

Brownian moment and thermophoresis effects on unsteady liquid film flow of MHD Carreau fluid over a stretching sheet

K.Avinash¹ R.Hemadri Reddy^{2*}

¹Department of Mathematics, Sreyas Institute of Engineering and Technology, Hyderabad-500068, India.

²Department of Mathematics, VIT University, Vellore- 632014, India.

Abstract

*In this study, we analyzed the flow, heat and mass transfer nature of magnetohydrodynamic (MHD) steady/unsteady liquid film flow of Carreau fluid past a stretching sheet in the presence of thermophoresis and Brownian moment effects. Similarity transformation is used to convert the partial differential equations to nonlinear ordinary differential equations. Further, the transformed equations are solved numerically by employing *bvp5c* Matlab package. The effects of various controlling flow parameters on the flow, heat and mass transfer are examined and discussed in detail in steady and unsteady cases. It is found that flow, thermal and concentration boundary layers are non-uniform for steady and unsteady cases. Brownian moment and thermophoresis parameters regulate the heat and mass transfer rates.*

Keywords: MHD, thin film flow, Carreau fluid, thermophoresis, Brownian motion.

1. Introduction

The heat and mass transfer phenomenon of the Carreau fluid flow past a stretching surface is very significant for its day to day growing industrial applications such as rubber sheets, glass blowing, plastic extrusion, drying/cooling of textiles and papers, textile production, crystal growing and fiber spinning. The eminence of ultimate industrial products muscally depends on the mass and heat transfer characteristics. Keeping this into a view, the study of 2D flow past a permeable stretching surface was initially examined by Crane [1]. Further, Zhu [2] examined the mass transfer and drag flow of non-Newtonian fluid over a group of Newtonian drops. Anderson [3] discussed the slip boundary condition towards a stretching surface. The heat transfer and MHD boundary layer flow of Carreau fluid past stretching surface was investigated by Khan and Hashim [4]. The Carreau fluid is also a non-Newtonian fluid, and it is differentiated by the mixture of Methylcellulose tylose (1%), hydroxyethyl-cellulose Natrosol HHX (0.3%) in glycerol solution and poly ethylene oxide was explained by Siska et al. [5] and Hyun et al. [6].

The 2D MHD peristaltic transport of an incompressible non-Newtonian fluid over a channel was studied by Hayat et al. [7] and found that the rising values of the magnetic field parameter depreciate the Axial induced magnetic field. An unsteady 3D flow of Carreau-Casson fluids over a stretching sheet was presented by Raju and Sandeep [8]. Akbar et al. [9] studied the numerical solution of peristaltic flow of a non-Newtonian fluid over a channel. Blood flow during a conical artery of a Carreau fluid in the presence of stenosis was discussed by Akbar and Nadeem [10]. Ibrahim and Shankar [11] investigated the boundary layer flow towards a stretching surface with radiation and Brownian motion effects and concluded that the increasing values of slip parameter depreciate the local Nusselt number. The effect of cross diffusion on the convection flow of a Carreau fluid past wedge with thermal radiation effects was explained by Raju et al. [12]. The impact of Brownian motion effects on heat transfer of a nanofluid over a plate was presented Haddad et al. [13]. Sheikholeslami and Ganji [14] examined the 3D mass and heat transfer of a nanofluid flow over a rotating disk. The effect of thermophoresis mass and heat transfer of a nanofluid towards a stretching surface was studied by Khan et al. [15].

JayachandraBabu and Sandeep [16] presented the MHD three dimensional flow of non-Newtonian fluid towards a stretching surface in the presence of Brownian motion effect and solved numerically by R-K and Newton's methods. Analytical solution of mass transfer in MHD nanofluid flow past a stretching surface with thermophoresis effect was pioneered by Freidoonimehr et al. [17]. De et al. [18] discussed the MHD convective radiative mass transfer of a non-Newtonian fluid towards a stretching surface with Brownian motion effect and numerically solved by using shooting method. The heat transfer in free convective MHD flow of a Carreau fluid towards a stretching surface was investigated by Sathish Kumar et al. [19]. Khan and Azam [20] discussed the unsteady mass and heat transfer MHD flow of non-Newtonian fluids induced in a stretching surface with thermophoresis effect. Very recently, the researchers [21-29] investigated the flow and heat transfer nature of the

magnetohydrodynamic Newtonian and non-Newtonian flows by considering the various flow geometries. The heat transfer nature of a nano-liquid film over an unsteady stretching surface was numerically investigated by Xu et al. [30].

By keeping in the view of all the above applications and the investigations, in this study, we analyzed the flow, heat and mass transfer behavior of magnetohydrodynamic (MHD) unsteady liquid film flow of Carreau fluid past a stretching sheet in the presence of thermophoresis and Brownian moment effects. Numerical solutions are obtained and the effects of various controlling flow parameters on flow, heat and mass transfer are examined and discussed in detail for steady and unsteady cases.

2. Mathematical formulation

Two-dimensional, electrically conducting unsteady flow, heat and mass transfer of Carreau fluid over stretching surface. The elastic sheet is located at the origin of a coordinate system (x, y) as shown in Fig.1. Here x -axis is taken along the stretching surface with stretched velocity $u_w(x, t) = bx / (1 - \alpha t)$, where b, α constants and y -axis is normal to it. The wall temperature and nanoparticle volume fraction are considered as $T_s(x, t) = T_0 - T_r(bx^2 / 2\nu_f)(1 - \alpha t)^{-1.5}$ and $C_s(x, t) = C_0 - C_r(bx^2 / 2\nu_f)(1 - \alpha t)^{-1.5}$, where T_0, C_0, T_r, C_r are the slit and reference temperatures and concentrations and ν_f is the kinematic viscosity of the base fluid. A magnetic field of strength $B(t) = B_0(1 - \alpha t)^{-0.5}$ is applied along the stretching sheet. The magnetic Reynolds number is assumed to be small. Thermophoresis and Brownian moment effects are taken into account.

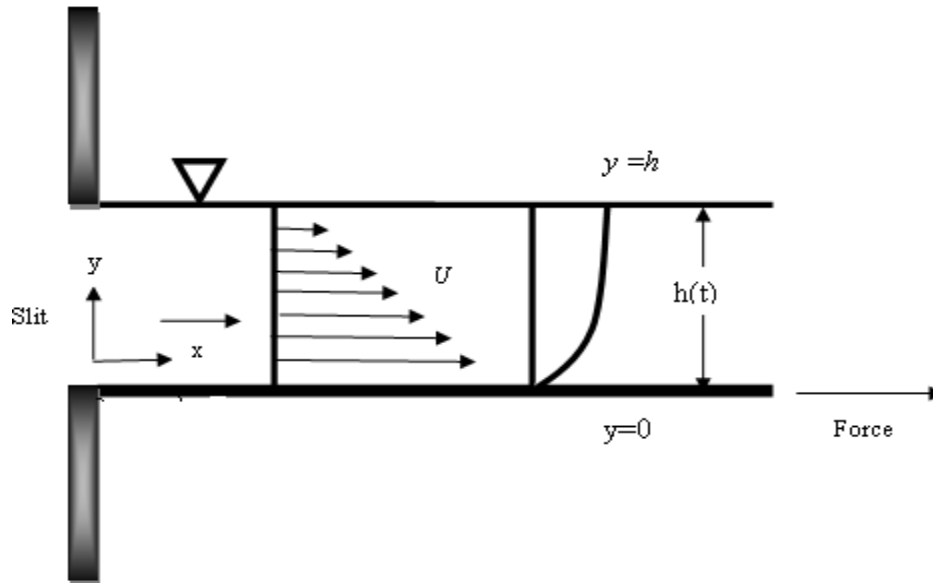


Fig. 1 Physical model of the problem

With the above assumptions, the governing conservation equations in unsteady state can be expressed as

$$u_x + v_y = 0, \quad (1)$$

$$u_t + uu_x + vv_y = \nu \left(1 + 1.5(n-1)\Gamma^2 (u_y)^2 \right) u_{yy} - \frac{\sigma B^2(t)}{\rho} u, \quad (2)$$

$$(\rho c_p)_f (T_t + uT_x + vT_y) = kT_{yy} + (\rho c_p)_s [D_B C_y T_y + (D_T / T_r) T_y^2], \quad (3)$$

$$C_t + uC_x + vC_y = D_B C_{yy} + (D_T / T_r) T_{yy}, \quad (4)$$

The boundary conditions for the present problem are

$$\begin{aligned} u = u_w, v = 0, T = T_s, C = C_s \quad \text{at } y = 0, \\ u_y = 0, v = h_t, T_y = 0, C_y = 0 \quad \text{at } y = h(t), \end{aligned} \quad (5)$$

where u and v are the velocity components along x and y directions respectively, ρ is the fluid density, μ is dynamic viscosity, σ is the electrical conductivity, $B(t)$ is the applied magnetic field, T is the fluid temperature, C is the fluid concentration, k represent the thermal conductivity, ρc_p represent the heat capacitance, D_B is the Brownian diffusion coefficient, Γ is the time constant and n is the power-law index and suffixes indicate the partial differentiation with respect to that independent variable.

We use the following similarity transformations

$$\begin{aligned} \eta = \frac{1}{\beta} \left(\frac{b}{v_f(1-\alpha t)} \right)^{0.5} y, \psi = \beta \left(\frac{v_f b}{(1-\alpha t)} \right)^{0.5} x f(\eta), \theta = \frac{(T - T_0)}{(T_s - T_0)}, \\ T_s = T_0 - T_r (bx^2 / 2v_f)(1-\alpha t)^{-1.5} \theta(\eta), \phi = \frac{(C - C_0)}{(C_s - C_0)}, \\ C_s = C_0 - C_r (bx^2 / 2v_f)(1-\alpha t)^{-1.5} \phi(\eta), \end{aligned} \quad (6)$$

we define the stream function ψ as $u = \psi_y$ and $v = -\psi_x$.

with the help of Eq. (6), the Eqs. (2) to (4) transformed as

$$\left(1 + 1.5(n-1)Wef''\right) f'''' + \lambda \left(ff'' - f'^2 - S \left(f' + \frac{\eta}{2} f'' \right) \right) - Mf' = 0, \quad (7)$$

$$\theta'' - Pr \lambda \left(2f'\theta + \frac{S}{2}(3\theta + \eta\theta') - f\theta' \right) + (Nb\theta'\phi' + Nt\theta'^2) = 0, \quad (8)$$

$$\phi'' - Sc\lambda \left(2f'\phi - f\phi' + \frac{S}{2}(3\phi + \eta\phi') \right) + \frac{Nt}{Nb} \theta'' = 0, \quad (9)$$

with the associated boundary conditions are

$$\begin{aligned} f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1, \\ f(1) = S/2, f''(1) = 0, \theta'(1) = 0, \phi'(1) = 0, \end{aligned} \quad (10)$$

where primes denotes the differentiation with respect to η , the characterized Prandtl number, magnetic field parameter and unsteadiness parameter, Weissenberg number, Brownian motion parameter, thermophoretic parameter which are defined respectively

$$Pr = \frac{v_f}{\alpha_f}, M = \frac{\sigma B_0^2}{b\rho_f}, S = \frac{\alpha}{b}, We = \frac{\Gamma^2 x^2 b^3}{(1-\alpha t)^3}, Nb = \frac{(\rho c_p)_s D_B C_s}{(\rho c_p)_f \alpha}, Nt = \frac{(\rho c_p)_s D_T T_s}{(\rho c_p)_f \alpha T_r}, \quad (11)$$

where $\lambda = \beta^2$ is the dimensionless film thickness and β is an unknown constant defined by

$$\beta = \left(\frac{hb}{v_f} \right) (1 - \alpha t)^{-0.5}, \quad (12)$$

The physical quantities of engineering interest, the skin friction at the surface C_{f_x} , local Nusselt number Nu_x and local Sherwood number Sh_x are given by

$$\text{Re}_x^{0.5} C_{f_x} = \left[f''(0) + 1.5(n-1)We(f''(0))^3 \right] f''(0) = 0, \text{Re}_x^{-0.5} Nu_x = -\theta'(0), \text{Re}_x^{-0.5} Sh_x = -\phi'(0), \quad (13)$$

where $\text{Re}_x = \frac{u_w x}{v_f}$ is the local Reynolds number.

3. Results and Discussion

The set of nonlinear ordinary differential equations (7)-(9) with the restrictions (10) is resolved numerically by employing `bvp5c` Matlab package. We reveal the results to keep up the influence of several non-dimensional parameters on the three usual profiles (velocity, temperature and concentration). Also we examined the same parameters on skin friction coefficient, heat and mass transfer rate with the aid of tables. In this paper, we consider the pertinent parameters as $Pr = 6.8, M = 1, Nb = 0.5, n = 1.5, We = 1.5, \lambda = 0.3, Nt = 0.5, Sc = 0.6$. These values are maintained as invariable in this study unless the varied parameters as depicted in the figures.

Figs. 2-4 depict the influence of magnetic field parameter on the momentum, thermal and concentration profiles. It is clear that boosting values of the external magnetic field suppress the momentum boundary layer and develops the thermal and concentration fields. In particular, the drag force generated by the external magnetic field is highly influenced the thermal and concentration fields in steady case when compared to unsteady case.

The effect of Weissenberg number on concentration, temperature and flow fields are displayed in Figs. 5-7. It is observed that the increasing values of the Weissenberg number suppress the thermal boundary layer thickness and enhances the momentum boundary layer thickness. But interestingly it declines the concentration field in unsteady case and enhances the same in unsteady case. This may be happen due to the fact that the rising value of We causes to change the viscosity of the fluid. This leads to decline the thermal field. A variation in the diffusion, energy and velocity fields at various values of the film thickness parameter is illustrated in Figs. 8-10. It is evident that enhancing the film thickness depreciates the temperature and flow fields. But it causes to boost the concentration profiles in both cases.

Figs. 11-14 displays the influence of thermophoresis and Brownian moment parameters on temperature and concentration fields. Physically, thermophoresis and Brownian moment parameters regulate the thermal and concentration fields due to the random motion of the particles. The similar behavior reflects by the graphs 11-14. That is rising values of the thermophoresis parameter enhances the temperature and concentration fields. But the Brownian motion parameter showed the mixed response in temperature and concentration fields.

The effect of Prandtl number on concentration and temperature fields are displayed in Figs. 15 and 16. It is evident that the rising values of the Prandtl number declines the thermal field and enhances the concentration field. This leads to enhance the heat transfer rate and suppress the mass transfer rate. The impact of Schmidt number on concentration field is depicted in Fig. 17. It is noticed that the rising values of the Schmidt number depreciate the concentration field.

Tables 1 and 2 illustrate the impact of various pertinent parameters on wall friction, heat and mass transfer rates for steady and unsteady flows. It is clear that the rising values of the thermophoresis parameter and magnetic field parameter depreciate the heat and mass transfer rate. The rising values of the Weissenberg number, Prandtl number and film thickness parameter boost the heat transfer rate and decline the mass transfer rate. The opposite trend to above has been observed for increasing values of Schmidt number and Brownian moment parameter. The increasing values of the magnetic field and film thickness parameters suppress the wall friction. Table 3 shows the validation of the present results by comparing with the published work under some special and limited case. We found a favorable agreement with the published work.

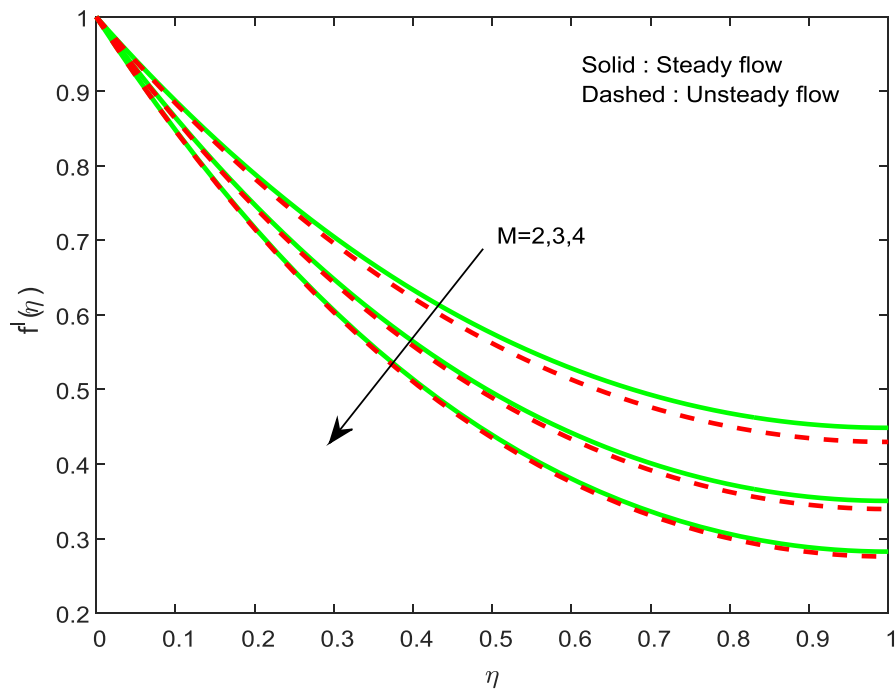


Fig.2. Effect of M on $f'(\eta)$.

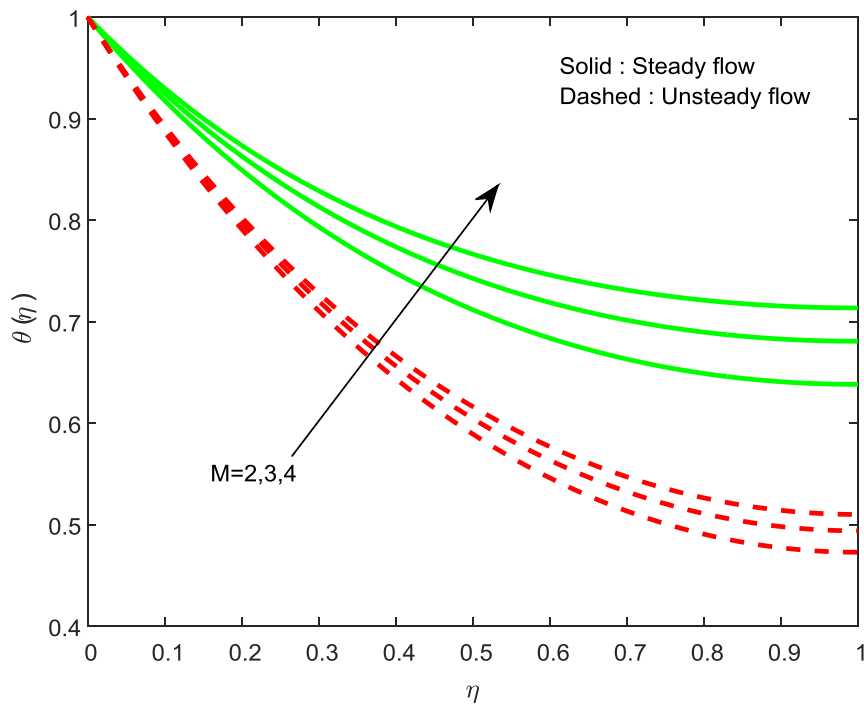


Fig.3. Effect of M on $\theta(\eta)$.

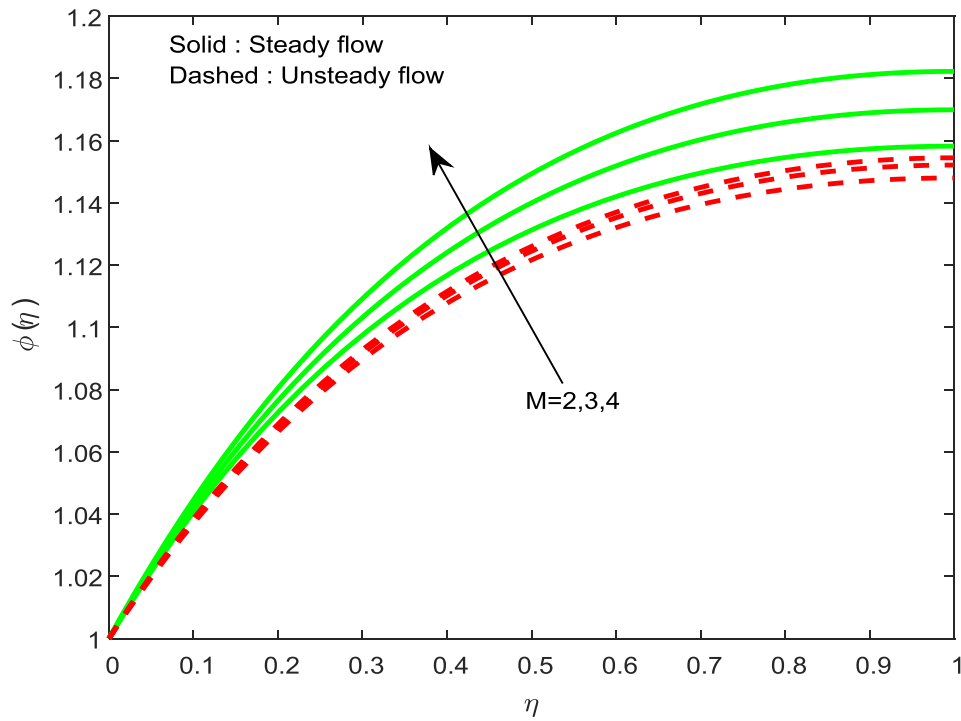


Fig.4. Effect of M on $\phi(\eta)$.

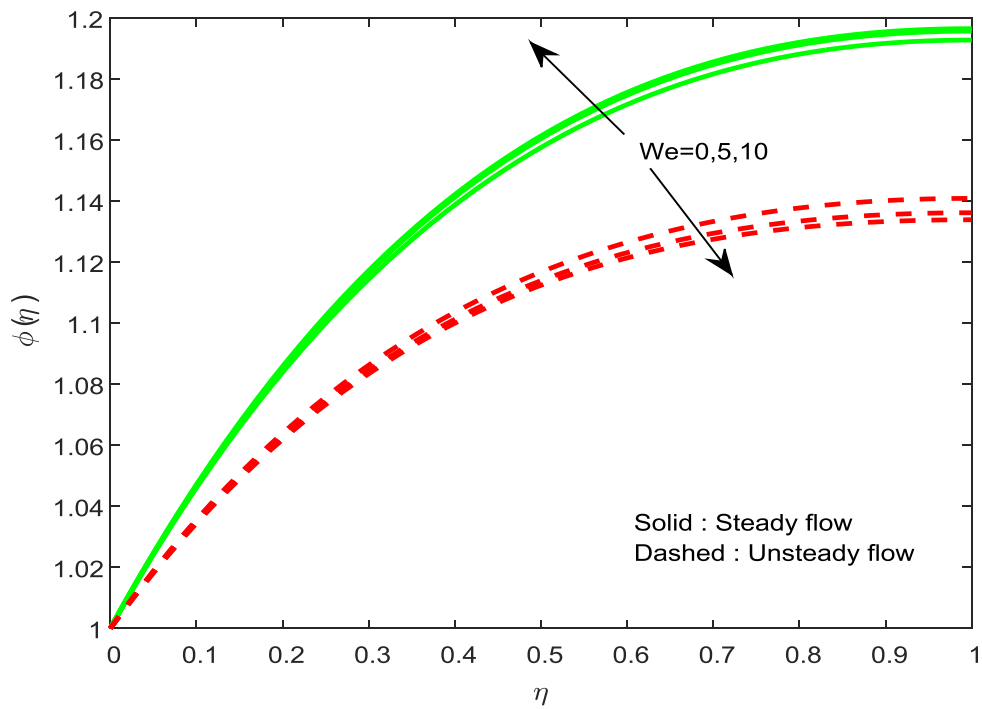


Fig.5. Effect of We on $\phi(\eta)$.

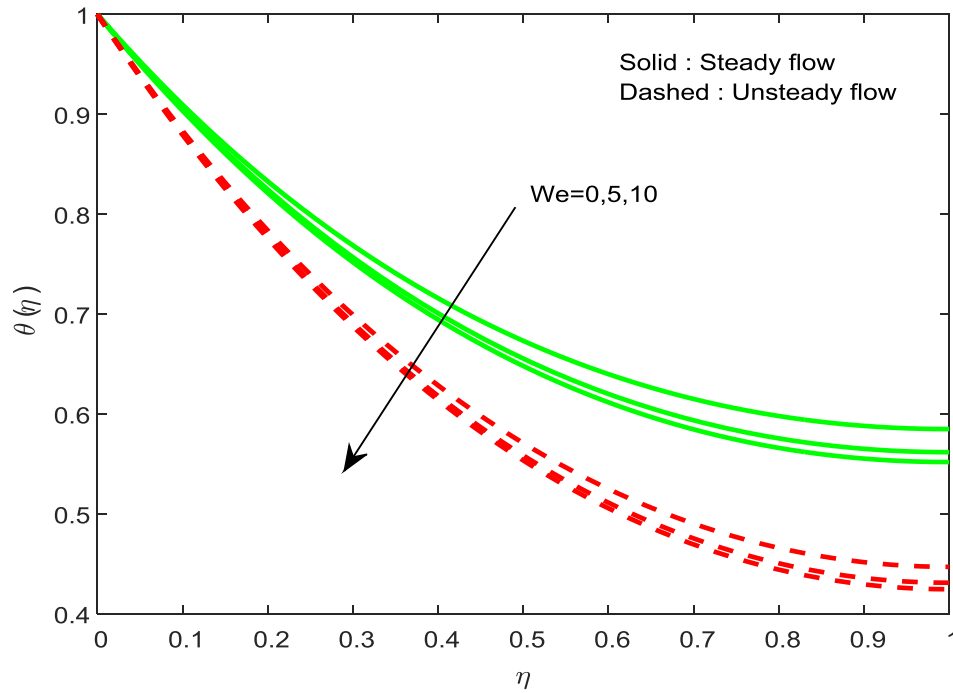


Fig.6. Effect of We on $\theta(\eta)$.

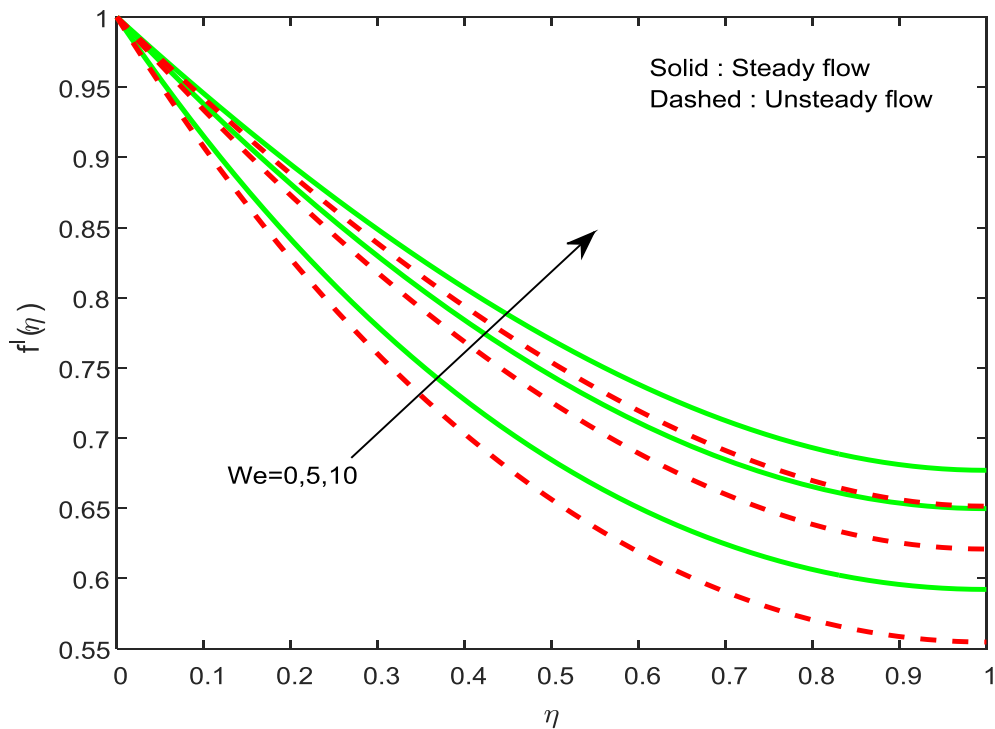


Fig.7. Effect of We on $f(\eta)$.

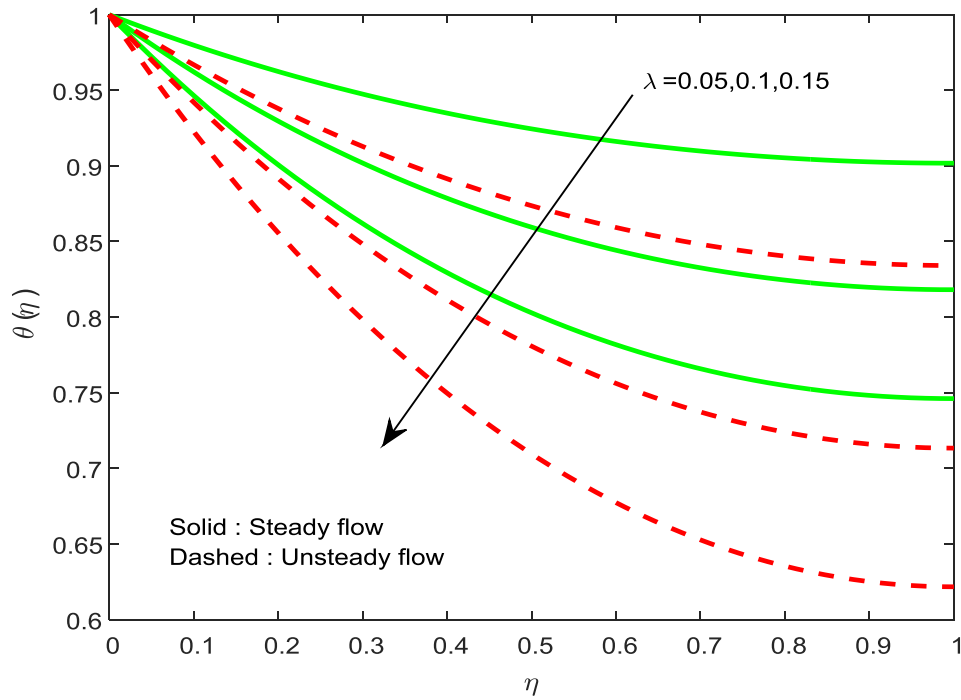


Fig.8. Effect of λ on $\theta(\eta)$.

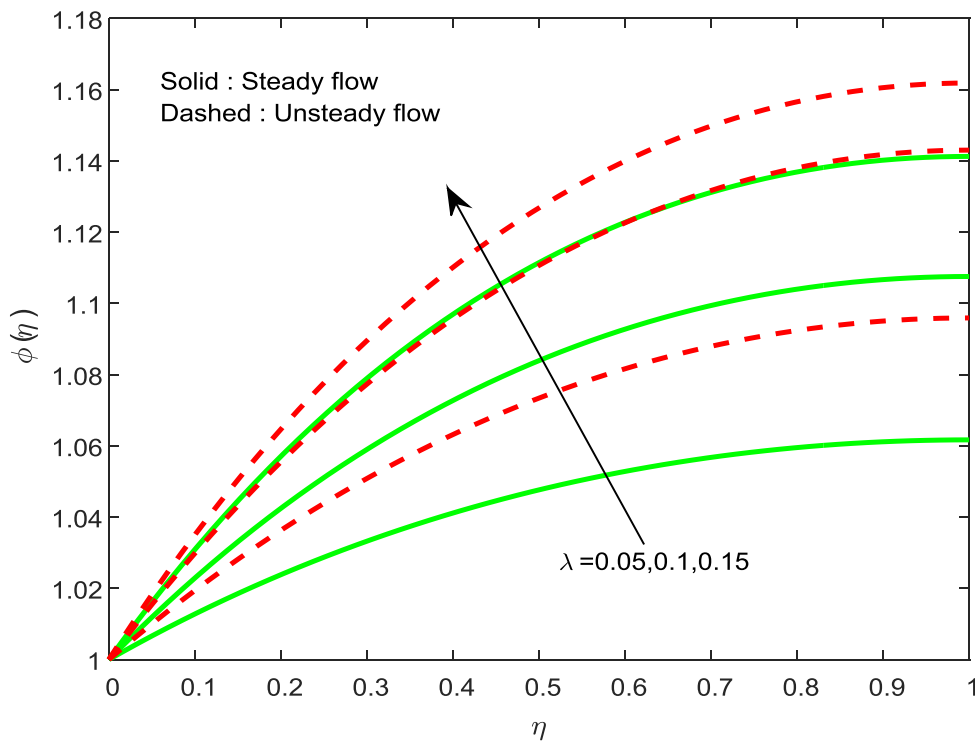


Fig.9. Effect of λ on $\phi(\eta)$.

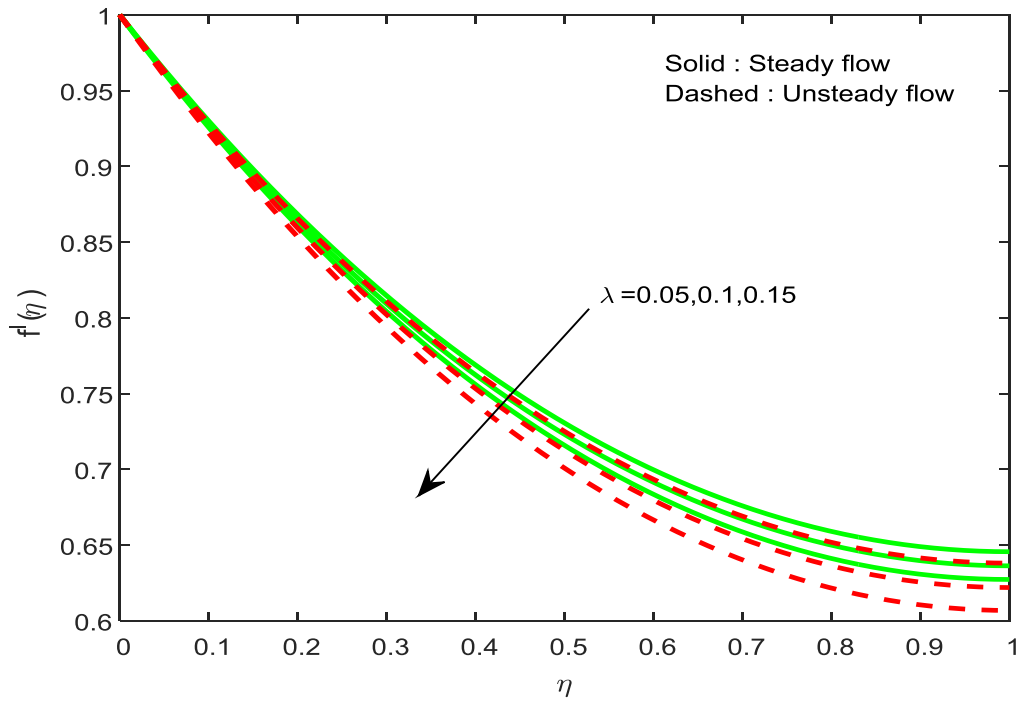


Fig.10. Effect of λ on $f'(\eta)$.

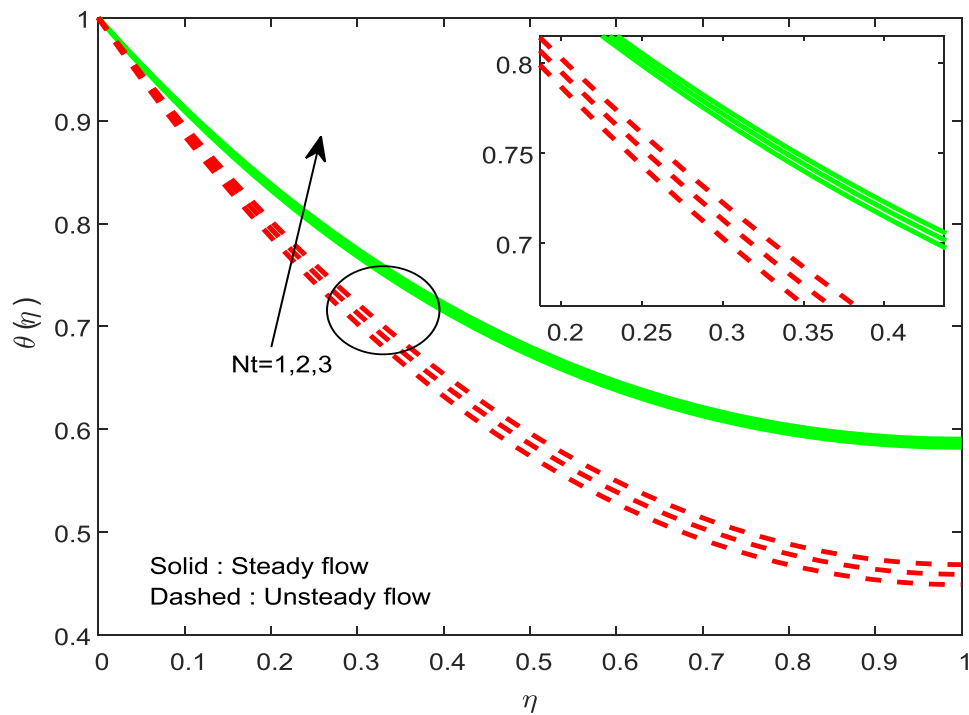


Fig. 11. Effect of Nt on $\theta(\eta)$.

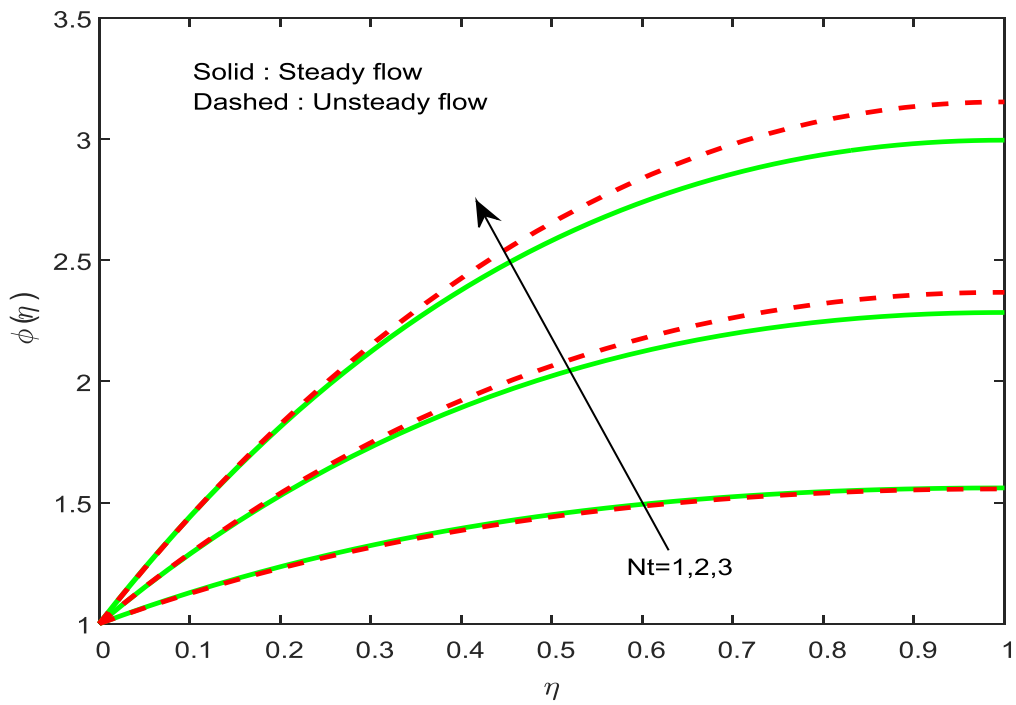


Fig. 12. Effect of Nt on $\phi(\eta)$.

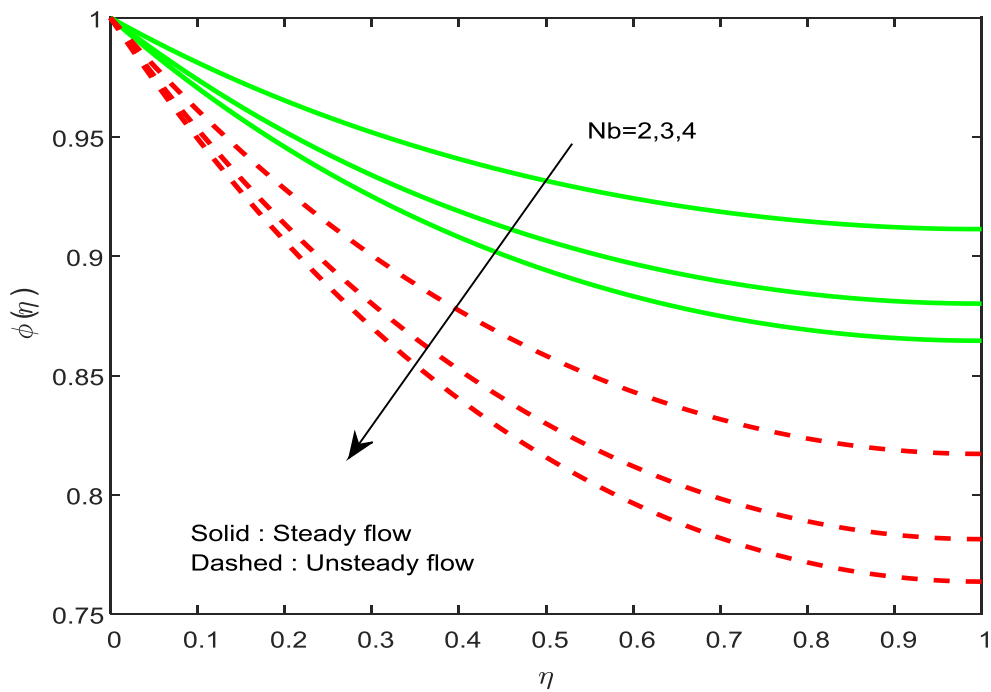


Fig. 13. Effect of Nb on $\phi(\eta)$.

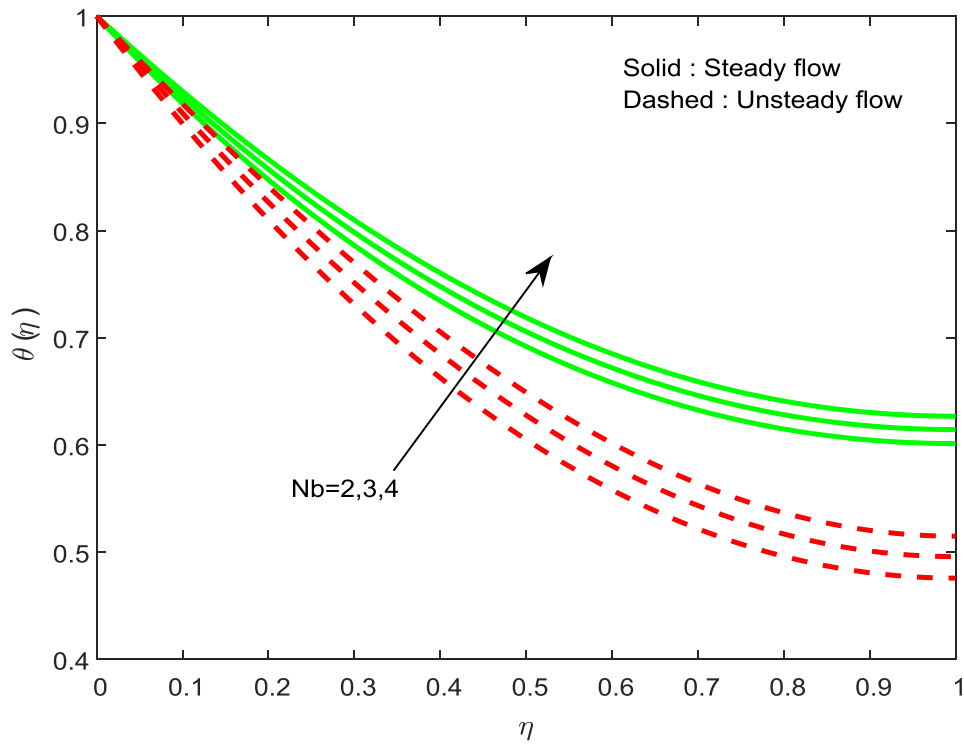


Fig. 14. Effect of Nb on $\theta(\eta)$.

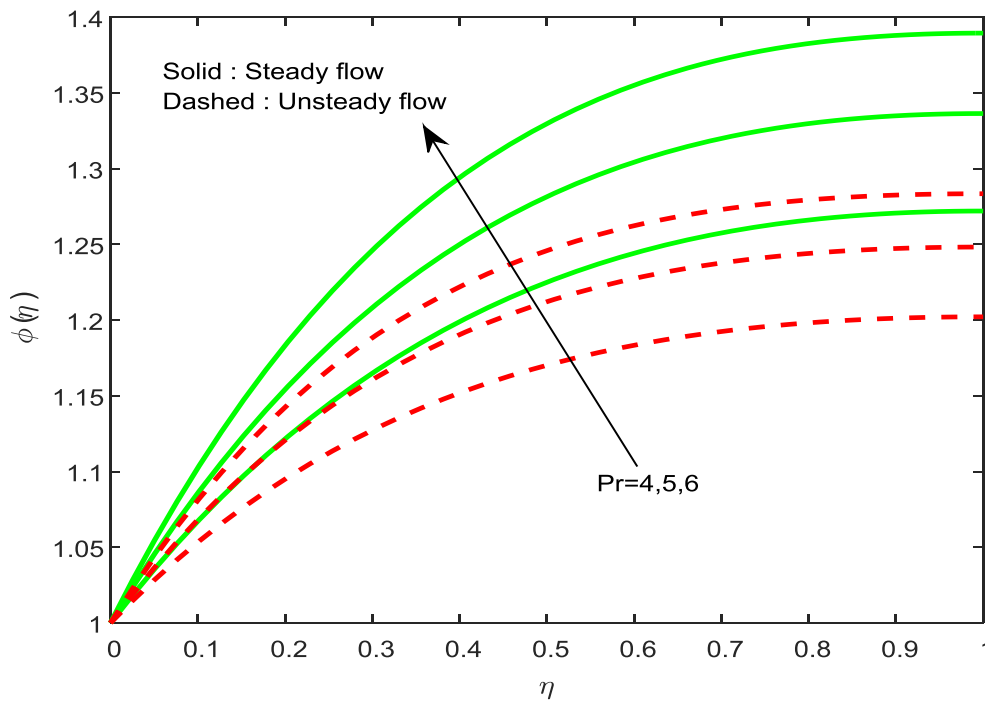


Fig. 15. Effect of Pr on $\phi(\eta)$.

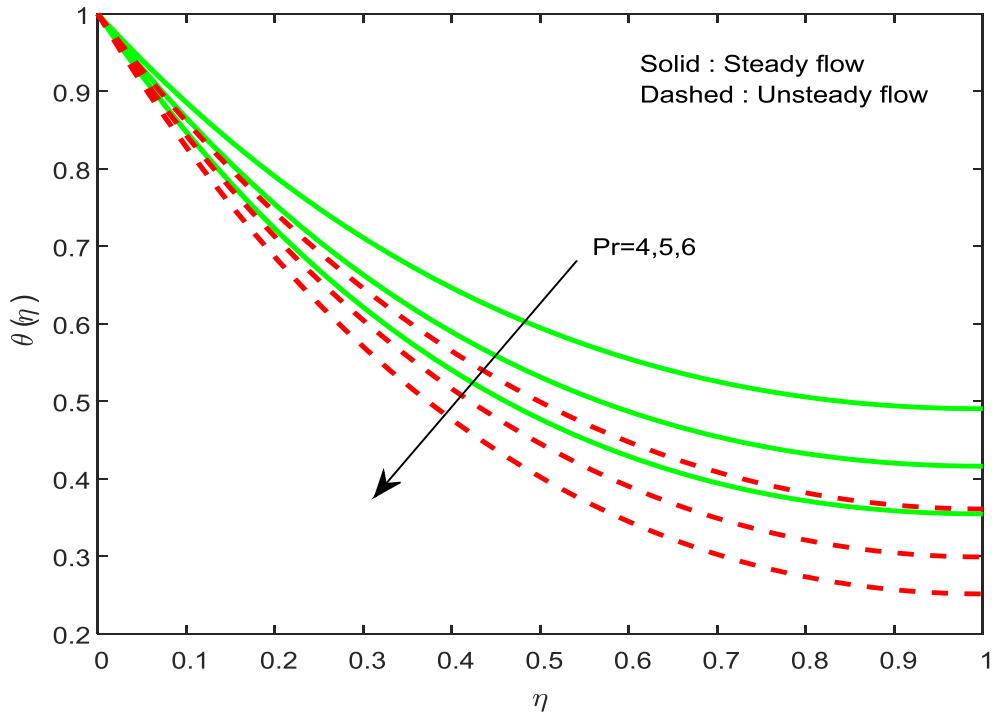


Fig. 16. Effect of Pr on $\theta(\eta)$.

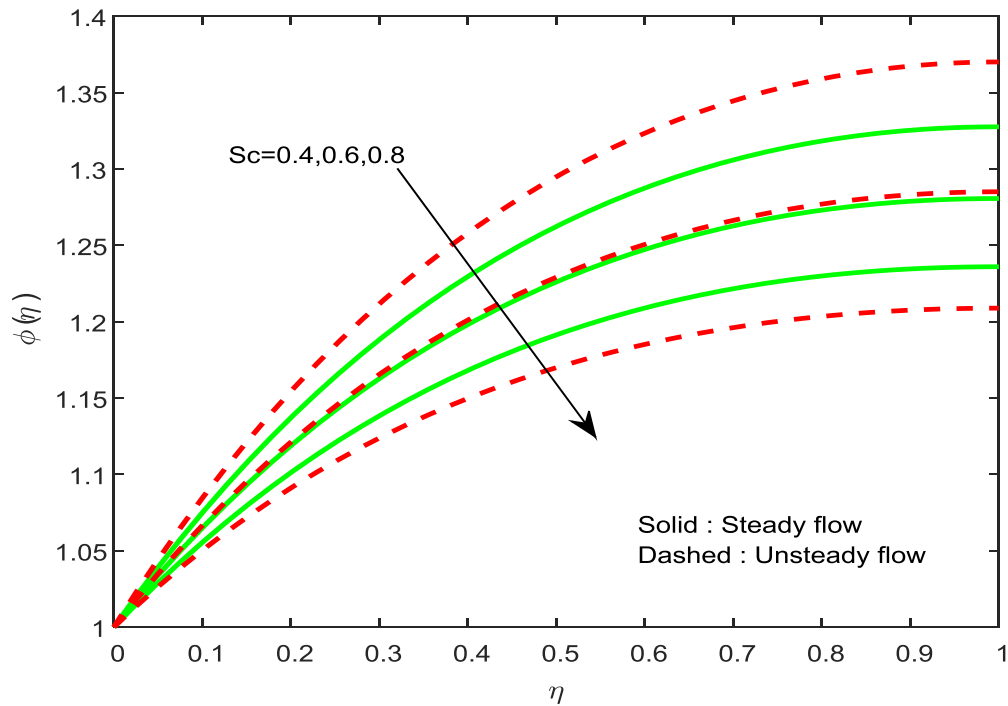


Fig. 17. Effect of Sc on $\phi(\eta)$

Table 1 Variations in $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for steady flow

M	We	λ	Nt	Nb	Pr	Sc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2							-1.199641	0.899752	-0.493325
3							-1.441252	0.832790	-0.474880
4							-1.625432	0.780380	-0.456901
	0						-0.913938	0.981838	-0.507463
	5						-0.646488	1.022844	-0.513722
	10						-0.564466	1.039116	-0.515054
		0.05					-0.744138	0.215578	-0.137288
		0.1					-0.766524	0.405438	-0.246897
		0.15					-0.788284	0.574417	-0.334833
			1				-0.850145	0.979078	-1.400729
			2				-0.850145	0.958603	-3.127221
			3				-0.850145	0.939153	-4.782542
				2			-0.850145	0.867956	0.203648
				3			-0.850145	0.796453	0.281272
				4			-0.850145	0.731870	0.319250
					4		-0.850145	1.241405	-0.742196
					5		-0.850145	1.467595	-0.952374
					6		-0.850145	1.673225	-1.143979
						0.4	-0.850145	1.021776	-0.818705
						0.6	-0.850145	1.010489	-0.711503
						0.8	-0.850145	0.999831	-0.608359

Table 2

Variations in $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for unsteady flow

M	We	λ	Nt	Nb	Pr	Sc	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2							-1.232950	1.223003	-0.414245
3							-1.458729	1.193216	-0.425103
4							-1.634468	1.170011	-0.431923
	0						-0.994970	1.257987	-0.397092
	5						-0.688764	1.284642	-0.388044
	10						-0.599858	1.294426	-0.383278
		0.05					-0.758808	0.351951	-0.207223
		0.1					-0.794271	0.617229	-0.320955
		0.15					-0.827716	0.826173	-0.379265
			1				-0.917851	1.231037	-1.361309
			2				-0.917851	1.171269	-3.137525
			3				-0.917851	1.117551	-4.734290
				2			-0.917841	1.055840	0.418535
				3			-0.917841	0.939967	0.505454
				4			-0.917841	0.839582	0.547094
					4		-0.917851	1.504813	-0.598314

					5		-0.917851	1.707928	-0.771189
					6		-0.917851	1.883942	-0.922832
						0.4	-0.917851	1.324781	-0.926376
						0.6	-0.917851	1.301879	-0.732618
						0.8	-0.917851	1.281597	-0.556600

Table 3 Comparison of the values of $-\theta'(0)$ for different values of S when $M = Nb = Nt = Sc = 0, Pr = n = 1.$

S	Xu et al. [30]	Present Results
1.0	2.677222162	2.67722216
1.2	1.999591426	1.99959142
1.4	1.447754361	1.44775436
1.6	0.956697844	0.95669784
1.8	0.484536632	0.48453663

4. Conclusions

Due to the numerous applications in glass blowing, plastic extrusion, drying/cooling of textiles and papers, textile production, crystal growing and fiberspinning, etc. In this study, we analyzed the flow, heat and mass transfer nature of magnetohydrodynamic (MHD) steady/unsteady liquid film flow of Carreau fluid past a stretching sheet in the presence of thermophoresis and Brownian moment effects. Similarity transformation is used to convert the partial differential equations to nonlinear ordinary differential equations. The findings of the present study are as follows:

- The flow, thermal and concentration boundary layers are non-uniform in steady and unsteady cases.
- Rising the film thickness enhances the heat transfer rate and declines the mass transfer rate.
- Brownian moment and thermophoresis parameters control the local Nusselt and Sherwood numbers.
- Weissenberg number enhances the heat transfer rate and declines the mass transfer rate.
- The influence of Lorentz force is high on steady flow when compared with unsteady flow.

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