Study on the Behaviour of Cosine Imprecise Functions

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Abstract: *We convert polynomial function of finite degree into Cosine imprecise function with the help of multiplication of cosine function. For some particular region we collect finite number of the points to define the most economical function called cosine imprecise function passing very close to those points in the region. Further we define area formula of the Cosine imprecise functions in terms of membership functions and the reference functions into summation form. We also study the conversion points of the different imprecise functions.*

Key Words*:* *Cosine imprecise function, imprecise function, conversion point, imprecise number, polynomial function.*

1. INTRODUCTION

Imprecise number is extension definition of fuzzy number. The term was first used in the article Theory of Believes and Realities [1]**.** Collection of all those number is called imprecise set. This set has a character same as classical set. Effect of the activity of any object is always formed an imprecise numberfor a certain interval. If we take the infinite intervals, it will form a function called imprecise function. It may be either sine imprecise function or cosine imprecise function. Thus study of the behavior of this type of function is very important. It will help us to solve many difficult problems. For example a loudness of sound produces from a device is imprecise number having maximum and minimum condition for some interval of time. If the process is continued in the infinite time bound will construct imprecise function. It can be modified according to our need with the help of multiplication of sine or cosine function. This resultant function is one of the imprecise function.

Our problems are not always expressible into algebraic function form. In most of the time it is obtained as an imprecise function. As for example receiver is getting response through the frequency of the wave, which is coming from the main place called server has a character of imprecise function. When this activity is completely imprecise form, then the system will be functioned properly. If it is not an imprecise function, the system is need to be modified. This phenomenon is called controllable or non-controllable form of the function. When this situation is uncontrollable form then we will suggest to minimize the force produces from the main server so that it will be transformed into controllable form. This experiment is done available in the field of science and technology. The value of the level for which a function is either controllable or not is called rate of convergence. Thus rate of convergence is very important for the application purpose. If we can identify this value, then what amount of force or energy is required for particular problem will be easily identified. Imprecise function is modifiable function, which can be controlled according to our need. So the polynomial function is converted into cosine imprecise function. Generally this type of imprecise function is obtained due to the multiplication another functions called multiplication factor.

To convert polynomial function into imprecise function in a certain region we will collect the finite number of points. And then with the help of these points we identify controllable function called imprecise function. Coefficients of the variable are defined according to the elementary transformation of matrix.

Area bounded by the imprecise function will help us to know about the variation of the effect of impresciness for any object in the different interval. Where the impreciseness is the membership function obtained by the effect of the activity of any object in the extension definition of fuzzy set.

The remaining sections of the article are obtained as, Section 2 contain about the preliminary definition of imprecise function, section 3 contain definition of new function called imprecise function, section 4 contain conversion of polynomial into Cosine imprecise function, section 5 contain area bounded by Cosine imprecise function of polynomial degree one, Section 6 contains area bounded by Cosine imprecise function of polynomial degree one, Finally section 7 is the conclusion and the summary of this article.

2. Preliminaries

Before starting of this article it is necessary to recall the definition of imprecise number, partial presence, membership value etc. over the real line which are discussed in the article of Baruah [9],[10],[12],[13] and Borgoyary [15], [16] ,[17] has also defined the same definitions in their article to apply in the field of transportation problems.

2.1 Imprecise Number*:* Imprecise number $N = [\alpha, \beta, \gamma]$ is divided into closed sub-intervals with the partial presence of element β in both the intervals.

2.2 Partial presence: Partial presence of an element in an imprecise real number $[\alpha, \beta, \gamma]$ is described by the present level indicator function $p(x)$ which is counted from the reference function $r(x)$ such that present level indicator for any x, $\alpha \leq x \leq \gamma$, is $(p(x) - r(x))$, where $0 \leq r(x) \leq p(x) \leq 1$

2.3 Membership value: If an imprecise number $N = [\alpha, \beta, \gamma]$ is associated with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \mu_1(x), & when\ \alpha \leq x \leq \beta \\ \mu_2(x), & when\ \beta \leq x \leq \gamma \\ 0, & otherwise \end{cases}$$

With a constant reference function 0 in the entire real line. Where $\mu_1(x)$ is continuous and non-decreasing in the interval $[\alpha, \beta]$, and $\mu_2(x)$ is a continuous and non-increasing in the interval $[\beta, \gamma]$with $\mu_1(\alpha) = \mu_2(\gamma) = 0$, then $(\mu_1(\beta) - \mu_2(\beta))$ is called membership value of the indicator function $\mu_N(x)$

2.3 Normal Imprecise Number*:* A normal imprecise number $N = [\alpha, \beta, \gamma]$ is associated with a presence level indicator function $\mu_N(x)$, where

$$\mu_N(x) = \begin{cases} \mu_1(x), & when\ \alpha \leq x \leq \beta \\ \mu_2(x), & when\ \beta \leq x \leq \gamma \\ 0, & otherwise \end{cases}$$

With a constant reference function 0 in the entire real line. Where $\mu_1(x)$ is continuous and non-decreasing in the interval $[\alpha, \beta]$ and $\mu_2(x)$ is a continuous and non-increasing in the interval $[\beta, \gamma]$ with

$$\mu_1(\alpha) = \mu_2(\gamma) = 0$$
$$\mu_1(\beta) = \mu_2(\beta) = 1$$

Here, the imprecise number is be characterized by, $\{x,\ \mu_N(x), 0 : x \in R\}$, R being the real line.

For any real line, $0 \leq \mu_1(x) \leq \mu_2(x) \leq 1$,a normal and subnormal imprecise number will be characterized in the common format as $\{x, \mu_1(x), \mu_2(x): x \in R\}$, where $\mu_1(x)$ is called membership function measured from the reference function $\mu_2(x)$ and $(\mu_1(x) - \mu_2(x))$ is called the membership value of the indicator function.

Here, the number is normal imprecise number when membership value of indicator function $\mu_N(x)$ is equal to 1. And it is a subnormal when the membership value of the indicator function less than 1.

2.4. Imprecise Functions

Imprecise number is an interval definable number such that interval is an area bounded by the impreciseness of object which can form a function. This type of function is known as imprecise function. This phenomenon is occurring presently available in the field of science and technology. For example possible of impreciseness of the electromagnetic wave in between the server and the receiver is an imprecise function. From this point of view it can be introduced some conditions for imprecise functions as follows:

(i) Function must be continuous in the certain region
(ii) Function should be oscillation in nature
(iii) Function must have finite number of maximum and minimum values within the interval.
(iv) Function should be semi-periodic/periodic in nature.

For example formation of impreciseness of the object, which can construct sine and cosine function is the interval definable number or the imprecise number for the different region. Thus, impreciseness of the object which is a uncontrollable function form can be converted into normal imprecise function with the help of multiplication factors like sine and cosine functions.

Conversion point: It is a point from where non-imprecise function starts to convert into imprecise function. For any object if we can identify this point, then we will get easily their applications. As for example loudness of sound causes brain and heart diseases. This type of sound produced by the different experiments can be transformed into human ear audible level by converting them into imprecise function form. Further at present scenery due to lake of the friction of Ozone layer, ultraviolet ray reaches the ground is the non-imprecise form of function. So, it may be benefitted to convert this ray into imprecise function form by inventing new device having Ozone layer character. And this may be possible due to the multiplication factor, which can convert this type of function into imprecise function. Normally these phenomena are already applied in the regulator of various purposes, which are normally seen available in the field of science and technology.

3. Conversion of polynomial into Cosine imprecise function:

In our mathematics most of the polynomials are freely floating as a graph without coming back repeatedly to the ground level. So it is called uncontrolled function. For example first degree, second degree polynomials etc. are the functions that can meet the ground level i.e. on the real line at most one, twice etc.

This type of function can also be transformed into oscillation form with the help of multiplication of cosine function. Thus the resultant function is called cosine imprecise function. This function is possible to meet the ground level repeatedly from a certain point. This point is known as conversion point. Some time the function is also known as a controllable function. This phenomenon is occurred due to the multiplication of another function. This multipliable function is called multiplication factor.

Mathematically to discuss, let us consider, $p_1(x_1, y_1)$, $p_2(x_2, y_2)$, $p_3(x_3, y_3)$ ……….$p_n(x_{n+1}, y_{n+1})$, be the n[th] collection of points, which are above and below the given polynomial of degree n,

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \dots\dots\dots\dots\dots + c_n x^n \dots \quad \textbf{(1)}$$

Graphically this is not a controllable function, which is discussed in the article [17]. To convert it into controllable form, this polynomial is multiplied by a cosine function.

Thus,

$$y = (c_0 + c_1 x + c_2 x^2 + c_3 x^3 \dots\dots + c_n x^n)\cos(lx);\ l\epsilon Z \quad \textbf{(2)}$$

will be a Cosine imprecise polynomial.

To obtain standard imprecise polynomial averagely passing closed to these points, this problem is written into the matrix form. Then we follow the rules of transpose matrix and the multiplication of matrices as follows as,

Here the points $p_1, p_2, \dots\dots\dots\dots, p_n$ are satisfy the given polynomial. So, by the law of roots, we will get n[th] set of simultaneous linear equation having arbitrary coefficients, $c_0, c_1, c_2, \dots\dots\dots\dots, c_n$ as follows:

$$y_1 = (c_0 + c_1 x_1 + c_2 {x_1}^2 + c_3 {x_1}^3 \dots\dots\dots + c_n {x_1}^n)\cos(lx_1)$$
$$y_2 = (c_0 + c_1 x_2 + c_2 {x_2}^2 + c_3 {x_2}^3 \dots\dots\dots + c_n {x_2}^n)\cos(lx_2)$$
$$y_3 = (c_0 + c_1 x_3 + c_2 {x_3}^2 + c_3 {x_3}^3 \dots\dots\dots + c_3 {x_3}^n)\cos(lx_3)$$

………………..…………………………………… ……………………………………………………… ……………

$$y_n = (c_0 + c_1 x_n + c_2 {x_n}^2 + c_3 {x_n}^3 \dots\dots\dots + c_n {x_n}^n)\cos(lx_n)$$

Which can be written in the matrix form-

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} \cos(lx_1) & x_1\cos(lx_1) & \dots\dots & {x_1}^n\cos(lx_1) \\ \cos(lx_2) & x_2\cos(lx_2) \dots\dots\dots & & {x_2}^n\cos(lx_2) \\ \cos(lx_3) & x_3\cos(lx_3) \dots\dots\dots & & {x_2}^n\cos(lx_2) \\ & \vdots & & \\ \cos(lx_n) & x_n\cos(lx_n) \dots\dots & & {x_n}^n\cos(lx_n) \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_3 \\ \vdots \\ c_n \end{bmatrix}$$
$$\cong AX = B\ (say) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(3)$$

$$\text{Where,} \quad A = \begin{bmatrix} 1 & x_1 {x_1}^2 & {x_1}^3 {x_1}^4 \dots\dots\dots\dots & {x_1}^n \\ 1 & x_2 {x_2}^2 & {x_2}^3 {x_2}^4 & \dots\dots\dots\dots & {x_2}^n \\ 1 & x_3 {x_3}^2 & {x_3}^3 {x_3}^4 & \dots\dots\dots\dots & {x_3}^n \\ & & \vdots & & \\ 1 & x_n {x_n}^2 & {x_n}^3 {x_n}^4 & \dots\dots\dots & {x_n}^n \end{bmatrix},$$

$$X = \begin{bmatrix} c_0 \\ c_1 \\ c_3 \\ \vdots \\ c_n \end{bmatrix},\quad B = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$$

To obtained solutions of the arbitrary constants, equation (1) is multiplied by transpose matrix A^T. Thus the equation becomes:

$$A^T(AX) = A^T B, \dots\dots\dots\dots\dots\dots\dots\dots \quad \textbf{(4)}$$

Where, $A^T A = (a_{ij})_{(n+1)\times(n+1)}$

and $A^T B = (b_i)_{(n+1)\times 1}$,

Such that, $a_{ij} = \sum_{k=1}^{m} [x_k^{i+j-2} \{\cos(lx_k)\}^2]$

and $b_i = \sum_{k=1}^{m} [y_k x_k^i \cos(lx_k)]$, $1 \le i, j \le (n+1)$

Here, n= order of the polynomial, m=no of collection of points, x_i and y_i ordinates and the abscissa of the given points.

Solution of the matrix for the arbitrary constants $c_0, c_1, c_2, \dots\dots\dots\dots\dots\dots, c_n$ can be obtained with the help of Gauss-elimination method and the elementary transformation of matrix, which is obtained in the following general form-

$$R_i^{'} \to R_i - \frac{R_j}{a_{(i-1)(i-1)}} \times a_{ij};\ (2 \le i \le n+1), (1 \le j \le n+1) \dots\dots\dots\dots\dots\textbf{(5)}$$

Steps of transformation will be done as follows:

$$(2 \le i \le n+1\ when\ j = 1)$$
$$(2 \le i \le n\ when\ j = 1)$$
$$(k \le i \le n+1\ \ when\ j = k-1)\ and\ (2 \le k \le n) \quad \dots\dots\dots\dots\dots\dots\dots\textbf{(6)}$$

Above operations are help us to get upper triangular matrix. By backward substitution the solution of the mentioned arbitrary constants can be obtained. Thus any polynomial can be transformed into controllable form with the multiplication of cosine function. And the resultant function is a cosine imprecise function.

(i) Cosine imprecise function of degree one

Here, the polynomial of degree one is transformed into imprecise function by the multiplication of cosine function to known as cosine imprecise function of degree one. Thus the given polynomial of degree one is multiplied by cosine function of angle $x = 2l; l \in Z^+$. Resultant of this cosine imprecise function creates a situation to oscillate the given polynomial from a certain point. These points are the left and right nearest value of the root of our given polynomial. Right nearest value is the average value on the x-axis between the root of the given polynomial and the nearest greater value for which the given multiplication factor, cosine function becomes zero. Left nearest value is the average value on the x-axis between the root of the given polynomial and the nearest smaller value for which the given multiplication factor, cosine function becomes zero. Later in the discussion, right and left nearest value on the x-axis will be called conversion along the positive x-axis and the negative x-axis respectively.

Example-1 (Cosine *imprecise function of polynomial of degree one with angle 2x*):

Let $y = c_0 + c_1 x$……………………………..**(7)**

be a polynomial of degree one. So, in particular

$for\ l = 2, y = (c_0 + c_1 x)\cos(2x)$………..………**(8)**

is an imprecise polynomial of degree one.

Let us consider (1,0.5), (2,-2.5),(3,2),(4,-4),(5,3.5),(6,-6),(7,7) be the points in the data collection.

So, $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5, x_6 = 6, x_7 = 7$

$y_1 = 0.5, y_2 = -2.5, y_3 = 2,$

$y_4 = -4, y_5 = 3.5, y_6 = -6, y_7 = 7$

By the law of roots this imprecise function satisfies the given collection of points to have linear simultaneous equation to form a matrix as follows:

$$From (3), (4), (5), (6), we\ have$$

$$AX = B$$

$$=> (A^T A)X = A^T B$$

Where, $A^T A = (a_{ij})_{(n+1)\times(n+1)}$

$$A^T B = (a_{ij})_{(n+1)\times 1},\quad X = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

Such that $a_{ij} = \sum_{k=1}^{m} x_k^{i+j-2}\{\cos(2x_k)\}^2$ and

$$b_i = \sum_{k=1}^{m} y_k x_k^i \cos(2x_k),\qquad 1 \le i \le (n+1)$$

Here, m=no. of points=7, n= degree of the polynomial=1

$$A^T A = (a_{ij})_{2\times 2} \text{and} A^T B = (b_i)_{2\times 1}$$

Where $a_{ij} = \sum_{k=1}^{7} x_k^{i+j-2}\{\cos(2x_k)\}^2$ and $b_i = \sum_{k=1}^{7} y_k x_k^i \cos(2x_k)$, $1 \le i \le 2$. Now,

$$a_{11} = \sum_{k=1}^{7} {x_k}^0 \{\cos(2x_k)\}^2$$

$$= cos^2(2x_1) + cos^2(2x_2) + cos^2(2x_3) + cos^2(2x_4) + cos^2(2x_5) + cos^2(2x_6) + cos^2(2x_7) = 4.85559,$$

$$a_{12} = a_{21} = \sum_{k=1}^{7} x_k \{\cos(2x_k)\}^2$$

$$= x_1 cos^2(2x_1) + x_2 cos^2(2x_2) + x_3 cos^2(2x_3) + x_4 cos^2(2x_4) + x_5 cos^2(2x_5) + x_6 cos^2(2x_6) + x_7 cos^2(2x_7) = 27.05897$$

$$a_{22} = \sum_{k=1}^{7} {x_k}^2 \{\cos(2x_k)\}^2$$

$$= {x_1}^2 cos^2(2x_1) + {x_2}^2 cos^2(2x_2) + {x_3}^2 cos^2(2x_3) + {x_4}^2 cos^2(2x_4) + {x_5}^2 cos^2(2x_5) + {x_6}^2 cos^2(2x_6) + {x_7}^2 cos^2(2x_7) = 134.39326,$$

$$b_1 = \sum_{k=1}^{7} y_k x_k \cos(2x_k)$$

$$= y_1 x_1 \cos(2x_1) + y_2 x_2 \cos(2x_2) + y_3 x_3 \cos(2x_3) + y_4 x_4 \cos(2x_4) + y_5 x_5 \cos(2x_5) + y_6 x_6 \cos(2x_6) + y_7 x_7 \cos(2x_7) = 15.20003$$

$$b_2 = \sum_{k=1}^{7} y_k {x_k}^2 \cos(2x_k)$$

$$= y_1 {x_1}^2 \cos(2x_1) + y_2 {x_2}^2 \cos(2x_2) + y_3 {x_3}^2 \cos(2x_3) + y_4 {x_4}^2 \cos(2x_4) + y_5 {x_5}^2 \cos(2x_5) + y_6 {x_6}^2 \cos(2x_6) + y_7 {x_7}^2 \cos(2x_7) = 152.75052$$

$$\text{Thus} \begin{bmatrix} 4.85 & 27.05 \\ 27.05 & 134.39 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 15.20 \\ 152.75 \end{bmatrix}$$

$$From\ the\ formula\ (11), we\ have -$$

$$R_i' \to R_i - \frac{R_j}{a_{(i-1)(i-1)}} \times a_{ij};\ (2 \le i \le 2), (1 \le j \le 1)$$

$$\begin{bmatrix} 4.85 & 27.05 \\ 0 & -16.47 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 15.20 \\ 67.97 \end{bmatrix}$$

$$=> 4.85c_0 + 27.05c_1 = 15.20, -16.47c_1 = 67.97$$

$$\text{So, } c_1 = -\frac{67.97}{16.47} = -4.12,$$

$$c_0 = \frac{1}{4.85}(15.20 - 27.05c_1)$$

$$= \frac{1}{4.85}(15.20 - 27.05 \times (-4.12)) = 27.69$$

$$\text{So, } y = (27.69 - 4.12x)\cos(2x)\ \ldots\ldots\ldots \mathbf{(9)}$$

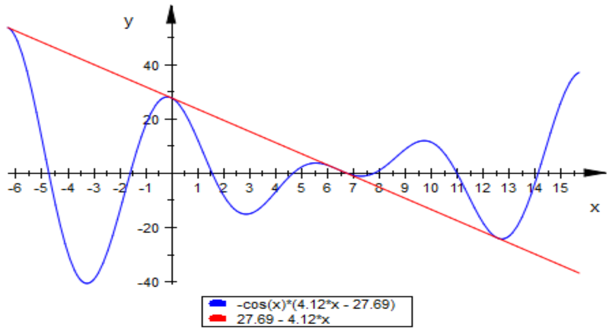

Fig.1. Graph of $y = (27.69 - 4.12x)$ and $y = (27.69 - 4.12x)\cos(2x)$

Here, the given cosine imprecise function meets the x-axis only when $y = 0$. For this condition we take $x = \frac{27.69}{4.12} = 6.720$ and $\frac{7\pi}{4} < 6.720 < \frac{9\pi}{4}$, for which $x = \frac{7\pi}{4}\ and\ \frac{9\pi}{4}$ are the nearest two values such that $\cos(2x) = 0$

So, the graph starts to oscillate from the point, $x = \frac{6.720\times4+9\pi}{2\times4} \simeq 6.8950(approx.)$ along the positive x-axis and $x = \frac{6.720\times4+7\pi}{2\times4} \simeq 6.109(approx)$. along the negative x-axis which is shown in the Fig1.

So, $\left(\frac{6.720\times4+9\pi}{2\times4}, \left(27.69 + -4.12\left(\frac{6.720\times4+9\pi}{2\times4}\right)\right)\cos\left(\frac{6.720\times4+9\pi}{4}\right)\right)$ is called conversion point along the positive x-axis.

Thus, the given function $y = (27.69 - 4.12x)\cos(2x)$ cuts the real line X-axis repeatedly to meet the ground level again and again. So, it is a cosine imprecise function.

In general for the cosine imprecise function $y = (a + bx)\cos(lx)\,; l \in Z^+$ conversion point is depend on the value of $x = -\frac{a}{b}$

Example-2 (Cosine *imprecise function of polynomial of degree two with angle 2x*)

Let $y = c_0 + c_1x + c_1x^2$……………………**(10)**

be a polynomial of degree two. So, in particular

$y = (c_0 + c_1x + c_1x^2)\cos(2x)\,, for\ l = 2$…………..**….(11)**

is an imprecise polynomial of degree two.

Let us consider (1,0.5), (2,-2.5),(3,2),(4,-4),(5,3.5),(6,-6),(7,7) be the points in the data collection.

$$\text{So, } x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5, x_6 = 6, x_7 = 7$$
$$y_1 = 0.5, y_2 = -2.5, y_3 = 2,$$
$$y_4 = -4, y_5 = 3.5, y_6 = -6, y_7 = 7$$
$$From (3), (4), (5), (6), we\ have$$
$$AX = B$$
$$=> (A^TA)X = A^TB$$

Where, $A^TA = (a_{ij})_{(n+1)\times(n+1)}$

and $A^TB = (a_{ij})_{(n+1)\times1}$,, $X = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix}$

Where $a_{ij} = \sum_{k=1}^{m} x_k^{i+j-2}\{\cos(2x_k)\}^2$ and

$$b_i = \sum_{k=1}^{m} y_k x_k^i \cos(2x_k), \qquad 1 \le i \le (n+1)$$

Here, m=no. of points=7 and n= degree of the polynomial=2

$A^TA = (a_{ij})_{3\times3}$ and $A^TB = (a_{ij})_{3\times1}$

Where $a_{ij} = \sum_{k=1}^{7} x_k^{i+j-2}\{\cos(2x_k)\}^2$ and

$$b_i = \sum_{k=1}^{7} y_k x_k^i \cos(2x_k), \qquad 1 \le i \le 3$$

$$a_{11} = \sum_{k=1}^{7} x_k{}^0\{\cos(2x_k)\}^2$$
$$= cos^2(2x_1) + cos^2(2x_2) + cos^2(2x_3) + cos^2(2x_4) + cos^2(2x_5) + cos^2(2x_6) + cos^2(2x_7) = 4.85594,$$

$$a_{12} = a_{21} = \sum_{k=1}^{7} x_k\{\cos(2x_k)\}^2$$
$$= x_1cos^2(2x_1) + x_2cos^2(2x_2) + x_3cos^2(2x_3) + x_4cos^2(2x_4) + x_5cos^2(2x_5) + x_6cos^2(2x_6) + x_7cos^2(2x_7) = 27.05897$$

$$a_{22} = a_{13} = a_{31} = \sum_{k=1}^{7} x_k{}^2\{\cos(2x_k)\}^2$$
$$= x_1{}^2cos^2(2x_1) + x_2{}^2cos^2(2x_2) + x_3{}^2cos^2(2x_3) + x_4{}^2cos^2(2x_4) + x_5{}^2cos^2(2x_5) + x_6{}^2cos^2(2x_6) + x_7{}^2cos^2(2x_7) = 134.39326,$$

$$a_{23} = a_{32} = \sum_{k=1}^{7} x_k{}^3\{\cos(2x_k)\}^2$$
$$= x_1{}^3cos^2(2x_1) + x_2{}^3cos^2(2x_2) + x_3{}^3cos^2(2x_3) + x_4{}^3cos^2(2x_4) + x_5{}^3cos^2(2x_5) + x_6{}^3cos^2(2x_6) + x_7{}^3cos^2(2x_7) = 749.24443,$$

$$a_{33} = \sum_{k=1}^{7} x_k{}^4\{\cos(2x_k)\}^2$$
$$= x_1{}^4cos^2(2x_1) + x_2{}^4cos^2(2x_2) + x_3{}^4cos^2(2x_3) + x_4{}^4cos^2(2x_4) + x_5{}^4cos^2(2x_5) + x_6{}^4cos^2(2x_6) + x_7{}^4cos^2(2x_7) = 4454.68738$$

$$b_1 = \sum_{k=1}^{7} y_k x_k \cos(2x_k)$$
$$= y_1x_1\cos(2x_1) + y_2x_2\cos(2x_2) + y_3x_3\cos(2x_3) + y_4x_4\cos(2x_4) + y_5x_5\cos(2x_5) + y_6x_6\cos(2x_6) + y_7x_7\cos(2x_7) = 15.2000$$

$$b_2 = \sum_{k=1}^{7} y_k x_k{}^2 \cos(2x_k)$$
$$= y_1x_1{}^2\cos(2x_1) + y_2x_2{}^2\cos(2x_2) + y_3x_3{}^2\cos(2x_3) + y_4x_4{}^2\cos(2x_4) + y_5x_5{}^2\cos(2x_5) + y_6x_6{}^2\cos(2x_6) + y_7x_7{}^2\cos(2x_7) = 152.75052$$

$$b_3 = \sum_{k=1}^{7} y_k x_k{}^3 \cos(2x_k)$$
$$= y_1x_1{}^3\cos(2x_1) + y_2x_2{}^3\cos(2x_2) + y_3x_3{}^3\cos(2x_3) + y_4x_4{}^3\cos(2x_4) + y_5x_5{}^3\cos(2x_5) + y_6x_6{}^3\cos(2x_6) + y_7x_7{}^3\cos(2x_7) = 1273.59811$$

Thus the simultaneous equation is converted into following simple matrix form:

$$\begin{bmatrix} 4.85 & 27.05 & 134.39 \\ 27.05 & 134.20 & 749.24 \\ 134.39 & 749.24 & 4454.68 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 15.20 \\ 152.75 \\ 1273.59 \end{bmatrix}$$
$$R_i' \to R_i - \frac{a_{ij}}{a_{jj}} \times a_{ij}\ ; (2 \le i \le 3, 1 \le j \le 2)$$

If $i = 2$, then $1 \le j \le 2 \approx \begin{bmatrix} 4.85 & 27.05 & 134.39 \\ 0 & -16.47 & -0.29 \\ 0 & -0.29 & 730.83 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 15.20 \\ 67.97 \\ 852.40 \end{bmatrix}$

If $i = 3,$ then $1 \le j \le 2 \approx \begin{bmatrix} 4.85 & 27.05 & 134.39 \\ 0 & -16.47 & -0.29 \\ 0 & 0 & 730.58 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} 15.20 \\ 67.97 \\ 851.18 \end{bmatrix}$

$$=> 4.85c_0 + 27.05 + 134.39c_1 = 15.20$$
$$=> -16.81c_1 - 0.29c_2 = 67.97$$
$$730.58c_3 = 851.18$$

Thus by backward substitution we will get,

$$c_0 = -5.91,\; c_1 = -4.14,\; c_2 = 1.16$$

So, $y = (-5.91 - 4.14x + 1.16x^2)\cos(2x)$ $\ldots\ldots\ldots\ldots\ldots(\mathbf{12})$

is a cosine imprecise function.

Thus for every data collection of points we can have controllable function called imprecise function.

Graph of this function is:

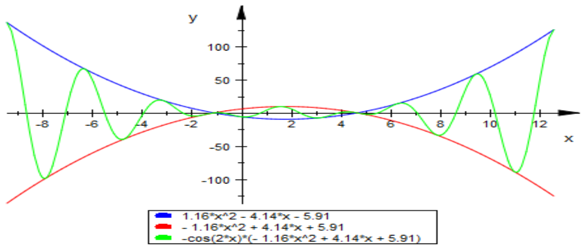

Figure 2: Graph of $(-5.91 - 4.14x + 1.16x^2), (5.91 + 4.14x - 1.16x^2) and (-5.91 - 4.14x + 1.16x^2)\cos(2x)$

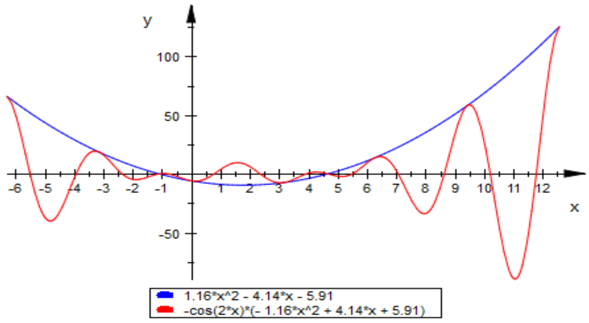

Figure 3: Graph of $(-5.91 - 4.14x + 1.16x^2) and (-5.91 - 4.14x + 1.16x^2)\cos(2x)$

Here, the given cosine imprecise function meets the x-axis only when $y = 0$. For this condition we take the solution of the given polynomial as follows-

$$x = \frac{-(-4.147) \pm \sqrt{(-4.147)^2 + 4 \times 1.16 \times 5.918}}{2 \times 1.16}$$
$$= 4.66(approx.), -1.0918(approx.)$$

We observe that $-\frac{3\pi}{4} < -1.0918 \text{ and } 4.66 < \frac{5\pi}{4}$ for which $x = -\frac{3\pi}{4} \text{ and } \frac{5\pi}{4}$ are the nearest two values of the given solution of the above polynomial such that $\cos(2x) = 0$

Here the given polynomial will start to oscillate from the average value of the respective interval along the part of the axes. So, the graph starts to oscillate from the point, $x = \frac{4.66\times4+5\pi}{8} \simeq 4.29(approx)$ along the positive x-axis and $x = \frac{-1.0918\times4-3\pi}{8} \simeq -1.72\,(approx)$ along the negative x-axis which is shown in the Fig 4.

So, $\left(\frac{4.66\times4+5\pi}{8}, (-5.91 - 4.14\left(\frac{4.66\times4+5\pi}{8}\right) + 1.16\left(\frac{4.66\times4+5\pi}{8}\right)^2)\cos(\frac{4.66\times4+5\pi}{4})\right)$ is called conversion point along the positive x-axis.

Thus, the given function $y = (-5.91 - 4.14x + 1.16x^2)\cos(2x)$ cuts the real line X-axis repeatedly to meet the ground level again and again. So, it is a cosine imprecise function.

In general for the cosine imprecise function $y = (c_0 + c_1x + c_1x^2)\cos(2x)\,; l \in Z^+$ conversion point is depend on the value of $x = \frac{-b\pm\sqrt{(-b)^2-4ac}}{2a}$

Further from the same point, imprecise function will come back to the original polynomial function. So this point is also known as a diversion point of the imprecise function with respect to the multiplication factor cosine.

Thus from the above calculations, we have observed that if the angle of the trigonometry function is increased then conversion point come closer to the origin of the co-ordinate axis system. It means that considered experiment starts to oscillate within the short period of time to have shorter wavelength and larger energy in the system.

For example if the wings of a Generator moves more angle then speed is increased and the energy output will be larger due to short wave length and the closer conversion point.

4. Area bounded by imprecise functions of polynomial of degree one:

To obtain area by the help of integration all the intervals are not allowed to take as a limit for the imprecise function. For example-

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(x)\,dx = 0,$$
$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos(2x)\,dx = 0 \quad \text{So on.}$$

For this reason to obtain area of an imprecise function our imprecise number will be taken as a limit of the integration. After defining area obtained by the imprecise number we will obtain area of the imprecise function for any interval in the summation form.

Thus for a cosine imprecise function, for $= (c_0 + c_1x)\cos(x), l = 1$, we have a section of imprecise numbers, $\left[\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right], \left[\frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}\right], \left[\frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}\right], \left[\frac{7\pi}{2}, 4\pi, \frac{9\pi}{2}\right]\ldots\ldots\ldots$ …… etc, having indicator function as follows:

$$\mu_N(x) = \begin{cases} \emptyset(x); \ \frac{n\pi}{2} \le x \le n\pi \\ \varphi(x); \ n\pi \le x \le \frac{(n+1)\pi}{2} \end{cases}$$

such that $\emptyset\left(\frac{n\pi}{2}\right) = \varphi\left(\frac{(n+1)\pi}{2}\right) = 0$

$and\ \emptyset(n\pi) = \varphi(n\pi)$…………………**(13)**

Here, function has maximum level at the point $= n\pi$.

So we can call, $\left[\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right], \left[\frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}\right], \left[\frac{5\pi}{2}, 3\pi, \frac{7\pi}{2}\right]$, $\left[\frac{7\pi}{2}, 4\pi, \frac{9\pi}{2}\right]$…………...……….etc. are imprecise numbers.

Similarly, for $y = (c_0 + c_1 x)\cos(2x), l = 2$, we have a section the intervals,

$\left[\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right], \left[\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right], \left[\frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right], \left[\frac{7\pi}{4}, 2\pi, \frac{9\pi}{4}\right]$………… ……...

etc, having indicator function as follows:

$$\mu_N(x) = \begin{cases} \emptyset(x); \ \frac{n\pi}{4} \le x \le \frac{n\pi}{2} \\ \varphi(x); \ \frac{n\pi}{2} \le x \le \frac{(n+2)\pi}{4} \end{cases}$$

such that $\emptyset\left(\frac{n\pi}{4}\right) = \varphi\left(\frac{(n+2)\pi}{4}\right) = 0$

$and\ \emptyset\left(\frac{n\pi}{2}\right) = \varphi\left(\frac{n\pi}{2}\right)$…………………**(14)**

Here, function has maximum level at the point $= \frac{n\pi}{2}$.

So we can call, $\left[\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right], \left[\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right], \left[\frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right]$, $\left[\frac{7\pi}{4}, 2\pi, \frac{9\pi}{4}\right]$……………...……. etc. are imprecise numbers.

In general, for $y = (c_0 + c_1 x)\cos(lx),\ l \epsilon Z$, we have a section the intervals,

$\left[\frac{\pi}{2l}, \frac{\pi}{l}, \frac{3\pi}{2l}\right], \left[\frac{3\pi}{2l}, \frac{2\pi}{l}, \frac{5\pi}{2l}\right], \left[\frac{5\pi}{2l}, \frac{3\pi}{l}, \frac{7\pi}{2l}\right], \left[\frac{7\pi}{2l}, \frac{4\pi}{l}, \frac{9\pi}{2l}\right]$………… …….

……………. etc. having indicator function as follows:

$$\mu_N(x) = \begin{cases} \emptyset(x); \ \frac{n\pi}{2l} \le x \le \frac{n\pi}{l} \\ \varphi(x); \ \frac{n\pi}{l} \le x \le \frac{(n+2)\pi}{2l} \end{cases}$$

such that $\emptyset\left(\frac{n\pi}{2l}\right) = \varphi\left(\frac{(n+2)\pi}{2l}\right) = 0$

$and\ \emptyset\left(\frac{n\pi}{l}\right) = \varphi\left(\frac{n\pi}{l}\right)$………………………**(15)**

Here, function has maximum level at the point $= \frac{n\pi}{2}$.

So $\left[\frac{\pi}{2l}, \frac{\pi}{l}, \frac{3\pi}{l}\right], \left[\frac{3\pi}{2l}, \frac{2\pi}{l}, \frac{5\pi}{2l}\right], \left[\frac{5\pi}{2l}, \frac{3\pi}{l}, \frac{7\pi}{2l}\right]$, $\left[\frac{7\pi}{2l}, \frac{4\pi}{l}, \frac{9\pi}{2l}\right]$…………..

……,……………. etc. are called imprecise numbers.

Since the indicator function of the imprecise number has the membership function and the reference function. The area of the cosine imprecise function will be measured separately for the respective function so that we can have better information how their character changes in the interval. Total area of the changes is the area of this cosine imprecise function. Here the limit of the integrations is taken in between maximum and minimum point, as the above mentioned membership function and the reference functions are counted in between this two points. For this purpose, let us define the area bounded by cosine imprecise function of degree one as follows:

(i) Area bounded by Cosine imprecise functionangle multiple of one

$$I_0 = \int_0^{\frac{\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$

$$= c_0 + \frac{c_1(\pi - 2)}{2} I_1$$

$$= \int_{\frac{\pi}{2}}^{\pi} (c_0 + c_1 x)\cos(x)\,dx$$

$$= -c_0 - \frac{c_1(\pi + 2)}{2}$$

$$I_2 = \int_{\pi}^{\frac{3\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx = -c_0 - \frac{c_1(3\pi - 2)}{2}$$

$$I_3 = \int_{\frac{3\pi}{2}}^{2\pi} (c_0 + c_1 x)\cos(x)dx = c_0 + \frac{c_1(3\pi + 2)}{2}$$

$$I_4 = \int_{2\pi}^{\frac{5\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx = c_0 + \frac{c_1(5\pi - 2)}{2}$$

$$I_5 = \int_{\frac{5\pi}{2}}^{3\pi} (c_0 + c_1 x)\cos(x)\,dx = -c_0 - \frac{c_1(5\pi + 2)}{2}$$

$$I_6 = \int_{3\pi}^{\frac{7\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx = -c_0 - \frac{c_1(7\pi - 2)}{2}$$

$$I_7 = \int_{\frac{7\pi}{2}}^{4\pi} (c_0 + c_1 x)\cos(x)\,dx = c_0 + \frac{c_1(7\pi + 2)}{2}$$

$$I_8 = \int_{4\pi}^{\frac{9\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx = c_0 + \frac{c_1(9\pi - 2)}{2}$$

so on.

Here, negative sing are shown area bounded by the imprecise function lower part of the x-axis.

Area of the membership imprecise function is

$$I_M = I_1 + |I_3| + I_5 + |I_7| = (4)c_0 + \frac{c_1}{2}\sum_{k=1}^{4}[(2k-1)\pi + 2(4)]$$

Area of the reference imprecise function is

$$I_R = I_0 + |I_2| + I_4 + |I_6| + I_8 = I_0 + (4)c_0 + \frac{c_1}{2}\sum_{k=1}^{4}[(2k+1)\pi - 2(4)]$$

Area of this imprecise function for the interval $\left[0, \frac{9\pi}{2}\right]$ is

$$I = I_M + I_R$$
$$= \int_0^{\frac{9\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$
$$= \int_0^{\frac{\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx + \int_{\frac{\pi}{2}}^{\frac{9\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$
$$= I_0 + c_0(8) + c_1\frac{\pi}{2}\sum_{k=1}^{4}[(2k+1) + (k-1)]$$
$$= I_0 + c_0(8) + c_1\pi\sum_{k=1}^{8} k$$
$$= I_0 + c_0(8) + c_1\pi\sum_{k=1}^{8} k \;; for\ \frac{9\pi}{2} - \frac{\pi}{2} = \frac{8\pi}{2}$$

Here our present imprecise function has maximum value at $x = \frac{\pi}{2}$

Thus the area of the imprecise this function in the interval $\left[0, \frac{n\pi}{2}\right]$ is

$$\int_0^{\frac{n\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$
$$= \int_0^{\frac{\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx + \int_{\frac{\pi}{2}}^{\frac{n\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$
$$= I_0 + c_0(n) + \frac{c_1}{1}\sum_{k=1}^{n-1}[k\pi]; for\ \frac{n\pi}{2} - \frac{\pi}{2}$$
$$= \frac{(n-1)\pi}{2}\ and\ l = 1 \quad \textbf{(16)}$$

Where, $I_0 = \frac{c_0}{1} + \frac{c_1}{2^{(2\times1-1)}}(\pi - 2); for\ l = 1$

(ii) Area bounded by cosine imprecise function multiple of angle two:

$$I_0 = \int_0^{\frac{\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx$$
$$= \frac{c_0}{2} + \frac{c_1(\pi - 2)}{8}$$
$$I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (c_0 + c_1 x)\cos(2x)\,dx$$
$$= -\frac{c_0}{2} - \frac{c_1(\pi + 2)}{8}$$
$$I_2 = \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx = -\frac{c_0}{2} - \frac{c_1(3\pi - 2)}{8}$$
$$I_3 = \int_{\frac{3\pi}{4}}^{\pi} (c_0 + c_1 x)\cos(2x)dx = \frac{c_0}{2} + \frac{c_1(3\pi + 2)}{8}$$
$$I_4 = \int_{\pi}^{\frac{5\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx = \frac{c_0}{2} + \frac{c_1(5\pi - 2)}{8} I_5$$
$$= \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (c_0 + c_1 x)\cos(2x)\,dx = -\frac{c_0}{2} - \frac{c_1(5\pi + 2)}{8}$$
$$I_6 = \int_{\frac{3\pi}{2}}^{\frac{7\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx$$
$$= -\frac{c_0}{2} - \frac{c_1(7\pi - 2)}{8} I_7$$
$$= \int_{\frac{7\pi}{4}}^{2\pi} (c_0 + c_1 x)\cos(2x)\,dx = \frac{c_0}{2} + \frac{c_1 7\pi + 2)}{8}$$
$$I_8 = \int_{2\pi}^{\frac{9\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx = \frac{c_0}{2} + \frac{c_1(9\pi - 2)}{8})$$

so on.

Here, negative sing are shown area bounded by the imprecise function lower part of the x-axis.

Area of the membership imprecise function is

$$I_M = |I_1| + I_3 + |I_5| + I_7$$
$$= (4)c_0 + \frac{c_1}{8}\sum_{k=1}^{4}[(2k-1)\pi + 2(4)]$$

Area of the reference imprecise function is

$$I_R = I_0 + |I_2| + I_4 + |I_6| + I_8$$
$$= I_0 + (4)c_0 + \frac{c_1}{8}\sum_{k=1}^{4}[(2k+1)\pi - 2(4)]$$

Area of this imprecise function for the interval $\left[0, \frac{9\pi}{4}\right]$ is

$$I = I_M + I_R$$
$$= \int_0^{\frac{9\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx$$
$$= \int_0^{\frac{\pi}{4}} (c_0 + c_1 x)\cos(2x)\,dx + \int_{\frac{\pi}{4}}^{\frac{9\pi}{4}} (c_0 + c_1 2x)\cos(x)\,dx$$
$$= I_0 + c_0(8) + c_1\frac{\pi}{8}\sum_{k=1}^{4}[(2k+1) + (2k-1)]$$
$$= I_0 + c_0(8) + c_1\frac{\pi}{2^2}\sum_{k=1}^{8} k; for\ \frac{9\pi}{4} - \frac{\pi}{4} = \frac{8\pi}{4}$$

Here our present imprecise function has maximum value at $x = \frac{\pi}{4}$

Thus the area of the imprecise this function in the interval $\left[0, \frac{n\pi}{4}\right]$ is

$$\int_0^{\frac{n\pi}{4}} (c_0 + c_1 x)\cos(x)\,dx$$

$$= \int_0^{\frac{\pi}{4}} (c_0 + c_1 x)\cos(x)\,dx + \int_{\frac{\pi}{4}}^{\frac{n\pi}{4}} (c_0 + c_1 x)\cos(x)\,dx$$

$$= I_0 + c_0(n) + c_1 \frac{\pi}{2^2}\sum_{k=1}^{n-1} k\,;\, for\ \frac{n\pi}{4} - \frac{\pi}{4} = \frac{(n-1)\pi}{4}\ and\ l = 2$$

Where, $I_0 = \frac{c_0}{2} + \frac{c_1}{2^{(2\times 2-1)}}(\pi - 2); for\ l = 2$

And also the area bounded by the imprecise function in the interval $\left[0, \frac{n\pi}{2}\right]$ is

$$\int_0^{\frac{n\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$

$$= \int_0^{\frac{\pi}{4}} (c_0 + c_1 x)\cos(x)\,dx + \int_{\frac{\pi}{4}}^{\frac{n\pi}{2}} (c_0 + c_1 x)\cos(x)\,dx$$

$$= I_0 + c_0(n) + c_1 \frac{\pi}{2^{(2\times 2-2)}}\sum_{k=1}^{2(n-1)} k\,;for\ \frac{n\pi}{2} - \frac{\pi}{2} = \frac{(n-1)\pi}{2} = \frac{2(n-1)\pi}{4}\ and\ l = 2$$

Where, $I_0 = \frac{c_0}{2} + \frac{c_1}{2^{(2\times 2-1)}}(\pi - 2); for\ l = 2$

In general area bounded by the imprecise function $y = (c_0 + c_1 x)\cos(lx)\,; l \in Z^+$ is

$$I = I_0 + c_0(n) + \frac{c_1}{l^{(2\times l-2)}}\sum_{k=1}^{n-1}[(2k-1)\pi]\,; for\ \frac{n\pi}{2} - \frac{\pi}{2} = \frac{(n-1)\pi}{2} = \frac{l(n-1)\pi}{2l}\ and\ l \in Z^+ \qquad \text{.......................................}\mathbf{(17)}$$

Where, $I_0 = \frac{c_0}{l} + \frac{c_1}{2^{(2l-1)}}(\pi - 2); for\ l = 2$

Here the present imprecise function has maximum value at $x = \frac{\pi}{2l}$

6. CONCLUSION

Obtaining a general form of Cosine imprecise function in various situations was the main objective of this article. So the general formula of cosine imprecise function is obtained by the help of finite numbers of data collection of points in the beginning. This formula is proved for the particular problems. Finally area of the cosine imprecise function is obtained by the help of integration and summation. Here the limits of the integration is taken from the imprecise number of different interval within the imprecise function and proposed a new formula of area at the last part of the article.

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