Cost Analysis of Two Identical Cold Standby System under the Influence of Snow Storm Causing digging out and Hospitalization

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Abstract

In this paper we want to study effect of snow storm as abnormal weather on two unit similar cold standby system. In this system after snow storm rescue operation starts on failed unit during rescue operation first digging out start from the snow and then hospitalization of the system (human) starts as a repair. There is a single repairman with the system has different rates. The failure rate due to snow storm follow exponential distributions and different repair rates follow different rates follow different general time distributions. The system is analyzed by making use of semi-Markov process, regenerative point technique and the following measures of system effectiveness such as mean time to system failure, study state availability, busy period of the repairman, expected number of visit periods by repairman are obtained. Profit is also evaluated using the above measures. In the end, numerical result, various graphs have been plotted for a particular case and interesting explanations have been made.

Keywords**:** *Cold stand by system, digging out, hospitalization.*

1. INTRODUCTION

A large number of papers have been analyzed by various researchers in the field of reliability to improve the reliability of the systems. Osaki [1] and Taneja etal. [2] and Tuteja and taneja [3] studied reliability models of such systems with different failure rate and different repair facilities. R. Subramanian & V. Anantharaman [4] reliability analysis of a complex standby redudent system. Singh and Taneja [5] and Malhotra and Taneja [9] investigated comparative study of the systems. Taj and Taneja et al.[10] analyzed reliability and modeling of a single machine subsystem of a cable plant. In the abnormal weather condition standby systems are affected very much. Environmental conditions cannot be control which may fluctuate due to changing climate. Therefore, Goel and Sharma[6], Gupta and Goel [7] and L.R. Goel, Ashok Kumar, A.K. Rastogi [8] have obtained reliability measures of cold standby repairable systems operating under different weather conditions.

In January 2017, snow storm hit an army camp in Gurez sector of Bandipora district near the Line of Control which trapped several soldiers. Rescue operations were launched immediately and seven soldiers, including a junior commissioned officer, were pulled out. Later, total 10 bodies were retriened by rescue team from the spot of the incident and search and rescue operations were on to find the missing soldiers. Major Amit Sagar of the High Altitude Warfare School died when an Army camp of 115 Battalion was hit by a snow storm at Sonmarg in central Kashmir's Ganderbal district. The snow storm struck two shelters occupied by two officers and four soldiers. While one officer and four soldiers were rescued and sent to hospital for treatment, Major Sagar succumbed.

While considering above facts and practical situations in mind, here reliability measures of a system of two identical units operating under ice storm obtained using semi-Markov process and regenerative point technique. In such situation we can improve the reliability of the system if the operative unit become failed due to snow storm rescue operation start on the failed unit first digging out the army man (failed unit) who under heavy snow and then hospitalization of the system (human) starts immediately by specialist doctor team as a repair.

This paper is organized as follows:

Briefly mentioned all sections and subsections.

Model and transition probabilities and mean sojourn times have been developed and they are given below:

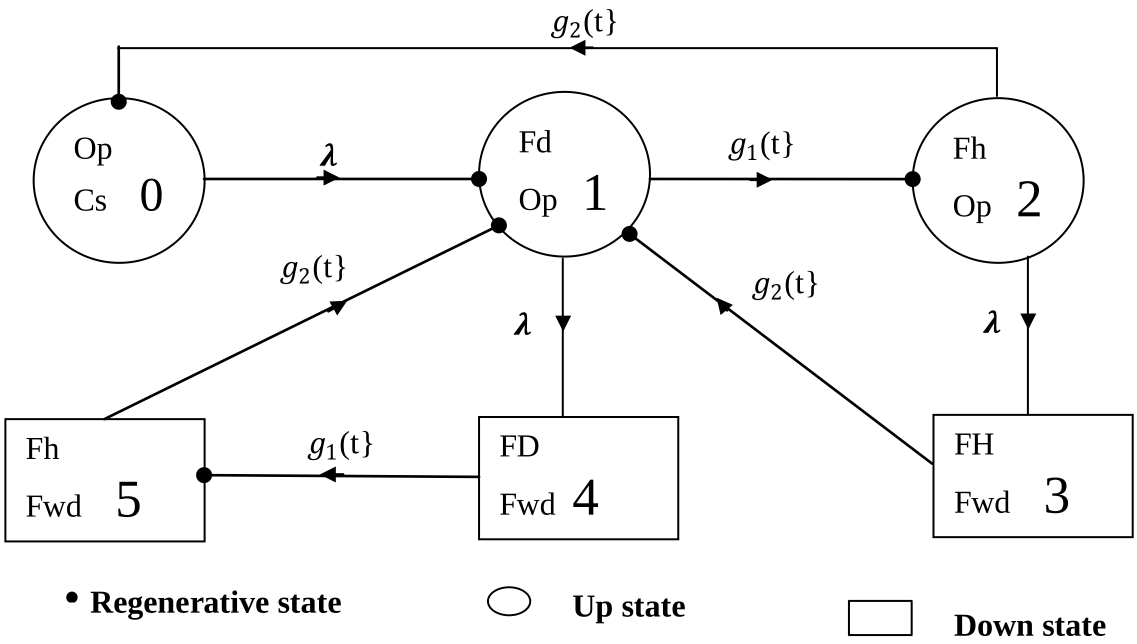
- Mean Times to system failure
- Study state availability analysis
- Expected busy period analysis of the repairman under digging out from the snow
- Expected busy period analysis of the repairman under hospitalization
- profit analysis
- particular cases

The assumptions for the model are given below:

- both units are identical
- cold stand by system
- the unit of the system fail due to snow strom
- first the failed unit goes to rescue operation first digging out from the snow after then hospitalization for repair
- complete the hospitalization the unit becomes operative
- Only one repairman with the system

2. NOTATIONS

λ	:	Failure rate of operative unit due to snow strom
⬭	:	up state
□	:	failed state
$G_1(t)$, $G_2(t)$, $g_1(t)$ $g_2(t)$	:	C.d.f. and P.d.f. of the repair rate of digging out from the snow and hospitalization of failed unit respectively.
Op	:	operative unit
cs	:	cold standby
fd	:	failed unit is digging out from the snow
FD	:	failed unit is under digging out from the snow continuing on the unit
Fh	:	failed unit is under hospitalization after digging out from the snow on the unit
FH	:	failed unit is under hospitalization continuing after digging out from the snow on the unit
Fwd	:	waiting for digging out from the snow



State Transition Diagram Fig.1.

3. TRANSITION PROBABILITIES & MEAN SOJOURN TIME

$dQ_{01}(t) = \lambda e^{-\lambda t}\, dt$

$dQ_{12}(t) = e^{-\lambda t} g_1(t)dt$

$dQ_{14}(t) = \lambda e^{-\lambda t}\, \overline{G}_1(t)\, dt$

$dQ_{15}^{(4)}(t) = (\lambda e^{-\lambda t} © 1) g_1(t)dt$

$dQ_{20}(t) = e^{-\lambda t} g_2(t)dt$

$dQ_{23}(t) = \lambda e^{-\lambda t}\, \overline{G}_2(t)\, dt$

$dQ_{21}^{(3)}(t) = (\lambda e^{-\lambda t} © 1) g_2(t)dt$

$dQ_{12}(t) = g_2(t)dt$

Taking Laplace Stieltjes Transformation we get

$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \lim_{s\to 0} Q_{ij}^{**}(s)$

$p_{01} = 1$,

$p_{12} = g_1^*(\lambda)$, $p_{14} = (1 - g_1^*(\lambda))$, $p_{15}^4 = (1 - g_1^*(\lambda))$

$p_{20} = g_2^*(\lambda)$, $p_{23} = (1 - g_2^*(\lambda))$, $p_{21}^3 = (1 - g_2^*(\lambda))$

$p_{51} = g_2^*(0)$

By these transition probabilities, it can be verified that

$p_{01}= 1$

$p_{12} + p_{15}^{(4)}=p_{12} + p_{14} = 1$

$p_{20} + p_{21}^{(3)}=p_{20} + p_{21} = 1$

$p_{51}= 1$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as the same time of stay in that state before transition to any other state. If T denote the sojourn in the regenerative state 'i', then

μ_i= E(T)= Pr(T>t)

$\mu_0= \frac{1}{\lambda}$

$\mu_1= \frac{1}{\lambda}\{1- g_1^*(\lambda)\}$

$\mu_2= \frac{1}{\lambda}\{1- g_2^*(\lambda)\}$

The unconditional mean time

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it counted from epoch of entrance into state 'i' is mathematical stated as : -

$m_{ij} = \int_0^\infty t\, q_{ij}(t)dt = -q_{ij}^{*'}(0)$

$m_{01} = \mu_0$

$m_{12} + m_{15}^3 = -g_1^{*'}(0) = k_1(\text{say}),\ m_{12} + m_{14} = \mu_1$

$m_{20} + m_{21}^3 = -g_2^{*'}(0) = k_2(say),\ m_{20} + m_{23} = \mu_2$

$m_{51} = k_2$

4. ANALYSIS OF MEAN TIME TO SYSTEM FAILURE

Regarding the failed states as absorbing states and applying the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ are obtained

$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t)$

$\phi_1(t) = Q_{14}(t) + Q_{12}(t)Ⓢ\phi_2(t)$

$\phi_2(t) = Q_{23}(t) + Q_{20}(t)Ⓢ\phi_0(t)$

Taking Laplace- Stieltjes Transforms (L.S.T) of these relations and solving them by Cramer's rule for $\phi_0^{**}(s) = \frac{N(S)}{D(S)}$

Where , N(S) = $Q_{01}^{**}(s)(Q_{14}^{**}(s)+ Q_{23}^{**}(s)\, Q_{12}^{**}(s))$

D(S)=1-$Q_{01}^{**}(s)\, Q_{20}^{**}(s)\, Q_{12}^{**}(s)$

Now, the mean time to system failure (MTSF) when the system starts from the state '0' is

$$T_0 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \lim_{s\to 0} \frac{1-\frac{N(S)}{D(S)}}{s} = \lim_{s\to 0} \frac{D(s)-N(s)}{sD(s)} = \frac{D'(0)-N'(0)}{D(0)} = \frac{N}{D}$$

Where N $=\mu_0 + p_{12}\mu_2 + \mu_1$

And

D=1-$p_{12}p_{20}$

5. AVAILABILITY ANALYSIS

The availability of a system is defined as the probability that the system is operating and provides service when requested. Using the probabilistic argument and A_i(t) as the probability of unit entering into up state at time t, given that the unit entered is regenerative state i at t=0, the following recursive relation are obtained.

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$$

$$A_1(t) = M_1(t) + q_{12}(t) © A_2(t) + q_{15}^{(4)}(t)) © A_5(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{21}^{(3)}(t)) © A_1(t)$$

$$A_5(t) = q_{51}(t) © A_1(t)$$

Where M_0(t)= $e^{-\lambda t}\, dt$, M_1(t)= $e^{-\lambda t}\overline{G_1}$(t)dt and M_2(t)= $e^{-\lambda t}\overline{G_2}$(t)dt

Taking Laplace transforms(L.T.) of these relations and solving them by crammer rule for A_0^*(s), we obtain

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

Where,

$$N_1(s) = M_0^*(s)(1 - q_{15}^{*(4)}(s)\, q_{51}^*(s)) - M_0^*(s) q_{12}^*(s) q_{21}^{*(3)}(s) + M_1^*(s) q_{01}^*(s) + q_2^*(s) q_{01}^*(s) q_{12}^*(s)$$

$$D_1(s) = 1 - q_{15}^{*(4)}(s)\, q_{51}^*(s) - q_{12}^*(s) q_{21}^{*(3)}(s) - q_{01}^*(s) q_{12}^*(s) q_{20}^*(s)$$

The steady state availability of the system is given by

$$A_0 = \lim_{s\to 0}(sA_0^*(s)) = \lim_{s\to 0}\left(s\frac{N_1(s)}{D_1(s)}\right) = \frac{N_1(0)}{D_1'(0)} = \frac{N_1}{D_1}$$

$$N_1 = \mu_0 p_{12} p_{20} + \mu_1 + \mu_2 p_{12}$$

$$D_1 = k_1 + k_2 + \mu_0 p_{12} p_{20}$$

Where, k_1 and k_2 is already specified.

6. BUSY PERIOD ANALYSIS OF THE REPAIR MAN DURING DIGGING OUT

B_i^D (t) = Probability that the repair man is busy during digging out in resque operation at instant t, given that the system entered regenerative state i at t=0,

$$B_0^D(t) = q_{01}(t) © B_1^D(t)$$

$B_1^D(t) = W_1(\text{t}) + q_{12}(\text{t})$ © B_2^D (t) +$q_{15}^{(4)}$ (t))© B_5^D (t)

$B_2^D(t) = q_{20}(\text{t})$ © B_0^D (t) +$q_{21}^{(3)}$ (t))© B_1^D (t)

$B_5^D(t) = q_{51}(\text{t})$ © B_1^D (t)

Where W_1(t)= $e^{-\lambda t}\overline{G_1}$(t)dt+$\lambda$ $e^{-\lambda t}\overline{G_1}$(t)dt

Taking Laplace transforms (L.T.) of these relations and solving them by applying crammer's rule for

B_0^{*D} (s), we obtain

B_0^{*D} (s)=$\frac{N_2(s)}{D_1(s)}$

Where

$N_2(s)$= q_{01}^*(s) W_1^*(s)

And D_1(s) is already specified.

In steady state, the total fraction of time for which the system is under repair is given by

$$B_0 = \lim_{s\to 0}(sB_0^{*D}s)) = \lim_{s\to 0}(s\frac{N_2(s)}{D_1(s)}) = \frac{N_2(0)}{D_1'(0)} = \frac{N_2}{D_1}$$

Where,

N_2=W$_1$

WhereW_1=W_1^*(0) and D_1 is already specified.

7. BUSY PERIOD ANALYSIS OF THE REPAIR MAN UNDER HOSPITALIZATION

B_i^H (t) = Probability that the repair man is busy under resque operation at instant t, given that the system entered regenerative state i at t=0,

$B_0^H(t) = q_{01}(\text{t})$ © B_1^H (t)

$B_1^H(t) = q_{12}(\text{t})$ © B_2^H (t) +$q_{15}^{(4)}$ (t))© B_5^H (t)

$B_2^H(t) = W_2(\text{t}) + q_{20}(\text{t})$ ©B_0^H (t) +$q_{21}^{(3)}$ (t))© B_1^H (t)

$B_5^H(t) = q_{51}(\text{t})$ © B_1^H (t)

Where W_2(t)= $e^{-\lambda t}\overline{G_2}$(t)dt+$\lambda$ $e^{-\lambda t}\overline{G_2}$(t)dt and W_5(t)= $\overline{G_2}$(t)dt

Taking Laplace transforms (L.T.) of these relations and solving them by applying crammer's rule for B_0^H B_0^{*H} (s), we obtain

B_0^{*H} (s)=$\frac{N_3(s)}{D_1(s)}$

Where

$$N_3(s)= q_{01}^{*}(s)\, q_{15}^{*(4)}(s)\, W_5^{*}(s) + q_{01}^{*}(s) q_{12}^{*}(s)\; W_2^{*}(s)$$

And $D_1(s)$ is already specified.

In steady state, the total fraction of time for which the system is under repair is given by

$$B_0^H = \lim_{s\to 0}(sB_0^{*H}(s)) = \lim_{s\to 0}(s\frac{N_3(s)}{D_1(s)}) = \frac{N_3(0)}{D_1'(0)} = \frac{N_3}{D_1}$$

Where,

$$N_3 = W_5 p_{15}^{(4)} + p_{12} W_2(t)$$

Where $W_2 = W_2^{*}(0)$, $W_5 = W_5^{*}(0)$ and D_1 is already specified.

8. EXPECTED NUMBER OF VISITS BY THE REPAIR MAN

We define

$V_0(t)$ = expected number of visits by the repair man in (0,t], given that the system started from the regenerative state i at t=0

$$V_0(t) = q_{01}(t) Ⓢ (1+ V_1(t)$$

$$V_1(t) = q_{12}(t) Ⓢ V_2(t) + q_{15}^{(4)}(t)) Ⓢ V_5(t)$$

$$V_2(t) = q_{20}(t) Ⓢ V_0(t) + q_{21}^{(3)}(t)) Ⓢ V_1(t)$$

$$V_5(t) = q_{51}(t) Ⓢ V_1(t)$$

Taking Laplace – Stieltjes Transforms (L.S.T.) of these relations and solving them by applying crammer rule for $V_0^{**}(s)$, we obtain

$$V_0^{**}(s) = \frac{N_3(s)}{D_1(s)}$$

Where

$$N_4(s) = Q_{01}^{**}(s) - Q_{01}^{**}(s)\, Q_{15}^{**(4)}(s)\, Q_{51}^{**}(s) - Q_{01}^{**}(s) Q_{12}^{**}(s)\; Q_{21}^{**(3)}(s)$$

And $D_1(s)$ is already specified.

$$V_0 = \lim_{s\to 0}(sV_0^{*}(s)) = \lim_{s\to 0}(s\frac{N_4(s)}{D_1(s)}) = \frac{N_4(0)}{D_1'(0)} = \frac{N_4}{D_1}$$

Where,

$$N_4 = p_{10}^{(8)} + p_{40}^{(5)} + p_{40}^{(5)} p_{10}^{(8)}$$

And D_1 is already specified.

9. COST- BENEFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P=C_0A_0 - C_{11}B_0^D - C_{12}B_0^H - C_2V_0$$

Where

C_0= revenue per unit up time of the system

C_{11}= cost per unit time for which repairman is busy under resque operation

C_{12}= cost per unit time for which repairman is busy under hospitibility

C_2= cost per visit of the repairman.

10. PARTICULAR CASES

Numerical result for the particular cases the following case is considered. Let us assume that the repair rate are exponentially distributed as under :

$g_1(t)= \alpha_1 e^{-\alpha_1 t}$ and $g_2(t)= \alpha_2 e^{-\alpha_2 t}$

$p_{01} = 1$,$p_{12}=\frac{\alpha_1}{\lambda+\alpha_1}$, $p_{14} = \alpha_1/(\lambda+\alpha_1)$, $p_{15}^{(4)} =\frac{\lambda}{\lambda+\alpha_1}$

$p_{20} = \alpha_2/(\lambda+\alpha_2)$, $p_{23} = \lambda/(\lambda+\alpha_2)$, $p_{21}^{(3)} =\frac{\lambda}{\lambda+\alpha_2}$, $p_{51} = 1$

$\mu_0=1/\boldsymbol{\lambda}$, $\mu_1=1/(\lambda+\alpha_1)$, $\mu_2=\frac{1}{\lambda+\alpha_2}$, $k_1=1/\alpha_1$, $k_2=1/\alpha_2$, $\mu_5=1/\alpha_2$

11. GRAPHICAL INTERPRETATION

By giving some numerical values to the parameters involved , various graphs has been plotted using particular case and the following interpretations have been drawn.

MTSF Versus Failure Rate ($\boldsymbol{\lambda}$) for Different Value of Repair Rate ($\boldsymbol{\alpha_1}$)

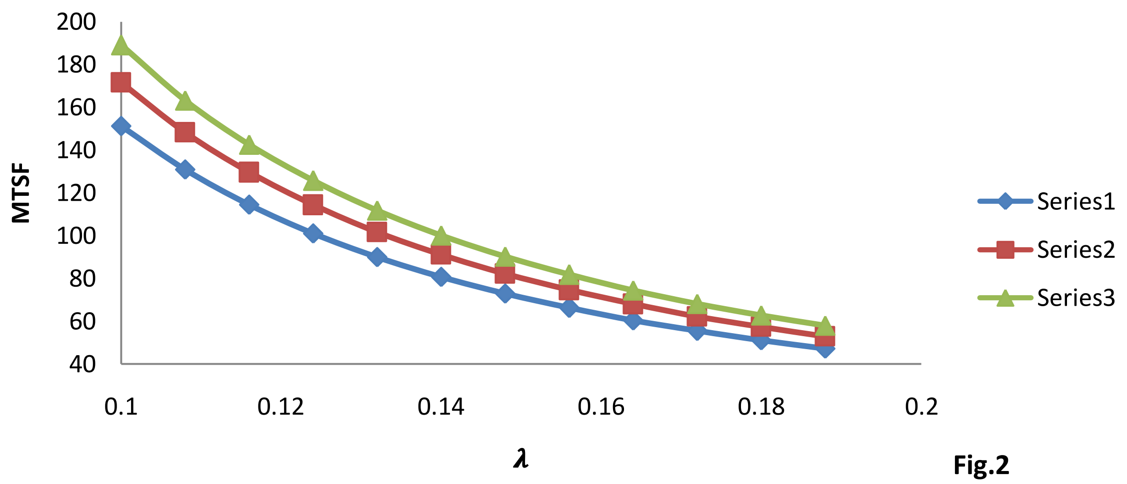


Fig.2

Fig. 2 : Shows the behaviour operative unit for different values of repair rate (α_1).From the graph, we can see of MTSF with respect to failure rate λ of the that the MTSF decreases as λ increases, but has higher values for higher values of α_1.

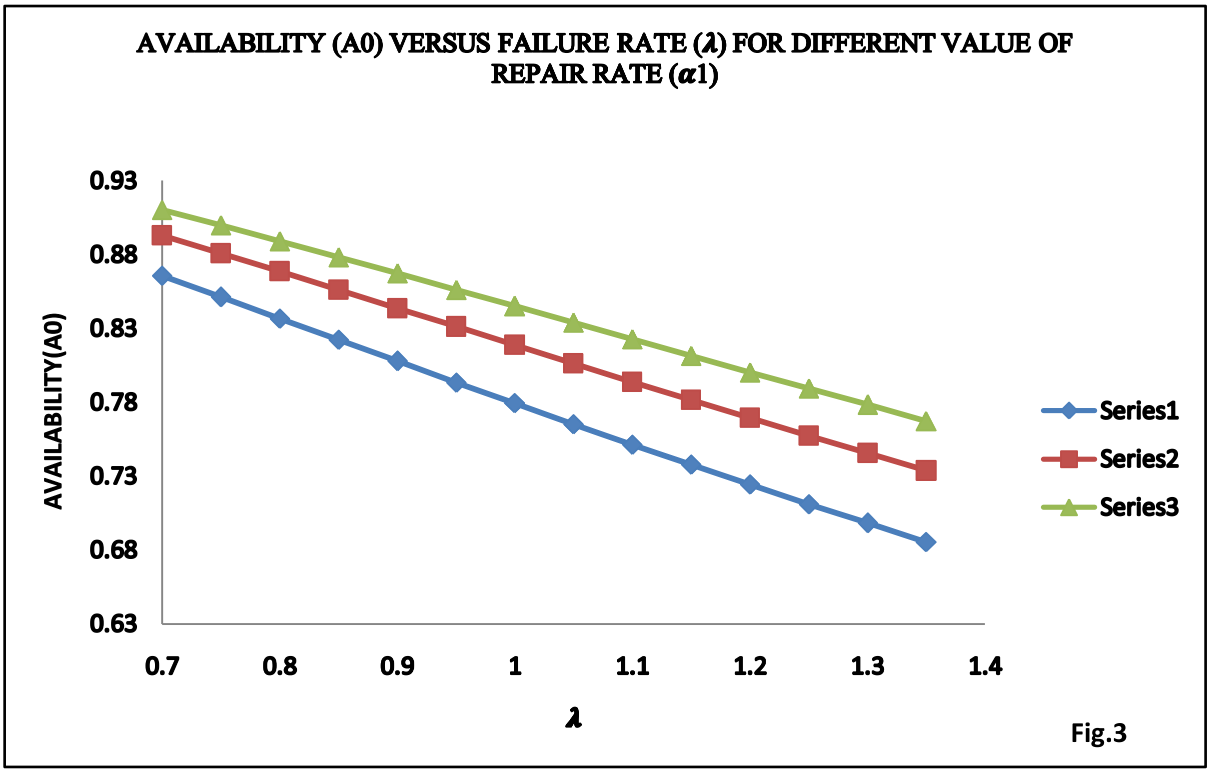


Fig. 3 : Shows the behaviour of availability (A_0) with respect to failure rate λ of the operative unit for different values of repair rate (α_1).From the graph, we can see that the A_0 decreases as λ increases, but has higher values for higher values of α_1.

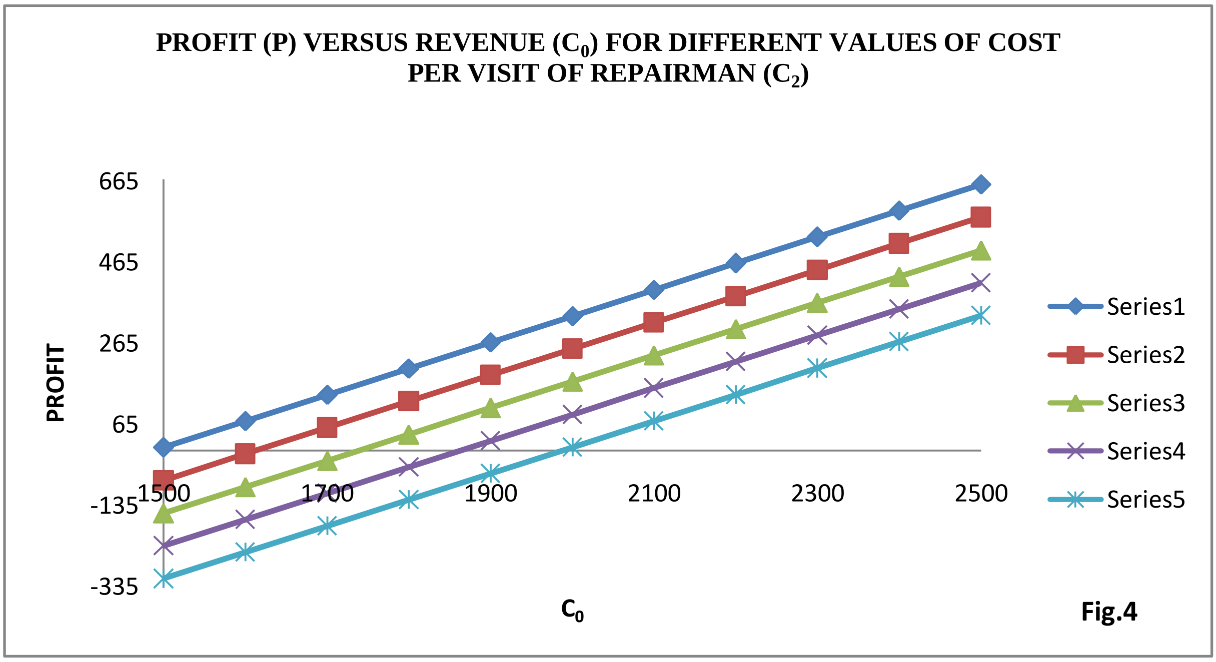


Fig.4 : Shows the behavior of profit (P) with respect to cost per unit visit up revenue (C_0) for different values of cost per visit (C_2) of the repairman. It reveals that profit (P) increase with increase in the values of C_0,but it gets lowered for higher of C_2.

Following conclusions can also be drawn from the graph.

(i) For series 1, C_2=200,the profit (P) is always $\geq$ 7.587059.

(ii) For series 2, C_2=400,the profit (P) is > or= or< 0 according as C_0 is > or= or< 1612.987.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 1612.987.

(iii) For series3, C_2=600,the profit (P) is > or= or< 0 according as C_0 is > or= or< 1737.662.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 1737.662.

(iv) For series 4, C_2=800, the profit (P) is > or= or< 0 according as C_0 is > or= or< 1862.337.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 1862.337.

(v) FOR series 5, C_2=1000, the profit (P) is > or= or< 0 according as C_0 is > or= or< 1987.013.Thus, the price of profit should be fixed in such a way so that the revenue is atleast 1987.013.

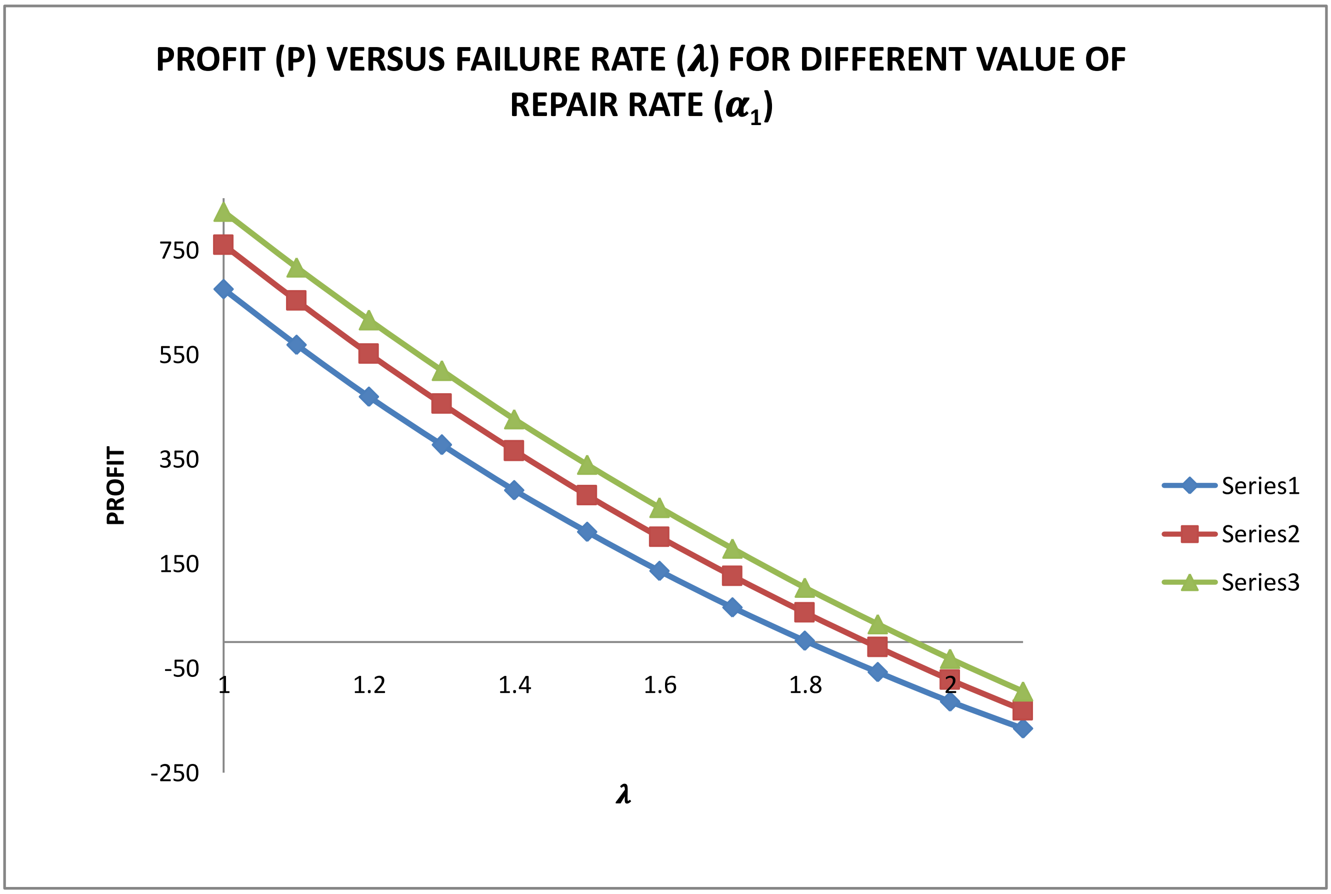


Fig.5 shows the behaviour of profit (P_2) with respect to failure rate (□) of the operative unit for different values of repair rate (□$_□$).From the graph, we can see that the P decreases as □ increases, but has higher values of or higher values of □$_□$□

Following conclusions are drawn from Fig.5.

(i) If for series 1, □$_□$=2,then profit (P) is > or =or < 0 according as failure rate (□) *is* < or = or > 1.80378.Therefore, the quality of the unit should be such that it has failure rate (□) less than 1.80378, otherwise the system will give negative profit.

(ii) If for series 2, $\square_{\square}$=2.5,then profit (P) is > or =or < 0 according as failure rate (□) is < or = or > 1.88455.Therefore, the quality of the unit should be such that it has failure rate (□) less than 1.88455, otherwise the system will give negative profit.

(iii) If for series 3,$\square_{\square}$=3,then profit (P) is > or =or < 0 according as failure rate (□) is < or = or > 1.9518.Therefore, the quality of the unit should be such that it has failure rate (□) less than 1.9518, otherwise the system will give negative profit.

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