is an IFGCS in (Y, σ). Hence f^{-1} is an IFG continuous mapping. Therefore, the mapping f is an IFG homeomorphism.

Theorem 3.15: Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IFπGS continuous mapping, then the following are equivalent.

(i) f is an IFπGS closed mapping.

(ii) f is an IFπGS open mapping.

(iii) f is an IFπGS homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be a bijective mapping and f be an IFπGS continuous mapping.

(i) ⇒ (ii): let f be an IFπGS closed mapping. This implies that f^{-1}: (Y, σ) → (X, τ) is IFπGS continuous mapping. Since every IFOS in (X, τ) is an IFπGSOS in (Y, σ). Hence f^{-1} is an IFπGS open mapping.

(ii) ⇒ (iii): let f be an IFπGS open mapping. This implies that f^{-1}: (Y, σ) → (X, τ) is IFπGS continuous mapping. Hence f and f^{-1} are IF π GS continuous mappings. Therefore, f is an IFπGS homeomorphism.

(iii) ⇒ (i): Let f is an IFπGS homeomorphism. Then, f and f^{-1} are IFπGS continuous mappings. Since every

IFCS in (X, τ) is an IFπGSCS in (Y, σ), f is an IFπGS closed mapping.

Remark 3.16: The composition of two IF π GS homeomorphisms need not be an IF π GS homeomorphism in general. This can be shown from the following example.

Example 3.17: Let X = {a, b}, Y = {c, d} and Z = {u, v}. Let G_1 = ⟨x, (0.81, 0.612), (0.221, 0.432)⟩, G_2 = ⟨ y, (0.61, 0.12), (0.42, 0.31) ⟩, G_3 = ⟨ z, (0.21, 0.41), (0.51, 0.41) ⟩, G_4 = ⟨ z, (0.11, 0.31), (0.31, 0.41) ⟩, G_5 = ⟨ z, (0.11, 0.31), (0.51, 0.41) ⟩, G_6 = ⟨ z, (0.21, 0.41), (0.31, 0.41) ⟩, and G_7 = ⟨ z, (0.41, 0.41), (0.31, 0.41) ⟩. Then τ = {$0_\sim$, G_1, $1_\sim$}, σ = { $0_\sim$, G_2, $1_\sim$} and η = { $0_\sim$, G_3, G_4, G_5, G_6, G_7, $1_\sim$} are IFTs on (X, τ), (Y, σ) and (Z, η) respectively. Define bijection mapping f : (X, τ) → (Y, σ) by f (a) = c , f (b) = d and g : (Y, σ) → (Z, η) by f (c) = u , f (d) = v. Then, f and f^{-1} are IFπGS continuous mappings. Also g and g^{-1} are IFπGS continuous mappings. Hence f and g are IFπGS homeomorphisms. But the composition g o f: (X, τ) → (Z, η) is not an IFπGS homeomorphism since g o f is not an IFπGS continuous mapping.

4. INTUITIONISTIC FUZZY Iπ GENERALIZED SEMI HOMEOMORPHISMS

Definition 4.1: A bijection mapping f : (X, τ) → (Y,σ) is called an intuitionistic fuzzy i π generalized semi homeomorphism (IFiπGS homeomorphism in short) if f and f^{-1} are IFπGS irresolute mappings.

We denote the family of all IFi π GS-homeomorphism of a topological space (X, τ) onto itself by **IFiπGS-*h*(X, τ)**.

Theorem 4.2: Every IFi π GS homeomorphism is an IFπGS homeomorphism but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IFi π GS homeomorphism. Then, f and f^{-1} are IFπGS irresolute mappings. Since every IFπGS irresolute mapping is an IFπGS continuous mapping. Therefore, f and f^{-1} are IFπGS continuous mappings and hence f is an IFπGS homeomorphism.

Example 4.3: Let X = {a, b}, Y = {u, v} and G_1 = ⟨ x, (0.21, 0.41), (0.51, 0.41) ⟩, G_2 = ⟨ x, (0.11, 0.31), (0.31, 0.41) ⟩, G_3 = ⟨ x, (0.11, 0.31), (0.51, 0.41) ⟩, G_4 = ⟨ x, (0.21, 0.41), (0.31, 0.41) ⟩, G_5 = ⟨ x, (0.41, 0.41), (0.31, 0.41) ⟩ and G_6 = ⟨ y, (0.21, 0.11), (0.41, 0.51) ⟩. Then τ = {$0_\sim$, G_1, G_2, G_3, G_4, G_5, $1_\sim$} and σ = { $0_\sim$, G_6, $1_\sim$} are IFTs on X and Y respectively. Define bijection mapping f: (X, τ) → (Y, σ) by f (a) = u and f (b) = v. Then, f is an IF π GS homeomorphism. But f is not an IFi π GS homeomorphism. For an IFS G = ⟨ y, (0, 0.3), (0.5, 0.4) ⟩ in (Y, σ), G is an IFπGSCS in (Y, σ), but f^{-1}(G) is not an IFπGSCS in (X, τ) and therefore f is not an IFπGS irresolute mapping.

Theorem 4.4: If f: (X, τ) → (Y, σ) is an IFi π GS homeomorphism, then, πgscl(f^{-1}(B)) = f^{-1}(πgscl(B)) for every IFS B in (Y, σ).

Proof: Since f is an IFiπGS homeomorphism, f : (X, τ) → (Y, σ) is an IFπGS irresolute mapping. Consider an

IFS B in (Y, σ). Clearly, πgscl(B) is an IFπGSCS in (Y, σ). Since f is an IFπGS irresolute mapping, f^{-1}(πgscl(B)) is an IFπGSCS in (X, τ). Since $f^{-1}(B) \subseteq f^{-1}$(πgscl(B)), πgscl($f^{-1}$(B)) ⊆ πgscl($f^{-1}$(πgscl(B))) = f^{-1}(πgscl(B))). This implies that πgscl (f^{-1}(B)) ⊆ f^{-1}(πgscl(B)).Since f is an IFiπGS homeomorphism, f^{-1}: (Y, σ) → (X, τ) is an IFπGS irresolute mapping. Consider an IFS f^{-1}(B) in (X, τ). Clearly, πgscl(f^{-1}(B)) is an IFπGSCS in (X, τ). This implies that $(f^{-1})^{-1}$(πgscl(f^{-1}(B))) = f (πgscl(f^{-1}(B))) is an IFπGSCS in (Y, σ). Clearly, B = $(f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}$(πgscl($f^{-1}$(B))) = f (πgscl(f^{-1}(B))). Therefore, πgscl(B)⊆ πgscl(f (πgscl(f^{-1}(B)))) = f (πgscl(f^{-1}(B)), since f^{-1} is an IFπGS irresolute mapping. Hence f^{-1}(πgscl(B)) ⊆ f^{-1}(f (πgscl(f^{-1}(B)) = πgscl(f^{-1}(B)). Therefore, f^{-1}(πgscl(B)) ⊆ πgscl(f^{-1}(B)). This implies that πgscl(f^{-1}(B)) = f^{-1}(πgscl(B)).

Corollary 4.5: If f: (X, τ) → (Y, σ) is an IFiπGS homeomorphism, then πgscl(f (B)) = f (πgscl(B)) for every IFS B in (X, τ).

Proof: Since f is an IFiπGS homeomorphism, f^{-1} is an IFiπGS homeomorphism. Let B be an IFS in (X, τ). By Theorem 6.3.4, πgscl(f (B)) = f (πgscl(B)) for every IFS B in (X, τ).

Corollary 4.6: If f : (X, τ) → (Y, σ) is an IFiπGS homeomorphism, then f(πgsint(B)) = πg sint(f (B)) for every IFS B in (X, τ).

Proof : For any IFS B in (X, τ), πgsint(B) = (πgscl(B^c))^c, By Corollary 6.3.5, f (πgsint(B)) = f(πgscl(B^c))^c) = (f(πgscl(B^c)))^c = (πgscl(f(B^c)))^c. This implies that f(πgsint(B)) = (πgscl (f (B))^c)^c = πgsint(f (B)).

Corollary 4.7: If f : (X, τ) → (Y, σ) is an IFiπGS homeomorphism, then f^{-1} (sint(B)) = sint (f^{-1}(B)) for every IFS B in (X, τ).

Proof : Since f^{-1}: (Y, σ) → (X, τ) is also an IFiπGS homeomorphism, the proof follows from Corollary 4.6.

Proposition 4.8: If f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are IFi π GS homeomorphisms then their composition g o f : (X, τ) → (Z, η) is also an IFiπGS homeomorphisms.

Proof: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) be any two IFiπGS homeomorphisms. Therefore, f, f^{-1}, g and g^{-1} are IFπGS irresolute mappings. By Theorem, g o f and (g o f)$^{-1}$ = f^{-1}o g^{-1} are IFπGS irresolute mappings and (g o f) is an IFiπGS homeomorphism.

Proposition 4.9: The set IFiπGS-h(X, τ) is a group under the composition of maps.

Proof : Define a binary operation * : IFiπGS-h(X, τ) x IFiπGS-h(X, τ) → IFiπGS-h(X, τ) by $f * g = g \circ f$ for all f, g ∈ IFiπGS-h(X, τ) and ∘ is the usual operation of composition of maps.

(i) **Closure Property**: Let f ∈ IFiπGS-h(X, τ) and g ∈ IFiπGS-h(X, τ). By Theorem 6.3.8, g ∘ f ∈ IFiπGS-h(X, τ).

(ii) **Associative property** : We know that the composition of mappings is associative.

(iii) **Existence of identity** : The identity mappings I : (X, τ) → (X, τ) belonging to IFiπGS-*h*(X, τ) servers as the identity element.

(iv) **Existence of inverse** : If $f \in$ IFiπGS-*h*(X, τ), then $f^{-1} \in$ IFiπGS-*h*(X, τ) such that $f^{-1} * f = f \circ f^{-1} = I$. Therefore, inverse exists for each element of IFiπGS-*h*(X, τ).

Therefore, (IFiπGS-*h*(X, τ) , ∘) is a group under the operation of composition of maps.

Theorem 4.10: Let f: (X, τ) → (Y, σ) be a IFiπGS homeomorphism. Then f induces an isomorphism from the group IFiπGS-*h*(X, τ) onto the group IFiπGS-*h*(Y, σ).

Proof : Let f: (X, τ) → (Y, σ) be a IFiπGS homeomorphism. We define a map θ_f : IFiπGS-*h*(X, τ) → IFiπGS-*h*(Y, σ), by θ_f (h): $f \circ h \circ f^{-1}$ for every h ∈ IFiπGS-*h*(X, τ), using the mapping f.

Clearly, we have to prove that θ_f is a homeomorphism. $\theta_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1) \circ (h_2 \circ f^{-1}) = (f \circ h_1) \circ (f^{-1} \circ f) \circ (h_2 \circ f^{-1}) = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f(h_1) \circ \theta_f(h_2)$, for all $h_1, h_2 \in$ IFiπGS-*h*(X, τ). Therefore, θ_f is a homeomorphism. Hence f induces an isomorphism from the group IFiπGS-*h*(X, τ) onto the group IFiπGS-*h*(Y, σ).

REFERENCES

[1] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.

[2] C.L.Chang, ″Fuzzy topological spaces″, J.Math.Anal.Appl, 24(1968), 182-190.

[3] D.Coker, ″An introduction to intuitionistic fuzzy topological spaces″, Fuzzy sets and systems, 88(1997), 81-89.

[4] El-Shafhi, M.E., and Zhakari, A., Semi generalized continuous mappings in fuzzy topological spaces, J. Egypt. Math. Soc. 15 (1) (2007), 57-67.

[5] H.Gurcay, A.Haydar and D.Coker, ″On fuzzy continuity in intuitionistic fuzzy topological spaces″, jour.of fuzzy math, 5(1997), 365-378.

[6] Joung Kon Jeon, Young Bae Jun and Jin Han Park, ″ Intuitionistic fuzzy alpha -continuity and intuitionistic fuzzy pre continuity″, International journal of Mathematical and Mathematical Sciences, 2005, 3091-3101.

[7] Neelamegarajan Rajesh et al On $\hat{g}$ semi homeomorphism in topological spaces. Annals of University of cralova math.comp.sci. ser. 33(2006), 208-215.

[8] Santhi, R., and Sakthivel.K., Alpha Generalized semi homeomorphism in Intuitionistic fuzzy topological spaces, NIFS 17 (2011), 1, 30–36.

[9] S.Maragathavalli and K.Ramesh, A note on intuitionistic Fuzzy πGeneralized Semi Irresolute Mappings, International journal of Mathematical Archive-3(3),2012,1-7.

[10] S.Maragathavalli and K.Ramesh ,Intuitionistic fuzzy π -generalized semi continuous mappings, International Journal of Computer Applications, 37 (2012), 30-34.

[11] S.S.Thakur and Rekha Chaturvedi, ″R.G-closed sets in intuitionistic fuzzy topological spaces″, Universitatea Din Bacau Studii Si Cercertar Stiintifice, 6(2006), 257-272.

[12] L.A.Zadeh, ″Fuzzy sets″, Information control, 8 (1965), 338-353.