Some Characterization of Jump Graphs

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Abstracts: *We present a charactization of jump graphs which are totally disconnected, complete and star. Also we establish the upper and lower bounds for the jump graphs..*

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I. Introduction

The graph considered here are finite, undirected without loops or multiple edges. Here we begin with the definition of jump graph as given in [1] and study some basic properties on this.

For a graph G with edge set E(G) we construct another graph namely jump graph J(G) as follows; Let G be a non- empty graph. The jump graph J(G) of G is the graph whose vertices are edges of G, and where two vertices of J(G) are adjacent if and only if they are not adjacent in G. Equivalently, the Jump graph J(G) of G is the complement of line graph of G.

We require the following definition.

A graph G^* is the end edge of the graph of a graph G, if G^* is obtained from G by joining the end edge u_i u_j' at each vertex u_i of graph G.

We state some elementary properties of jump graphs.

Theorem1.1. Let G be a (p ,q) graph Then,

(1) J(G) is a $\left(q,\ \binom{q+1}{2} - \frac{1}{2}\sum d_i^2\right.$ graph. $\left.\right)$
Where d_i is the degree of the vertex v_i in G.

(2) Δ (J(G))= q+1 – δ_e(G)
Where δ_e(G) is the minimum edge degree of graph G

(3) Δ(J(G)) = q+1 - Δ_e(G)
Where Δ_e(G) is the maximum edge degree of G.RESULTS;

Theorem 1.2, The jump graph J(G) is totally disconnected if and only if G is a star.

PROOF; Assume jump graph J(G) is totally disconnected. Then all vertices of J(G) are non adjacent . But these vertices are mutually adjacent pendent edges of G. Hence g is a star.

Conversely, suppose G is a star. Then each edge is adjacent to every other edges of G. By definition of jump graph J(G) , the edges of G becomes the vertices of J(G) of degree zero. Hence J(G) is totally disconnected. Hence the proof.

Here we establish the charactization of jump graphs which are complete.

Theorem 1.3. For any graph G, the jump graph J(G) is complete if and only if G= nK_2 where n$\geq$ 1.

PROOF: Assume jump graph J(G) is complete. Then all vertices of J(G) are adjacent in each other. But these vertices are mutually non-adjacent edges of G. Hence G is nK_2.

Conversely, suppose G=nK_2. Then every edge of G is independent in each other. By definition of jump graph J(G), the edges of G becomes the vertices of J(G) which are mutually adjacent to each other. Hence J(G) is complete. Hence the proof

We now characrize the graphs whose jump graphs are star graph

Theorem 1.4 . For any graph G, the jump graph J(G) is star if and only if

G = $K_2 \cup K_{1,n}$

PROOF; Assume J(G) is star. Then we have following cases.

Case 1. Let v be the vertex of maximum degree of J(G). Which is adjacent to every other vertices of J(G). Then vertex v forms an independent edge. i .e., K_2 in graph G.

Case 2. The pendent vertices of J(G) are not adjacent. But these vertices are mutually adjacent pendent edges in G. Hence they form star graph i.e., K_2 in G

Therefore from above cases we conclude that G = $K_2 \cup K_{1,n}$

Conversely, suppose G = $K_2 \cup K_{1,n}$ Then an edge K_2 is not adjacent to any edge of $K_{1,n}$. Then by the definition of jump graph J(G), the vertex corresponds to an edge of K_2 Thus the resulting graph is a star graph.

The girth of a graph is denoted by g(G), is the length of a shortest cycle (if any) in G. Note that this term is undefined if G has cycles.

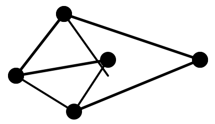
Theorem 1.5. For any connected graph G

1) g(J(G)) =3, if G has three mutually nonadjacent edges (or $3K_2$ is an induced sub graph of G)
2) $4 \leq$ g(J(G))≤ 6. Otherwise.

PROOF: Let G be a connected graph . we consider the following cases.

Case 1. If $3K_2$ is an induced sub graph of G. Then the vertices corresponding to these $3K_2$are not adjacent in L(G) where as they are mutually adjacent in J(G). Thus forms a triangle in J(G). Hence g(J(G))= 3.

Case 2. If $3k_2$ is not an induced sub graph of G. Then g(J(G)) > 3. Now assume that, g(J(G)) =7 Then $\bar{F}$(see the figure) is an induced sub graph of J(G)), a contradiction. There fore g(J(G)) ≤ 6



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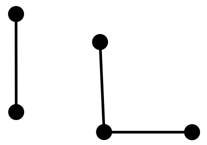

$\bar{F}$

Hence $4 \leq$ g(J(G))≤ 6.

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