On Construction of Codes by using Fuzzy Sets and Soft Sets

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Abstract :

The objective of this paper is to study the codes arising from Fuzzy sets and Soft sets.Properties of fuzzy linear codes and fuzzy cyclic codes are discussed by means of fuzzy linear space. P-fuzzy sets are considered as mapping from an arbitrary non-empty set S *into a partially ordered set* P *which determines a binary block code* V *of length* n *. At last, the codes developed by using soft sets,called soft codes(soft linear codes) are discussed with examples.*

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1. INTRODUCTION

Coding theory is concerned with reliability of communication over noisy channel. Algebraic codes [15] are used for data compression, error correction and for network coding. The theory of error-correcting codes was first introduced by Claude Shannon in 1948 and then gradually developed by time to time by different researchers. Messages in the form of bit strings are encoded by translating them intolonger bit strings, called codeword. A set of codeword is called a code. There are many types of codes which is important to its algebraic structures such as Linear block codes, Hamming codes, BCH codes and so on. The most common type of code is a linear code over the finite field F_q .More literature can be studied in [15,16,17,19,21].

Fuzzy sets were introduced by Lotfi A. Zadeh [3] as an extension of crisp set (classical set).Fuzzy set theory permits the gradual assessment of themembership of elements in a set which is described in the interval $[0,1]$It can be used in a wide range of domains where the information is partial and vague. Fuzzy set perhaps is the most suitable framework to model uncertain data. Fuzzy sets have a several applications in the areas such as signal processing, decision making, control theory, pattern recognition, computer version and so on.The complexities of modeling uncertain data are the main problem in engineering, environmental science, social sciences, health and medical sciences etc. The fuzzy set theory [2], probability theory, rough set theory [10] etc. are well known and useful mathematical tools which describes uncertainty but each of them has its own limitation pointed out by Molodstov. Therefore, Molodstov introduced the theory of soft sets [1] to model vague and uncertain information.A soft set is a parameterized collection of subsets of a universe of discourse.A huge amount of literature can be seen in [1,2,3,5,9,10,11].

In this paper,codes arising from fuzzy linear spaceand P -fuzzy sets are discussed. P -fuzzy sets are considered as mapping from an arbitrary non-empty set S into a partially ordered set P , which determines a binary block-code V of length n . Further, soft codes (soft linear codes) are discussed through the application of soft sets which is an approximated collection of codes.

The paper is organized as follows: In section 2, basic concepts of fuzzy sets, soft sets and linear codes are presented. In section 3, codes arising from fuzzy linear space and P -fuzzy sets are discussed.In section 4, codes from soft sets, called soft codes (soft linear codes) are presented along with their fundamental definitions and examples.

2.*Preliminaries :*

This section has two subsections. In the first section, we recapitulate some underlying definitions and basics ofFuzzy sets and Soft sets. The second section recalls all the basicnotion oflinear algebraic codes.

2.1. *Fuzzy Sets and Soft Sets :*

Definition 2.1.1: [3]A fuzzy set A in R (real line)is defined to be a set of ordered pairs,

$A = \{(x, A(x)) / x \in R\}$ where $A(x)$ is called the membership function for the fuzzy set.

Definition 2.1.2:[3]The**α-cut** of α -level set of fuzzy set A is a set consisting of those elements of the universe X whose membership values exceed the threshold level α.
That is $A_\alpha = \{x \in X / A(x) \geq \alpha\}$.

Definition 2.1.3:[14] Let S be a non-empty set and $(P, \leq)$ a partially ordered set.
Any function $A : S \to P$ is a P-fuzzy set on S.
Also, for $p \in P$, $A : S \to \{0,1\}$
So that for $x \in S$, $A_p(x) = 1$, iff $A(x) \geq p$. Here $A_p(x)$ is a characteristic function of a p-level subset (or a p-cut). That is $A_p = \{x \in S / A(x) = 1\}$.

Remark 1:[14] Let $A : S \to P$ be a P-fuzzy set on S, and ~ a binary relation on P, then for $p, q \in P$, $p \sim q$ if and only if $A_p = A_q$.
Obviously, ~ is an equivalence relation on P.

Lemma 1 :[14]Let $A : S \to P$ be a fuzzy set. For every $x \in S$, if $A(x) = p$, then p is a supremum of the class to which it belongs, that is, $p = \vee[p]\sim$

Theorem 1:[14](Decomposition of P-fuzzy sets)
If $A : S \to P$ is a P-fuzzy set on S, For $x \in S$, $A(x) = \vee(p \in P / A_p(x) = 1)$.
That is, the supremum on the right exists in $(P, \leq)$ for every $x \in S$, and is equal to $A(x)$.

Definition 2.1.4 :[18]Let V_n be a n-dimensional linear space on a field F_q, A a fuzzy subset of V_n, if for any $x, y \in V_n$, $\alpha \in F_q$, we have

(1) $A(x + y) \geq \min\{A(x), A(y)\}$; (2) $A(\alpha x) \geq A(x)$

Then, A the fuzzy linear subspace of V_n on F_q.

Definition 2.1.5 :[1] A pair (F, A) is called a soft set over U, where F is mapping given by $F : A \to P(U)$. Here U refers to an initial universe and E to be a set of parameters. $P(U)$ denote the power set of U and $A \subseteq E$.

A soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$is considered to be set of e- elements of the soft set (F, A) or as the set of e- approximate elements of the soft set.

Definition 2.1.6 :[1]Let(F, A) and (G, B)be the two soft sets over a common universe U, then (F, A) is called a soft subset of (G, B) denoted by $(F, A) \tilde{\subseteq} (G, B)$ if,

(1) $A \subseteq B$
(2) $\forall e \in A$, $F(e)$and$G(e)$ are identical approximations.

2.2 :*Linear Algebraic Codes :*

Definition 2.2.1 :[21]A code$C$ is any non-empty subset of F_q^n. The code C is called linear,if it is an F_q-linear subspace of F_q^n. The number n is the length of the code.

Definition 2.2.2 :[21]The Hamming distance d on F_q^n is given by $d(x, y) := |\{i / 1 \leq i \leq n, x_i \neq y_i\}|$ where $x = (x_1, \ldots x_n)$ and $y = (y_1, \ldots y_n)$.

Definition 2.2.3 :[21]The minimum distance of a code $C \subseteq F_q^n$ is given by $d(C) := \min\{d(x, y) : x, y \in C, x \neq y\}$.

Definition 2.2.4 :[21]The linear code C is called linear $[n, k]$-code, if $\dim(C) = k$.

Definition 2.2.5 : [21] Let C be a linear $[n, k]$-code. Let G be a $k \times n$ matrix whose rows forms a basis of C. Then G is called generator matrix of the code C.

Definition 2.2.6 :[16] [21] Let C be an $[n, k]$-code over F_q. Then the dual code of C is denoted as

$C^{\perp}$ and is defined to be $C^{\perp} = \{y \in F_q^n : x \cdot y = 0, \forall x \in C\}$.

Definition 2.2.7 :[16] [21]A code C is called self-orthogonal code , if $C \subset C^{\perp}$.

3. *Codes arising from Fuzzy Sets:*

This section deals with construction of codes from fuzzy set by means of fuzzy linear space. Block-code from P-fuzzy sets arealso discussed.

Definition 3.1 :[18] Let F_2 be a binary symmetric channel, then the fuzzy linear subspace A of V_n is called a fuzzy linear code, where V_n is the $n-$ dimensional linear space over F_2.

Lemma 2 : A is the fuzzy linear subspace, if for any $\alpha \in [0,1]$,If $A_\alpha \neq \phi$, A_α is a linear subspace of V_n.

Definition 3.2 : A is a fuzzy linear code, iff for any $\alpha \in [0,1]$,If $A_\alpha \neq \phi$, A_α is a linear code.

Definition 3.3 :A fuzzy linear subspace A of V_n is called a fuzzy cyclic code, if for any $(a_0, a_1, ..., a_{n-1}) \in V_n$, we have $A((a_{n-1}, a_0, ..., a_{n-2})) \geq A((a_0, a_1, ..., a_{n-1}))$.

Lemma 3: A is a fuzzy cyclic code, iff for any $\alpha \in [0,1]$,If $A_\alpha \neq \phi$, A_α is a cyclic code.

Lemma 4 : Let A and B be two fuzzy cyclic codes, then (1) $A \cap B$ and (2) $A + B$ are fuzzy cyclic codes.

$P-$Fuzzy sets are considered to be mapping from an arbitrary non-empty set S into a partially ordered set P.Let $S = \{1,2,...n\}$ and let $(P, \leq)$ be a finitepartially ordered set. Every $P-$fuzzy set on S determines a binary block-code V of length n in the following way :

For every class $[p]\sim(p \in P)$, there corresponds a codeword $v_{[p]} = x_1 x_2 ... x_n$, such that $x_i = j$ if and only if, $A_p(i) = j$, for $i \in S$ and $j \in [0,1]$.

Forany $x, y \in V$, $x = x_1....x_n$, $y = y_1...y_n$, $x \leq y$ if and only if, $y_1 \leq x_1, ..., y_n \leq x_n$ ....**(@)**

Where $\leq$ is the ordinary ordering relation on thelattice $(\{0,1\}, \leq) : 0 < 1$.

Theorem 2 :[14]Every finite partially ordered set $(P, \leq)$ determines a block-code V , such that $(P, \leq)$ is isomorphic to $(V, \leq)$.

Proof : Let $P = \{p_1, ..., p_n\}$ and let $A : P \rightarrow P$ be the mapping as a P-fuzzy set on P.

The decomposition of A gives a family $\{A_p / p \in P\}$ which is the required code underthe above defined ordering relation. Now considerthe mapping $f : P \rightarrow \{A_p / p \in P\}$, such that $f(p) = A_p$.By **Lemma 1**, every (~)- class contains exactly one element and thus f is one - to-one. If $p \leq q$, then $A_q \subseteq A_p$. By **(@)** follows that $A_p \leq A_q$, and f is an isomorphism.

Theorem 3 : Let $V = \{v_1, ..., v_n\} \subseteq \{0,1\}^n$ be a binary block-code, such that for every $i \in \{1, ..., n\}$ at least one codeword has a nonzero i^{th}-coordinate. Then there is a $P-$fuzzy set which corresponds to V if and only if, for every $i \in \{1, ..., n\}$, $\vee (v \in V / v(i) = 1) \in V$.

The Hamming distance $d(x, y)$ between x and y from $\{0,1\}^n$ is the number of coordinates in which x and y differ. That is, $d(x, y) := \{i / x_i \neq y_i\}$, $x, y \in \{0,1\}^n$.

The code distance of $V \subseteq \{0,1\}^n$ is the minimum Hamming distance between two codewordsin V denoted by $d(V)$ and is defined as,

$$d(V) := \min\{d(x, y) : x, y \in \{0,1\}^n, x \neq y\}.$$

4. ***Codes from SoftSets(SoftLinear Codes) :***

In this section the codes evolved from soft sets, called soft codes (soft linear codes) are discussed along with examples.

Definition 4.1 :[1][13]Let F_q be a finite field and $V = F_q^n$ be a vector space over F_q, where n is a positive integer. Let $P(V)$ be the power set of V and (F, A) be a soft set over V. Then (F, A) is called soft linear code over V if and only if , $F(e)$ is subspace of V which is a linear code.

Example 1 . Let $F_q = F_2$ and $V = F_2^3$ is a vector space over F_2 and let (F, A) be a soft set over $V = K_2^3$. Then (F, A) is a soft linear code over $V = K_2^3$, where $F(e_1) = \{000,111\}$, $F(e_2) = \{000,101,011,110,111\}$.

Definition 4.2 :[13] Let (F, A) be a soft code over $V = F_q^n$. Then D_s is called soft dimension of (F, A) if $D_s = \{\dim(F(e)), \forall e \in A\}$.

Example 2 . Let (F, A) be a soft code defined as in above **Example 1.** Then the soft dimension is given by

$$D_s = \{\dim(F(e_1)) = 1, \dim(F(e_2)) = 2\} = \{1,2\}$$

Definition 4.3 :[13] A soft linear code (F, A) over F_q^n of soft dimension D_s is called soft linear $[n, D_s]$-code.

Definition 4.4 : Let (F, A) be a soft code over $V = F_q^n$. Then the soft minimum distance of (F, A) is denoted by $S_d(F, A)$ and is defined to be $S_d(F, A) = d(F(e)) : d(F(e))$ is the minimum distance of the code $F(e)$; for all $e \in A$

Example 3. Let $F_q = F_2$ be a finite field and $V = F_2^3$ is a vector space over F_2 and let (F, A) be a soft set over $V = F_2^3$. Then clearly (F, A) is a soft code over $V = F_2^3$, where $F(e_1) = \{000,111\}$, $F(e_2) = \{000,101,110,011\}$.The minimum distance of the code $F(e_1) = 3$ and $F(e_2) = 2$. Thus the soft minimum distance of the soft code (F, A) is given as $S_d(F, A) = \{d(F(e_1)) = 3, d(F(e_2)) = 2\} = \{3,2\}$

Definition 4.5 : Let (F, A) be a soft linear $[n, D_s]$-code. Let G_s be the n-matrix whose elements are the generator matrices of the soft code (F, A) corresponding to each $e \in A$ where n is the number of parameters in A.

The matrix G_s is termed as the soft generator matrix of the soft linear code (F, A).

Definition 4.6 : Let (F, A) be a soft $[n, D_s]$-code over the field F_q and let the vector space $V = F_q^n$. Then the soft dual code of (F, A) is defined to be

$(F, A)^\perp = F(e)^\perp : F(e)^\perp$ is the dual code of $F(e)$, for all $e \in A$.

Example 4. Let $F_q = F_2$ be a finite field and $V = F_2^3$ is a vector spaceover F_2 and let (F, A) be a soft set over $V = F_2^3$. Then clearly (F, A) is a soft code over $V = F_2^3$, where $F(e_1) = \{000,111\}$ and $F(e_2) = \{000,101,110,011\}$.

Then the soft dual code of (F, A) is denoted as $(F, A)^\perp$ where $F(e_1)^\perp = \{000,110,101,011\}$

and $F(e_2)^\perp = \{000,111\}$.

Definition 4.7 :A soft linear code (F, A) in $V = F_q^n$ over the field F_q is called soft self-dual code if $(F, A)^\perp = (F, A)$.

Definition 4.8 :[13] A soft linear code (F, A) in $V = F_q^n$ over the field F_q is called complete-soft code, if for all $e \in A$, the dual of $F(e)$ also exists in (F, A).

Example 5. Let $F_q = F_2$ be a finite field and $V = F_2^3$ is a vector space over F_2 and let (F, A) be a soft set over $V = F_2^3$. Then (F, A) is a soft code over $V = F_2^3$, where $F(e_1) = \{000,111\}$ and $F(e_2) = \{000,110,101,011\}$.

Then $(F, A)$is acomplete -soft code. Since the dual of $F(e_1)$ and $F(e_2)$andalso thedual of $F(e_2)$ is $F(e_1)$.

Theorem 4. [13]All complete-soft codes are trivially soft codes but the converse is not true in general.

5. Conclusion :

In this paper, codes from Fuzzy sets and Soft sets are discussed at the basic level. The advantage of soft codes (soft linear code) is that it can send n-messages to n -persons.

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