pgrw-Locally Closed Sets in Bitopological Spaces

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Abstract - *The aim of this paper is to introduce a new class of closed sets called pgrw-locally closed sets, pgrw-lc*- sets, pgrw-lc**- sets in bitopological spaces. A subset A of a bitopological space (X,τ_1,τ_2) is called (τ_i,τ_j)-pgrw-locally closed if $A=U\cap V$ where U is a τ_i-pgrw-open set and V is a τ_j-pgrw-closed set. Examples are provided to illustrate the behaviour of these new classes of sets and maps.*

Keywords- pgrw-locally closed set, pgrw-lc*-set, pgrw-lc**-set, pairwise pgrw-lc-continuous maps , pairwise pgrw-lc-irresolute maps

I. INTRODUCTION

According to Bourkbaki[1], a subset of a topological space X is called locally closed in X if it is the intersection of an open set and a closed set in X. In 1963, Kelly [2] defined a bitopologicalspace (X, τ_1, τ_2) to be a set X equipped with two topologies τ_1, τ_2 on X and initiated a systematic study of bitopological spaces. ω-Locally closed set in a bitopologicalspace is introduced by S. S. Benchalliand et. al. [3].In the present paper we define pgrw-locally closed sets, pgrw-lc*-sets, pgrw-lc**-sets and investigate some of their properties.

II. PRELIMINARIES

Throughout this paper,X, Y and Z represent bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2), i, j∈ {1, 2} and i≠j.

Definition: A subset A of a topological space (X, τ) is called a pregeneralised regular weakly (pgrw)closedset[5] if $pcl(A)\subseteq U$ whenever $A \subseteq U$ and U is a rw-open set.

Definition: A subset A of a topological space (X, τ) is called a rw-closed [4] if $cl(A)\subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X.

Definition:A subset A of a bitopological space (X, τ_1,τ_2) is called a(τ_i,τ_j)-pgrw closed set if τ_j-$pcl(A)\subseteq G$ whenever $A \subseteq G$ and G is a τ_i-rw open set where $i,j\in\{1,2\}$ $i \neq j$.

III.(τ_i,τ_j)-pgrw-LOCALLY CLOSED SETS

3.1Definition: A subset A of a bitopological space (X,τ_1,τ_2) is called (τ_i,τ_j)-pgrw-locally closed if $A=U\cap V$ where U is a τ_i-pgrw-open set and V is a τ_j-pgrw-closed set.

3.2Notation: (τ_i,τ_j)-pgrw-LC(X) represents the collection of all (τ_i,τ_j)-pgrw-lc subsets of (X,τ_1,τ_2).

3.3Example:X={a,b,c,d},
τ_1={X,Φ,{a},{b},{a,b},{a,b,c}}τ_2={X,Φ,{a},{c,d},{a,c,d}}

τ_1-pgrw-open sets are X,Φ,{a,b,d},{a,b,c},{a,d},{a,b},{b,c},{a,c},{a},{b},{c}.

τ_2-pgrw-closedsets are X,Φ,{b},{c},{d},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,c,d}, {a,b,d}. The set {a,b,c} is τ_1-pgrw-open and {b,c,d} is τ_2-pgrw-closed.

{a,b,c}∩{b,c,d} ={b,c} is (τ_1, τ_2)-pgrw-locally-closed.

3.4 Example: In example3.3{d}={a,b,d}∩{d}where {a,b,d} is a τ_1-pgrw-open set and {d} is a τ_2-pgrw-closed set. ∴ {d} is a (τ_1, τ_2)-pgrw-locally-closed set.

Also {d}={a,d}∩{b,d} where {a,d} is a τ_1-pgrw-open set and {b,d} is a τ_2-pgrw-closed set. ∴ U and V of definition 3.1are not unique.

3.5 Theorem: A subset A of a bitopological space (X,τ_1,τ_2) is (τ_i, τ_j)-pgrw-locally closed iff X-A is the union of a τ_i-pgrw- closed set and a τ_j-pgrw-open set.

Proof: A is a (τ_i,τ_j)- pgrw- locally closed subset of X.

⇒ A = U∩V where U is τ_i-pgrw-open and V is τ_j-pgrw-closed.

⇒ X-A = (X-U)∪(X-V) where X-U is τ_i-pgrw-closed and X-V is τ_j-pgrw-open.

Conversely

X-A = G∪F where G is τ_i-pgrw-closed and F is τ_j-pgrw-open.

⇒ A = (X-G) ∩ (X-F) where X-G is τ_i-pgrw-open and X-F is τ_j-pgrw-closed.

⇒ A is a (τ_i,τ_j)- pgrw- locally closed set.

3.6 Theorem:

1) Every τ_i-pgrw-open subset of (X,τ_1,τ_2) is a (τ_i, τ_j)- pgrw-locally closed set.
2) Every τ_j-pgrw-closed subset of (X,τ_1,τ_2) is a (τ_i, τ_j)- pgrw-locally closed set.

Proof:

1) A is a τ_i-pgrw-open subset of a bitopological space (X,τ_1,τ_2).

   ⇒A = A∩X where A is τ_i-pgrw-open and X is $\tau_j$pgrw-closed.

   ⇒A is (τ_i,τ_j)-pgrw-locally closed.

2) A is a τ_j-pgrw-closed subset of a bitopological space (X,τ_1,τ_2).

   ⇒A = X∩A where X is τ_i-pgrw-open and A is τ_j-pgrw-closed.

   ⇒A is a (τ_i, τ_j)-pgrw-locally closed set.

The converses are not true.

3.7 Example: In example 3.3 {d} = {a,b,d}∩{d} is a (τ_1, τ_2)-pgrw-locally-closed set, but not τ_1-pgrw-open. The set {a} = {a,b}∩{a,c,d} is (τ_1, τ_2)-pgrw-locally-closed set, but not τ_2-pgrw-closed.

3.8 Corollary:

1) Every τ_i-open subset of (X,τ_1,τ_2) is a (τ_i, τ_j)- pgrw- locally closed set.
2) Every τ_j-closed subset of (X,τ_1,τ_2) is a (τ_i, τ_j)-pgrw-locally closed set.

Proof :

1) A is a τ_i-open subset of (X,τ_1,τ_2).

   ⇒ A is τ_i-pgrw-open.

   ⇒ A is a (τ_i,τ_j)- pgrw-locally closed set.

2) A is a τ_j-closed subset of (X,τ_1,τ_2).

   ⇒ A is τ_j-pgrw-closed.

   ⇒ A is a (τ_i,τ_j)- pgrw-locally closed set.

3.9 Theorem: Every(τ_i , τ_j)-lc-set of a bitopological space (X,τ_1,τ_2) is (τ_i, τ_j)- pgrw-lc-set.

Proof: A is a (τ_i, τ_j)- locally closed set in X.

⇒ A = G∩F where G is τ_i- open and F is τ_j-closed.

⇒ A = G∩F where G is τ_i- pgrw-open and F is τ_j-pgrw- closed.

⇒ A is a (τ_i,τ_j)-pgrw- locally closed set in X.

Converse is not true.

3.10 Example: In example 3.3 {d} is (τ_1, τ_2)-pgrw-lc, but not (τ_1, τ_2)-lc.

3.11 Definition : A subset A of a bitopological space (X,τ_1,τ_2) is called (τ_i, τ_j)- pgrw- lc* if there exist a

τ_i- pgrw-open set S and a τ_j- closed set F in (X,τ_1,τ_2) such that A=S∩F.

3.12 Example: Consider 3.3.{b,c} = {a,b,c}∩{b,c,d} where {a,b,c} is τ_1-pgrw-open and {b,c,d} is τ_2-closed.

So {b,c} is (τ_1, τ_2)-pgrw-lc*.

3.13Definition: A subset A of a bitopological space (X,τ_1,τ_2) is called (τ_i , τ_j)- pgrw- lc** if there exist a τ_i- open set S and a τ_j-pgrw- closed set F in (X,τ_1,τ_2) such that A = S∩F.

3.14 Example: In 3.3 {a,b} = {a,b}∩{a,b,d} where {a,b} is τ_1-open and { a,b,d } is τ_2-pgrw-closed. So {a,b} is (τ_1, τ_2)-pgrw-lc**.

3.15 Notation:

i. (τ_i ,τ_j)-pgrw-LC*(X) = The collection of all (τ_i , τ_j)-pgrw-lc* subsets of (X,τ_1,τ_2).

ii. (τ_i ,τ_j)-pgrw-LC** (X) = The collection of all (τ_i , τ_j)-pgrw-lc** subsets of (X,τ_1,τ_2).

3.16 Theorem: Every τ_i-pgrw-open subset of X is a

(τ_i ,τ_j)-pgrw-lc* set.

Proof: A isτ_i-pgrw-open in a bitopological space X.

⇒ A = A∩X where A is τ_i-pgrw-open and X is τ_j -closed.

⇒ A is (τ_i ,τ_j)-pgrw-lc*.

3.17 Corollary: Every τ_i -open subset of X is a (τ_i ,τ_j)- pgrw- lc* set.

Proof: A is τ_i-open. ⇒ A is τ_i - pgrw-open. ⇒ A is (τ_i ,τ_j)- pgrw-lc*.

3.18 Theorem: Every τ_j-pgrw-closed subset of X is a (τ_i ,τ_j)- pgrw-lc** set.

Proof: A is a τ_j-pgrw-closed subset of a bitopological space X.

⇒A=X∩A where X is τ_i-open and A is τ_j-pgrw-closed.

⇒ A is (τ_i ,τ_j)-pgrw-lc**.

3.19 Corollary: Every τ_j-closed subset of X is a (τ_i , τ_j)-pgrw-lc** set.

Proof:A is τ_j-closed.

⇒A is τ_j-pgrw-closed.⇒A is (τ_i ,τ_j)- pgrw-lc**.

3.20 Theorem: (τ_i ,τ_j)-pgrw-LC*(X)⊆(τ_i , τ_j)-pgrw-LC(X).

Proof:A∈(τ_i , τ_j)-pgrw-LC*(X). ⇒ A is a (τ_i ,τ_j)-pgrw-lc* set in X.

⇒ A=S∩F where S is τ_i-pgrw-open and F is τ_j-closed.

⇒ A=S∩F where S is τ_i-pgrw-open and F is τ_j-pgrw-closed.

⇒ A is a (τ_i ,τ_j)-pgrw-lc set.

⇒ A∈(τ_i , τ_j)-pgrw-LC(X).

∴ (τ_i ,τ_j)-pgrw- LC*(X)⊆(τ_i , τ_j)-pgrw-LC(X).

The converse is not true.

3.21 Example: Inexample 3.3 {a,c,d} is a (τ_1, τ_2)-pgrw-lc set , but not (τ_1, τ_2)-pgrw-lc*.

3.22Theorem: Every(τ_i,τ_j)-pgrw-lc**-subsetofa bitopological space (X,τ_1,τ_2) is (τ_i ,τ_j)-pgrw-lc-set.

Proof: A is a (τ_i ,τ_j)-pgrw-lc**-set in X.

⇒ A=S∩F where S is τ_i-open and F is τ_j-pgrw-closed.

⇒ A=S∩F where S is τ_i-pgrw-open and F is τ_j-pgrw-closed.

⇒ A is a (τ_i ,τ_j)-pgrw- lc set.

The converse is not true.

3.23 Example: In 3.3 the set {a, d} is a (τ_1, τ_2)-pgrw-lc set ,but not (τ_1, τ_2)-pgrw-lc**.

3.24 Remark: (τ_i, τ_j)-pgrw-lc*(X) and (τ_i, τ_j)-pgrw-lc**(X) are independent of each other.

3.25 Example : Consider example 3.3.{a,c,d} is (τ_1, τ_2)-pgrw-lc**, but not (τ_1, τ_2)-pgrw-lc*.

{a,d} is (τ_1, τ_2)-pgrw-lc*, but not (τ_1, τ_2)-pgrw-lc**.

3.26 Theorem : A subset A of a bitopological space (X,τ_1,τ_2) is (τ_i , τ_j)-pgrw-lc* iff A = G∩τ_j-cl(A) for some τ_i- pgrw-open set G.

Proof: A is a (τ_i , τ_j)-pgrw-lc* subset of a bitopological space (X,τ_1,τ_2).

⇒ A = G∩F where G is τ_i-pgrw- open and F is τ_j-closed.

A ⊆ G and A ⊆τ_j-cl(A). ∴ A ⊆ G∩τ_j-cl(A)....................(i)

Next A ⊆ F and F is τ_j-closed. ⇒τ_j-cl(A) ⊆F

$\Rightarrow G\cap\tau_j\text{-cl}(A) \subseteq G\cap F \Rightarrow G\cap\tau_j\text{-cl}(A)\subseteq A$....................(ii)

From (i) and (ii) $A = G\cap\tau_j\text{-cl}(A)$ where G is a τ_i-pgrw-open set.

Conversely

$A = G\cap\tau_j\text{-cl}(A)$ for some τ_i-pgrw-open set G.

$\Rightarrow$ $A = G\cap\tau_j\text{-cl}(A)$ where G is τ_i-pgrw-open and τ_j-cl(A) is τ_j-closed.

$\Rightarrow$ A is (τ_i,τ_j)-pgrw-lc*.

3.27 Theorem: If A is a subset of (X,τ_1,τ_2) such that

$A \cup (X-\tau_j\text{-cl}(A))$ is τ_i-pgrw open, then A is (τ_i, τ_j)-pgrw-lc*.

Proof: For every subset A of (X,τ_1,τ_2),

$A = A\cup\Phi = A\cup((X-\tau_j\text{-cl}(A) \cap (\tau_j\text{-cl}(A)))$

$= (A \cup (X-\tau_j\text{-cl}(A)) \cap (A\cup\tau_j\text{-cl}(A))$

$= (A \cup(X-\tau_j\text{-cl}(A)) \cap\tau_j\text{-cl}(A)$, because $A\subseteq\tau_j\text{-cl}(A)$.

So if $A\cup(X-\tau_j\text{-cl}(A))$ is τ_i-pgrw open, then A is the intersection of a τ_i-pgrw-open set and a τ_j-closed set.

$\therefore$ A is (τ_i,τ_j)-pgrw-lc*.

3.28 Corollary: If A is a subset of (X,τ_1,τ_2) such that $[\tau_j\text{-cl}(A)]\text{-}A$ is τ_i-pgrw-closed, then A is (τ_i, τ_j)-pgrw-lc*.

Proof: For any sub-set A of X,

$\tau_j\text{-cl}(A)\text{-}A=(\tau_j\text{-cl}(A))\cap A^c=((X-\tau_j\text{-cl}(A))\cup A)^c$

$\therefore$ $(\tau_j\text{-cl}(A)\text{-}A)$ is τ_i-pgrw-closed.

$\Rightarrow$ $A\cup(X-\tau_i\text{-cl}(A))$ is τ_i-pgrw-open.

$\Rightarrow$ A is (τ_i,τ_j)-pgrw-lc*.

3.29 Theorem: If $A\in(\tau_i, \tau_j)$-pgrw-LC*(X) and B is τ_j-closed, then $A\cap B\in(\tau_i, \tau_j)$-pgrw-LC*(X).

Proof: $A\in (\tau_i, \tau_j)$-pgrw-LC*(X) and B is τ_j-closed.

$\Rightarrow$ $A=G\cap F$ where G is τ_i-pgrw-open and F is τ_j-closed and B is τ_j-closed.

$\Rightarrow$ $A\cap B= (G\cap F) \cap B=G\cap (F\cap B)$ where G is τ_i-pgrw-open and $F\cap B$ is τ_j-closed.

$\Rightarrow$ $A\cap B\in(\tau_i,\tau_j)$-pgrw-LC*(X).

3.30 Theorem: If $A\in(\tau_i, \tau_j)$-pgrw-LC** (X) and B is

τ_i-open, then $A\cap B\in(\tau_i, \tau_j)$-pgrw-LC**(X).

Proof: $A\in(\tau_i,\tau_j)$-pgrw-LC** (X) and B is τ_i-open.

$\Rightarrow$ $A=G\cap F$ where G is τ_i-open and F is τ_j-pgrw-closed and B is τ_i-open.

$\Rightarrow$ $A\cap B= (G\cap F)\cap B=(G\cap B)\cap F$ where $G\cap B$ is τ_i-open and F is τ_j-pgrw-closed.

$\Rightarrow$ $A\cap B\in(\tau_i,\tau_j)$-pgrw-LC** (X).

IV.PAIRWISE pgrw-lc-CONTINUOUS MAPS AND PAIRWISE pgrw-lc-IRRESOLUTE MAPS

4.1Definition: A map $f: (X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is called pairwise pgrw-lc-continuous (resp. pairwise pgrw-lc*-continuous, pairwise pgrw-lc**-continuous) if $f^{-1}(V)\in(\tau_i, \tau_j)$ -pgrw-LC(X)(resp. (τ_i,τ_j)-pgrw-LC*(X), (τ_i,τ_j)-pgrw-LC**(X)) $\forall\sigma_i$-open set V in (Y,σ_1,σ_2).

4.2 Example: $X=\{a,b,c,d\}$, $\tau_1=\{X,\Phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$, $\tau_2=\{X,\Phi,\{a\},\{c,d\},\{a,c,d\}\}$

τ_1-pgrw-opensets are $X,\Phi,\{a,b,d\},\{a,b,c\},\{a,d\},\{a,b\},\{b,c\},\{a,c\},\{a\},\{b\},\{c\}$.

τ_2 -closed sets are $X,\Phi,\{b,c,d\},\{a,b\},\{b\}$

τ_2-pgrw-closed setsare $X,\Phi,\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{b,c,d\},\{a,c,d\},\{a,b,d\}$

$Y = \{a,b,c,d\}$, $\sigma_1 = \{Y,\Phi,\{b,c\},\{b,c,d\},\{a,b,c\}$,

$\sigma_2 = \{Y,\Phi,\{a,b\},\{c,d\}\}$

Define $f : X\rightarrow Y$ as $f(a)=b$, $f(b)=c$, $f(c)=d$, $f(d)=a$.

Pre images of σ_1-open sets are $X, \Phi,\{a,b\},\{a,b,d\},\{a,b,c\}$ which are (τ_1, τ_2) -pgrw-lc sets $((\tau_1, \tau_2)$ -pgrw-lc* sets and (τ_1, τ_2)-pgrw-lc** sets).

∴ f is (τ_1, τ_2) -pgrw-lc continuous((τ_1, τ_2) -pgrw-lc* continuous and (τ_1, τ_2)-pgrw-lc** continuous.)........(i)

τ_1 –pgrw-closed sets: X,Φ,{c},{d},{b,c},{c,d},{a,d},{b,d},{b,c,d},{a,c,d},{a,b,d}

τ_2 –pgrw-open sets: X,Φ, {a,c,d},{a,b,d},{a,b,c},{c,d},{a,d},{a,c},{d},{a},{b},{c}

τ_1 –closed sets: X,ø,{b,c,d},{a,c,d},{c,d},{d}

Pre images of σ_2-open sets are X, Φ,{a,d},{b,c} which are (τ_2, τ_1) -pgrw-lc sets ((τ_2, τ_1) -pgrw-lc* sets and

(τ_2, τ_1)-pgrw-lc** sets).

∴ f is (τ_2, τ_1) -pgrw-lc continuous((τ_2, τ_1) -pgrw-lc* continuous,(τ_2, τ_1) -pgrw-lc** continuous.).......... (ii)

So from (i) and (ii) it follows that f is pairwise pgrw-lc-continuous (pairwise pgrw-lc*-continuous , pairwise pgrw-lc**-continuous).

4.3Definition: A map f: $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is called pairwise pgrw-lc-irresolute (resp. pairwise pgrw-lc*- irresolute, pairwise pgrw-lc**-irresolute) if

$f^{-1}(V)\in(\tau_i, \tau_j)$- pgrw-LC(X)(resp. (τ_i, τ_j)-pgrw-LC*(X), (τ_i, τ_j)- pgrw-LC**(X)) ∀ $V\in(\sigma_i,\sigma_j)$-pgrw-LC(Y)(resp. (σ_i,σ_j))-pgrw-LC*(Y), (σ_i,σ_j)-pgrw-LC**(Y)).

4.4 Theorem: If a map f : $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is pairwise-lc-continuous, then it is pairwise-pgrw-lc-continuous.

Proof: A mapf: $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is pairwise-lc-continuous.

⇒ $f^{-1}(V)\in(\tau_i, \tau_j)$-LC (X) $\forall\sigma_i$- open set V in (Y,σ_1,σ_2).

⇒$f^{-1}(V)\in(\tau_i, \tau_j)$-pgrw-LC(X) $\forall\sigma_i$-open set V in (Y,σ_1,σ_2).

⇒ f: $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is pairwise pgrw-lc-continuous.

The converse is not true.

4.5 Example: X = {a,b,c}, τ_1 = {X,Φ,{a},{a,c}} , τ_2 = {X,Φ,{a},{b,c}}

Y = {a,b,c}, σ_1 = {Y,Φ, {a},{b},{a,b}},

σ_2 = {Y,Φ,{b},{c},{b,c}}

σ_1-pgrw-open sets are Y,Φ, {a},{b},{a,b}, σ_2-pgrw-closed sets are Y,Φ,{b},{c},{b,c}. (τ_1, τ_2)-pgrw-lc sets: All subsets of X. Define a map f: X→Y by f(a)=c , f(b)=b , f(c)= a. f is pairwise pgrw-lc-continuous.

(τ_1, τ_2)-lc sets : X , Φ,{a},{c},{b,c}, {a,c} f is not pairwise-lc-continuous. For $f^{-1}(\{b\})=\{b\}$ and is not

(τ_1, τ_2)-lc set.

4.6 Theorem:

(i)If f: $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is pairwise pgrw-lc*-continuous, then it is pairwise pgrw-lc-continuous.

(ii) If f: $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is pairwise pgrw-lc**-continuous, then it is pairwise pgrw-lc-continuous.

Proof:

(i) f: $(X,\tau_1,\tau_2)\rightarrow (Y,\sigma_1,\sigma_2)$ is pairwise pgrw-lc*-continuous.

⇒ $f^{-1}(V)\in(\tau_i, \tau_j)$-pgrw-LC*(X) $\forall\sigma_i$-open set V in (Y,σ_1,σ_2).

⇒ $f^{-1}(V)\in(\tau_i, \tau_j)$-pgrw-LC(X) $\forall\sigma_i$-open set V in (Y,σ_1,σ_2) .

⇒ f is pairwise pgrw-lc-continuous.

(ii) Proof is similar to (i).

The converse statements are not true.

4.7Example: Consider the bitopological spaces in example in 4.2.

The function f: X→Y defined by f(a)=b , f(b)=a , f(c)=c , f(d)=d is pairwise pgrw-lc-continuous. But f is not pairwise pgrw-lc*-continuous. For ({b,c,d} is σ_1-open in Y, but $f^{-1}(\{b,c,d\})=\{a,c,d\}$ is not (τ_1, τ_2)-pgrw-lc*.

4.8 Example: Consider the spaces in example in 4.2 in which the function f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ defined by f(a)=b, f(b)=a, f(c)=d, f(d)=c is a pairwise pgrw-lc-continuous. But it is not pairwise pgrw-lc**-continuous. For, {b,c} is σ_1-open in Y, but $f^{-1}(\{b,c\})=\{a,d\}$ is not (τ_1, τ_2)-pgrw-lc**.

4.9 Theorem: If a map f: $(X,\tau_1,\tau_2)\to(Y, \sigma_1, \sigma_2)$ is pairwise-pgrw-lc-irresolute, then it is pairwise-pgrw-lc-continuous.

Proof: V is σ_i-open in (Y,σ_1,σ_2) and f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc-irresolute.

$\Rightarrow$ V is (σ_i,σ_j)-pgrw-lc and f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc-irresolute.

$\Rightarrow$ $f^{-1}(V)\in(\tau_i,\tau_j)$- pgrw-LC(X) $\forall$ V $\in\sigma_i$

$\Rightarrow$ f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc-continuous.

Converse is not true.

4.10 Example: X = {a,b,c}, , τ_1 = {X,Φ,{a},{b,c}},

τ_2 = {X,Φ,{a},{a,c}}

τ_1-pgrw-open sets: X,Φ,{a},{b,c},

τ_2-pgrw-closed sets: X,Φ,{b},{b,c}

Y = {a,b,c},σ_1 = {Y,Φ,{a},{a,c}},σ_2 = Y,Φ,{a},{b},{a,b}}

σ_1- pgrw-open sets: Y,Φ,{a},{a,c},

σ_2-pgrw-closed sets: Y, Φ,{b,c},{a,c},{c}

A function f :X→Y defined by f(a)=b, f(b)= a, f(c)=c is (τ_1,τ_2) -pgrw-lc-continuous .

(σ_1,σ_2)-pgrw-lc sets of Y are Y , Φ, {a}, {c}, {a,c}, {b,c}. f is not (τ_1,τ_2)-pgrw-lc-irresolute.

For {b,c} is (σ_1, σ_2)-pgrw-lc set in Y, but $f^{-1}(\{b,c\})=\{a,c\}$ is not (τ_1, τ_2)-pgrw-lc.

4.11Theorem: If f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc*-irresolute, then it is pairwise-pgrw-lc*-continuous.

Proof: V is σ_i -open in (Y,σ_1,σ_2) and f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc*-irresolute.

$\Rightarrow$ V is (σ_i,σ_j)-pgrw-lc* and f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc*-irresolute .

$\Rightarrow$ $f^{-1}(V)\in(\tau_i,\tau_j)$- pgrw-LC*(X) $\forall$ V $\in\sigma_i$

$\Rightarrow$ f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc*-continuous.

The converseis not true. For example,consider the example in 4.10.

A function f:X→Y defined by f(a)=b, f(b)= a, f(c)=c is (τ_1,τ_2)-pgrw-lc*-continuous. (σ_1,σ_2)-pgrw-lc* sets of Y are Y , Φ,{a}, {c} , {b,c},{a,c}. f is not (τ_1,τ_2)-pgrw-lc*-irresolute. For {b,c} is (σ_1,σ_2)-pgrw-lc* set in Y,

$f^{-1}(\{b,c\})=\{a,c\}$ is not (τ_1, τ_2)-pgrw-lc*.

4.12Theorem: If f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc**-irresolute, then it is pairwise-pgrw-lc**-continuous.

Proof: Proof is similar to 4.11.

4.13 Corollary:

(i) If a map f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc*-irresolute, then it is pairwise pgrw-lc-continuous.

(ii) If a map f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc**-irresolute, then it is pairwise-pgrw-lc-continuous.

Proof:

(i)f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise-pgrw-lc*-irresolute.

$\Rightarrow$ f is pairwise-pgrw-lc*-continuous.

$\Rightarrow$pairwise pgrw-lc-continuous.

(ii) Proof is similar to (i).

V.COMPOSITION OF MAPS

5.1 Theorem: If f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a pairwise pgrw-lc-irresolute map and g: $(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is a pairwise pgrw- lc-continuous map , then gof is pairwise pgrw-lc-continuous.

Proof: f: $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a pairwise pgrw-lc-irresolute map and g: $(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is a pairwise pgrw- lc-continuous map.

$\Rightarrow\forall$ $V\in\eta_i$ $g^{-1}(V)\in(\sigma_i,\sigma_j)$-pgrw-LC(Y) and $f^{-1}(g^{-1}(V)))\in(\tau_i,\tau_j)$-pgrw-LC(X).

$\Rightarrow\forall$ $V\in\eta_i$ $(gof)^{-1}(V)\in(\tau_i,\tau_j)$-pgrw-LC(X).

$\Rightarrow$ gof is pairwise pgrw- lc-continuous.

5.2 Theorem: Iff: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is a pairwise pgrw-lc*-irresolute map and g:$(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ is a pairwise pgrw-lc*-continuous map, then gof is pairwise pgrw-lc*-continuous.

Proof:f:X→Yispairwisepgrw-lc*-irresoluteand g:Y→Zispairwisepgrw-lc*-continuous.

$\Rightarrow\forall$ V $\in\eta$ $g^{-1}(V)$ is (τ_i,τ_j)-pgrw-lc* and $f^{-1}(g^{-1}(V))$ is (τ_i,τ_j)-pgrw-lc*.

$\Rightarrow\forall$ V $\in\eta$ $(gof)^{-1}(V)$ is (τ_i,τ_j)-pgrw-lc*.

$\Rightarrow$gof: $(X,\tau_1,\tau_2)\rightarrow(Z,\eta_1,\eta_2)$ is pairwise pgrw-lc*-continuous.

5.3 Theorem: If f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is a pairwise pgrw-lc**-irresolute map and g : $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ is a pairwise pgrw-lc**-continuous map, then gof is pairwise pgrw-lc** continuous.

Proof:f:X→Y is pairwise pgrw-lc**-irresolute and g:Y→Z is pairwise pgrw-lc**-continuous.

$\Rightarrow\forall$V $\in\eta$ $g^{-1}(V)$ is (τ_i,τ_j)-pgrw-lc** and $f^{-1}(g^{-1}(V))$ is (τ_i,τ_j)-pgrw-lc**.

$\Rightarrow\forall$V $\in\eta$ $(gof)^{-1}(V)$ is (τ_i,τ_j)-pgrw-lc**.

$\Rightarrow$gof: $(X,\tau_1,\tau_2)\rightarrow(Z,\eta_1,\eta_2)$ is pairwise pgrw-lc**-continuous.

5.4 Theorem: If f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ are pairwise pgrw-lc-irresolute, then gof is pairwise pgrw-lc-irresolute.

Proof: f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ are pairwise pgrw-lc-irresolute.

$\Rightarrow\forall$ $V\in(\eta_i,\eta_j)$-pgrw-LC(Z) $g^{-1}(V)\in(\sigma_i,\sigma_j)$-pgrw-LC(Y) and $f^{-1}(g^{-1}(V))\in(\tau_i,\tau_j)$-pgrw-LC(X).

$\Rightarrow\forall$ $V\in(\eta_i,\eta_j)$-pgrw-LC(Z) $(gof)^{-1}(V)\in(\tau_i,\tau_j)$-pgrw-LC(X).

$\Rightarrow$ gof is pairwise pgrw-lc-irresolute.

5.5 Theorem:f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ are pairwise-pgrw-lc*-irresolutes $\Rightarrow$ gof is pairwise pgrw-lc*-irresolute.

Proof: f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g :$(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ are pairwise-pgrw-lc*-irresolutes.

$\Rightarrow\forall$ $V\in(\eta_i,\eta_j)$-pgrw-LC*(Z) $g^{-1}(V)\in(\sigma_i,\sigma_j)$-pgrw-LC*(Y) and $f^{-1}(g^{-1}(V))\in(\tau_i,\tau_j)$-pgrw-LC*(X).

$\Rightarrow\forall$ $V\in(\eta_i,\eta_j)$-pgrw-LC*(Z) $(gof)^{-1}(V)\in(\tau_i,\tau_j)$-pgrw-LC*(X).

$\Rightarrow$ gof is pairwise pgrw-lc*-irresolute.

5.6 Theorem: f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ are pairwise-lc**-irresolute.$\Rightarrow$ gof is pairwise pgrw-lc**-irresolute.

proof: f: $(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ and g: $(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ are pairwise-lc**-irresolute.

$\Rightarrow\forall$ $V\in(\eta_i,\eta_j)$-pgrw-LC**(Z) $g^{-1}(V)\in(\sigma_i,\sigma_j)$-pgrw-LC**(Y) and $f^{-1}(g^{-1}(V)))\in(\tau_i,\tau_j)$-pgrw-LC**(X).

$\Rightarrow\forall$ $V\in(\eta_i,\eta_j)$-pgrw-LC**(Z) $(gof)^{-1}(V)\in(\tau_i,\tau_j)$-pgrw-LC**(X).

$\Rightarrow$ gof is pairwise pgrw-lc**-irresolute.

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