Regular Mildly Generalized Continuous and Irresolute Functions in Topological Spaces

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Abstract

The aim of this paper is introduce and investigate new class of mappings called Regular Mildly Generalized Continuous (briefly RMG- Continuous) maps. Also introduced Regular Mildly Generalized Irresolute (briefly RMG-Irresolute) mappings which are stronger then RMG-continuous mappings are studied and the relationships between these mappings are investigated. Several properties of these new notations have been discussed and the connections between them are studied.

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1. Introduction

The concept of continuity is connected with the concept of topology. Some researchers studied weaker and stronger forms of continuous functions in topology using the sets stronger and weaker than the open and closed sets. In 1963 N. Levine [6] introduced semi-continuous functions using semi-open sets. Balachandran et. al[2], Mashour et. al. [4], N.Palaniappan [12] et. al. Nagaveni [8], M.Sheik John[14] and J. K. Park et. al. [13] have introduced g-continuity, pre-continuity, rg-continuity, wg- continuity, w-continuity, mildly-g-continuity respectively. In this paper, we introduce new class of continuous functions called RMG-continuous functions and RMG-irresolute functions in topological spaces and also we discuss their properties and characteristics.

2. Preliminaries

Throughout this paper,(X, τ), (Y, σ) and (Z, η)or simply X, Y and Z will always denote topological spaces on which no separation axioms are assumed unless explicitly stated. Int(A), Cl(A), RMG-cl(A), and RMG-int(A) denote the interior of A, closure of A, RMG-closure of A and RMG-interior of A respectively. $X-A$ or A^c denotes the complement of A in X. We recall the following definitions and results.

Definition2.1 A subset A of a topological space X is called

i) Regular open [15], if $A = \mathrm{int}(\mathrm{cl}(A))$and regular closed if$\mathrm{cl}(\mathrm{int}(A)) = A$.
ii) Pre-open [4], if $A \subseteq \mathrm{int}(\mathrm{cl}(A))$ and pre-closed if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$.
iii) Semi open [6], if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$ and semi-closed if $\mathrm{int}(\mathrm{cl}(A) \subseteq A$.
iv) α-open [9], if $A\subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$ and α-closed if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)) \subseteq A$.
v) Semi pre open [1], if $A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$ and semi pre closed if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq A$.
vi) π-open [3], if A is a finite union of regular open sets. The complement of π-open set is called the π-closed set.
Vii) A subset A of X is called δ-closed [16] if $A = \mathrm{cl}\delta(A)$, where $\mathrm{cl}\delta(A) = \{x \in X : \mathrm{int}(\mathrm{cl}(U)) \cap A \neq \emptyset, U \in A\}$.

Definition 2.2: A subset of a topological space (X, τ) is called

1. Generalized closed (briefly g-closed) [7] if $\mathrm{cl}(A) \subseteq U$ whenever $A\subseteq U$ and U is open in X.
2. Generalized α-closed (briefly gα-closed) [5] if α-cl(A) ⊆U whenever A⊆U and U is α-open in X.
3. Weakly generalized closed (briefly wg-closed) [8] if $\mathrm{cl}(\mathrm{int}(A)) \subseteq U$ whenever $A\subseteq U$ and U is open in X.

4. Strongly generalized closed (briefly g*-closed) [14] if cl(A) ⊆U whenever A⊆U and U is g-open in X.
5. Weakly closed (briefly w-closed) [14] if cl(A) ⊆U whenever A⊆U and U is semi-open in X.
6. Mildly generalized closed (briefly mildly g-closed) [13] if cl(int(A)) ⊆U whenever A⊆U and U is g-open in X.
7. Regular weakly generalized closed (briefly rwg-closed) [8] if cl(int(A)) ⊆U whenever A⊆U and U is regular open in X.
8. Weakly π-generalized closed (briefly wπg -closed) [11] if cl(int(A)) ⊆U whenever A⊆U and U is π-open in X.
9. Regular weakly closed (briefly rw-closed) [17] if cl(A)⊆U whenever A⊆U and U is regular semiopen in X.
10. A subset A of a space (X, τ) is called regular generalized closed (briefly rg-closed) [12] if cl(A)⊆U whenever A⊆U and U is regular open set in X.
12. π-generalized closed (briefly πg-closed)[3] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
11. Regular Mildly Generalized closed (briefly RMG-closed)[18] if cl(int(A)) ⊆ U whenever A ⊆ U and U is rg-open in X.

The complements of the above mentioned closed sets are their respective open sets.

Definition2.3: A function f: $(X, \tau)\rightarrow(Y, \sigma)$ is called

i) Completely continuous[17] if $f^{-1}(V)$ is regular closed set of (X, τ) for every closed set V of (Y, σ).
ii) g-continuous [2] if $f^{-1}(V)$ is g-closed set of (X, τ) for every closed set V of (Y, σ).
iii) g^*-continuous [14] if $f^{-1}(V)$ is g^* closed set of (X, τ) for every closed set V of (Y, σ).
iv) semi-continuous [6] if $f^{-1}(V)$ is semi-closed set of (X, τ) for every closed set V of (Y, σ).
v) $g\alpha$-continuous [14] if $f^{-1}(V)$ is $g\alpha$-closed set of (X, τ) for every closed set V of (Y, σ).
vi) semipre-continuous [17] if $f^{-1}(V)$ is semipre-closed set of (X, τ) for every closed set V of (Y, σ).
vii) pre- continuous [4] if $f^{-1}(V)$ is pre-closed set of (X, τ) for every closed set V of (Y, σ).
viii) w-continuous [14] if $f^{-1}(V)$ is w-closed set of (X, τ) for every closed set V of (Y, σ).
ix) rg-continuous [12]if $f^{-1}(V)$ is rg-closed set of (X, τ) for every closed set V of (Y, σ).
x) rw-continuous [17] if $f^{-1}(V)$ is rw-closed set of (X, τ) for every closed set V of (Y, σ).
xi) mildly-g-continuous(or wg^*)[13] if $f^{-1}(V)$ is mildly-g- closed set of (X, τ) for every closed set V of (Y, σ).
xii) wg-continuous [8] if $f^{-1}(V)$ is wg-closed set of (X, τ) for every closed set V of (Y, σ).
xiii) rwg- continuous [8] if $f^{-1}(V)$ is rwg-closed set of (X, τ) for every closed set V of (Y, σ).
xiv) $w\pi g$- continuous [11] if $f^{-1}(V)$ is $w\pi g$-closed set of (X, τ) for every closed set V of (Y, σ).

Definition 2.4: A function f: $(X, \tau)\rightarrow(Y, \sigma)$ is called

i) irresolute [17] $f^{-1}(V)$ is semi- open set of (X, τ) for each semi-open set V of (Y, σ).
ii) w-irresolute [14] $f^{-1}(V)$ is w- closed set of (X, τ) for each w-closed set V of (Y, σ).
iii) pre-irresolute [4] $f^{-1}(V)$ is pre- closed set of (X, τ) for each pre-closed set V of (Y, σ).
iv) $g\alpha$- irresolute [14] $f^{-1}(V)$ is $g\alpha$- closed set of (X, τ) for each $g\alpha$-closed set V of (Y, σ).
v) gc- irresolute [14] $f^{-1}(V)$ is g- closed set of (X, τ) for each g-closed set V of (Y, σ).
vi) rg- irresolute [12] $f^{-1}(V)$ is rg- closed set of (X, τ) for each rg-closed set V of (Y, σ).
vii) rw- irresolute [17] $f^{-1}(V)$ is rw- closed set of (X, τ) for each rw-closed set V of (Y, σ).
viii) wg- irresolute [8] $f^{-1}(V)$ is wg- closed set of (X, τ) for each wg-closed set V of (Y, σ).
ix) rwg- irresolute [8] $f^{-1}(V)$ is rwg- closed set of (X, τ) for each rwg-closed set V of (Y, σ).

Definition 2.5: A function f: $(X, \tau)\rightarrow(Y, \sigma)$ is called

i) strongly continuous[17] if $f^{-1}(V)$ is both open and closed set in (X, τ) for each set V of (Y, σ).
ii) strongly-w-continuous[17] if $f^{-1}(V)$ is open in (X, τ) for every w-open set V in (Y, σ).
iii) perfectly continuous[14] if $f^{-1}(V)$ is clopen in (X, τ) for every RMG-open set V in (Y, σ).

Lemma 2.6:i) Every closed set is RMG-closed.[18]
ii) Every pre-closed (respectively w-closed, $g\alpha$-closed) set is RMG-closed set in X.[18]
iii) Every RMG-closed is Mildly-g-closed set (respectively wg-closed, $w\pi g$-closed, rwg-closed) sets in X.[18]

Lemma 2.7: [19]If A and B are subsets of a space X. Then

i) RMG-cl(X) = X and RMG-cl(∅) = ∅.
ii) A ⊂RMG-cl(A).

iii) If $A \subset B$ then RMG-cl(A) $\subset$ RMG-cl(B).

Remark 2.8: i) RMG-closure of a set A is not always RMG-closed set.[19]

ii)If $A \subset X$ is RMG-closed, then RMG-cl(A) = A.[19]

Theorem 2.9:[19] Let A be a subset of X and x∈X. Then x∈RMG-cl(A) if and only if V∩A≠ ∅ for every RMG-open set V containing x.

Lemma 2.10:[19] A is RMG-open iff U⊂ int(cl(A)), whenever U is RMG-closed and U⊂A.

3. Regular Mildly Generalized Continuous Functions

Definition 3.1: A function $f: (X, \tau) \to (Y, \sigma)$ is said to be regular mildly generalized continuous (briefly RMG-Continuous) if the inverse image of every closed set in Y is RMG-closed set in X.

Example 3.2: Let X=Y={a, b, c, d}with the topology τ ={X, ∅, {a}, {b}, {a, b}, {a, b, c}} and σ ={Y, ∅, {a}, {b, c}, {a, b, c}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =c, f(b) =c, f(c) =d, f(d) =c. Thus f is RMG- continuous but not continuous, as inverse image of closed set {d} in Y is {c} which is not closed set in X. Then inverse image of closed set in Y is RMG-closed set in X.

Theorem 3.3: If A function $f: (X, \tau) \to (Y, \sigma)$ is pre-continuous, then it is RMG-continuous but not conversely.

Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be a pre-continuous map. Let F be a any closed set in Y. Then the inverse image $f^{-1}(F)$ is pre-closed set in X. By the lemma 2.6(ii), every pre-closed set is RMG-closed set in X. $f^{-1}(F)$ is RMG-closed set in X. Therefore f is RMG-continuous.

The converse of the above Theorem need not be true as seen from the following example.

Example 3.4: Let X={a, b, c, d}with the topology τ ={X, ∅, {a}, {b, c},{a, b, c}} and Y={x, y, z} σ ={Y, ∅, {x}, {x, z}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =x, f(b) =y, f(c) =x, f(d) =z. Then f is RMG- continuous but not pre-continuous, as inverse image of closed set {y} in Y is {b} which is not closed set in X.

Remark 3.5: From Mashhour et. al. [4], we know that every continuous map is pre-continuous map but not conversely. Also from Theorem 3.3, every pre-continuous map is RMG-continuous map but not conversely. Hence every continuous map is a RMG-continuous map but not conversely.

Example 3.6: Let X={a, b, c, d}with the topology τ ={X, ∅, {a}, {b, c},{a, b, c}} and Y={1, 2, 3} σ ={Y, ∅,{1, 3}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =2, f(b) =3, f(c) =1, f(d) =2. Then f is RMG- continuous but not continuous, as inverse image of closed set {3} in Y is {b} which is not closed set in X.

Remark 3.7: From Sheik John[14], we know that every w-continuous map is pre-continuous map but not conversely. Also from Theorem 3.3, every pre-continuous map is RMG-continuous map but not conversely. Hence every w-continuous map is a RMG-continuous map but not conversely.

Example 3.8: Let X={a, b, c, d}with the topology τ ={X, ∅, {a}, {b, c},{a, b, c}} and Y={x, y, z} σ ={Y, ∅, {x}, {x, y}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =z, f(b) =x, f(c) =y, f(d) =z. Then f is RMG- continuous but not w-continuous, as inverse image of closed set {z} in Y is {c} which is not w-closed set in X.

Remark 3.9: From Sheik John[14], we know that every gα-continuous map is pre-continuous map but not conversely. Also from Theorem 3.3, every pre-continuous map is RMG-continuous map but not conversely. Hence every gα-continuous map is a RMG-continuous map but not conversely.

Example 3.10: Let X={a, b, c, d}with the topology τ ={X, ∅, {a}, {b, c},{a, b, c}} and Y={p, q σ ={Y, ∅, {p}}. Let $f: (X, \tau) \to (Y, \sigma)$ be defined by f(a) =p, f(b) =q, f(c) =p, f(d) =p. Then f is RMG- continuous but not pre-continuous, as inverse image of closed set {q} in Y is {a, b, d} which is not gα-closed set in X.

Corollary 3.11: i) If a function $f : (X, \tau) \to (Y, \sigma)$ is δ-continuous, then it is RMG-continuous.

ii) If a function $f : (X, \tau) \to (Y, \sigma)$ is π-continuous, then it is RMG-continuous.

Proof: i) Let F be closed subset of Y. Since f is δ-continuous, $f^{-1}(F)$ is a δ-closed in X. From theorem 3.6.4[18], $f^{-1}(F)$ RMG-closed. Therefore f is RMG-continuous.

ii) Let F be closed subset of Y. Since f is π-continuous, $f^{-1}(F)$ is a π-closed in X. From Corollary3.6.5 of [18], $f^{-1}(F)$ RMG-closed. Therefore f is RMG-continuous.

Remark 3.12: The converse of Corollary 3.11 need not be true as shown in the following example. Let X= {a, b, c, d} with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and Y= {1, 2} with topology $\sigma = \{Y, \emptyset, \{1\}, \{2\}\}$. Let function

$f:(X,\tau)\rightarrow(Y,\sigma)$ be defined by $f(a)=2, f(b)=2, f(c)=1$ and $f(d)=2$. Now $f^{-1}(\emptyset)=\emptyset$,$f^{-1}(Y)=X, f^{-1}(\{1\})=\{c\}$ and $f^{-1}(\{2\})=\{a,b,d\}$ are RMG-closed sets in X. Hence, f is RMG-continuous function. However, since

i) {c} is not δ-closed set in X i.e. f is not δ-continuous on X.

ii) {c} is not π-closed set in X i.e. f is not π-contnuous on X.

Theorem 3.13: If a function f: (X, τ)→(Y, σ) is RMG-continuous, then it is mildly-g-continuous but not conversely.

Proof: Let f: (X, τ)→(Y, σ) be a RMG-continuous map. Let f be a any closed set in Y. Since F is RMG-continuous. $f^{-1}(F)$ is RMG-closed set in X. By the lemma 2.6(ii), every RMG-closed set is mildly-g-closed set in X. $f^{-1}(F)$ is mildly-g-closed set in X. Therefore f is mildly-g-continuous.

The converse of the above Theorem need not be true as seen from the following example.

Example 3.14: Let X={a, b, c, d}with the topology τ ={X, $\emptyset$, {a}, {b}, {a, b}, {a, b, c}} and Y={p. q} σ ={Y, $\emptyset$, {p}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =q, f(b) =q, f(c) =p, f(d) =q. Then f is mildly-g- continuous but not RMG-continuous, as inverse image of closed set {q} in Y is {a, b, d} in X, which is not a RMG-closed set in X.

Theorem 3.15: If a function f: (X, τ)→(Y, σ) is RMG-continuous, then it is wg-continuous but not conversely

Proof: Follows from the fact that every RMG-closed set is wg-closed(Lemma 2.6(iii)).

Example 3.16: Let X={a, b, c, d}with the topology τ ={X, $\emptyset$, {a}, {b}, {a, b}, {a, b, c}} and Y={1, 2, 3} σ ={Y, $\emptyset$, {1, 2}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =3, f(b) =3, f(c) =2, f(d) =3. Then f is wg- continuous but not RMG-continuous, as inverse image of closed set {3} in Y is {a, b, d}in X, which is not a RMG-closed set in X.

Theorem 3.17: If a function f: (X, τ)→(Y, σ) is RMG-continuous, then it is wπg-continuous but not conversely.

Proof: Follows from the fact that every RMG-closed set is wπg-closed(Lemma 2.6(iii)).

Example 3.18: Let X={a, b, c, d}with the topology τ ={X, $\emptyset$, {a}, {b}, {a, b}, {a, b, c}} and Y={m, n, o} σ ={Y, $\emptyset$, {m, n}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =m, f(b) =o, f(c) =m, f(d) =o. Then f is wπg-continuous but not RMG-continuous, as inverse image of closed set {o} in Y is {b, d} which is not a RMG-closed set in X.

Remark 3.19: From above Theorem 3.15, we know that every wg-continuous map is RMG-continuous but not conversely. Also From N. Nagaveni[8] every wg-continuous map is rwg-continuous map but not conversely. Hence every RMG-continuous map is a rwg-continuous map but not conversely.

Example 3.20: Let X={a, b, c, d}with the topology τ ={X, $\emptyset$, {a}, {b, c}, {a, b, c}} and Y={p, q, r} σ ={Y, $\emptyset$, {q}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =p, f(b) =p, f(c) =r, f(d) =q. Then f is rwg-continuous but not RMG-continuous, as inverse image of closed set {p, r} in Y is {a, b, c} which is not a RMG-closed set in X.

Theorem 3.21: If A function f: (X, τ)→(Y, σ) is completely-continuous, then it is RMG-continuous but not conversely.

Proof: Let f: (X, τ)→(Y, σ) be a completely-continuous map. Let F be a closed set in Y. Then $f^{-1}(F)$ is regular-closed set in X and $f^{-1}(F)$ is RMG-closed set in X. Therefore f is RMG-continuous.

Theorem 3.22: If A function f: (X, τ)→(Y, σ) is w-irresolute, then it is RMG-continuous.

Proof: Let f: (X, τ)→(Y, σ) be a w-irresolute map. Let V be an open set in Y. Then V is w-open in Y. Since f is w-irresolute, $f^{-1}(V)$ is w-open and hence $f^{-1}(V)$ is RMG-open set in X. Thus f is RMG-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.23: Let X={a, b, c, d}with the topology τ ={X, $\emptyset$, {a}, {b, c}, {a, b, c}} and Y={p, q, r} σ ={Y, $\emptyset$, {q, r}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =r, f(b) =p, f(c) =q, f(d) =r. Then f is RMG- continuous but not w-irresolute, as inverse image of w-closed set {p} in Y is {b} which is not a w-closed set in X.

Remark 3.24: The following examples shows that RMG-continuous and g-continuous, g*-continuous, πg-continuous, semi- continuous, semi-pre-continuous, rg-continuous and rw-continuous are independent.

Example 3.25: Let X={a, b, c, d} be with topology τ ={X, $\emptyset$, {a}, {b}, {a, b},{b, c}, {a, b, c}} and Y={1, 2, 3}be with topology σ ={Y, $\emptyset$, {1}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =2, f(b) =2, f(c) =3, f(d) =2. Then the inverse image of every closed set in Y is a RMG-closed set in X and hence f is RMG- continuous. Let {3} be a closed set in Y, $f^{-1}(\{3\})$ ={c} is not g-closed, g*-closed, πg-closed, rg-closed and rw-closed in X. Thus f is not g-continuous, g*-continuous, πg-continuous, rg-continuous and rw-continuous.

Example3.26: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b},{b, c}, {a, b, c}} and Y={1, 2, 3} be with topology σ ={Y, ∅, {1, 2}}. Let f :(X, τ)→(Y, σ) be defined by f(a) =2, f(b) =3, f(c) =1, f(d) =3. Then the inverse image of every closed set in Y is a g-closed, g^*-closed, g-closed, rg-closed and rw-closed in X and hence f is g-continuous, g^*-continuous, πg-continuous, rg-continuous and rw-continuous. Let {3} be a closed set in Y, f^{-1}({3})={b, d} is not RMG-closed set in X. Thus f is not RMG-continuous.

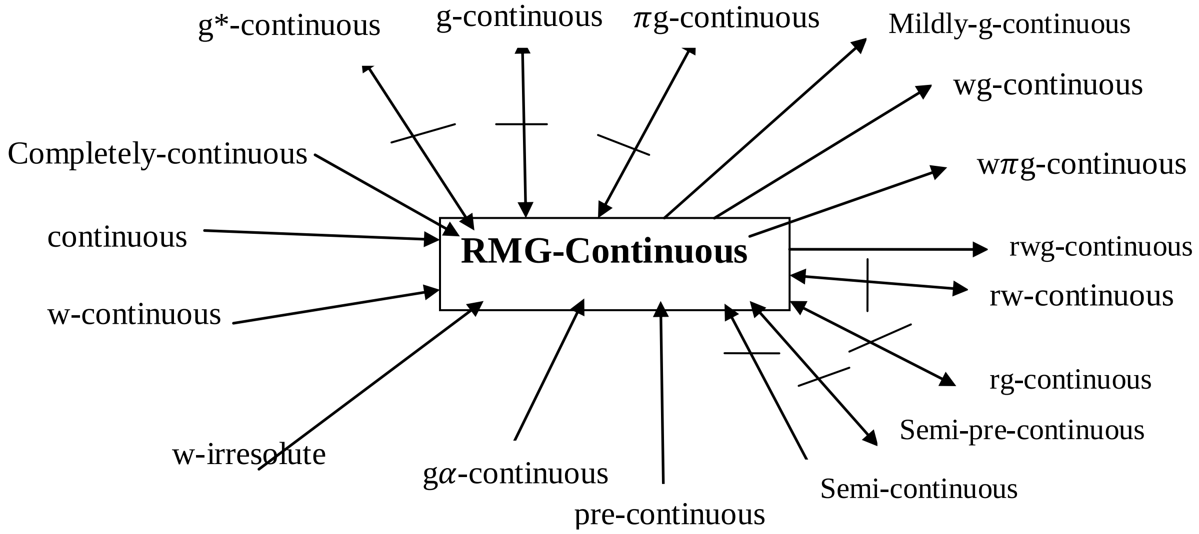
Example 3.27: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b},{b, c}, {a, b, c}} and Y={p, q} be with topology σ ={Y, ∅, {p}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =q, f(b) =q, f(c) =p, f(d) =q. Then the inverse image of every closed set in Y is a RMG-closed set in X and hence f is RMG- continuous. Let {q} be a closed set in Y, f^{-1}({q}) ={a, b, d} is not semi-closed and semi-pre-closed in X. Thus f is not semi-continuous and semi-pre-continuous.

3.2.28 Example: Let X={a, b, c, d} be with topology τ ={X, ∅, {a}, {b}, {a, b},{b, c}, {a, b, c}} and Y={p, q} be with topology σ ={Y, ∅, {q}}. Let f :(X, τ) →(Y, σ) be defined by f(a) =q, f(b) =p, f(c) =p, f(d) =q. Then the inverse image of every closed set in Y is a semi-closed and semi-pre-closed in X and hence f is semi-continuous and semi-pre-continuous. Let {p} be a closed set in Y, f^{-1}({p})={b, c} is not RMG-closed set in X. Thus f is not RMG-continuous.

Remark 3.29: From the above discussion and know results we have the following implications. In the following diagram, by

A $\longrightarrow$ B we mean A implies B but not conversely and

A $\not\longleftrightarrow$ B means A and B are independent of each other.



Remark 3.30: The composition of two RMG-continuous maps need not be RMG-continuous and this shown by the following example.

Example 3.31: Let X=Y=Z={a, b, c, d}. τ ={X, ∅, {a}, {b, c}, {a, b, c}}, σ ={Y, ∅, {a}, {b}, {a, b}, {a, b, c}} and η ={Z, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Let f: (X, τ)→(Y, σ) be defined by f(a) =d, f(b) =a, f(c) =b, f(d) =c and g: (Y, σ)→(Z, η) be identity map. Then f and g are RMG-continuous, but their composition gof : (X, τ)→(Z, η) is not RMG-continuous, because F={d} is closed in (Z, η), but $(gof)^{-1}$(F)={a} which is not RMG-closed in (X, τ)

Theorem 3.32: If f: (X, τ)→(Y, σ) is a RMG-continuous and g: (X, τ)→(Z, η) is continuous, then their composition gof : (X, τ)→(Z, η) is RMG-continuous.

Proof: Let F be any closed set in (Z, η). Since g is continuous, $(g)^{-1}$(F) is closed in (Y, σ). Since f is RMG-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is a RMG-closed set in X and gof is RMG-continuous.

Theorem 3.33: Let (X, τ), (Z, η) be any topological spaces and (Y, σ) be topological space " where every RMG-closed subset is closed". Then the composition gof : (X, τ)→(Z, η) of the RMG-continuous mps f: (X, τ)→(Y, σ) and g: (Y, σ)→(Z, η) is RMG-continuous.

Proof: Let F be any closed set of (Z, η). As g is RMG-continuous, $(\mathrm{g})^{-1}$(F) is a RMG-closed set in (Y, σ). By hypothesis, every RMG-closed set in (Y, σ) is closed, $(\mathrm{g})^{-1}$(F) is closed set in (Y, σ). Since f is RMG-continuous, $\mathrm{f}^{-1}(\mathrm{g}^{-1}(\mathrm{F}))$ is RMG-closed in (X, τ). But $\mathrm{f}^{-1}(\mathrm{g}^{-1}(\mathrm{F})) = (\mathrm{gof})^{-1}(\mathrm{F})$ and so gof is RMG-continuous.

Theorem 3.34: Let f: (X, τ)→(Y, σ) be function, then the following are equivalent.

i) f is RMG-continuous.

ii) The inverse image of each open set in Y is RMG-open in X.

iii) The inverse image of each closed set in Y is RMG-closed in X.

Proof: Suppose i) holds. Let U be an open set in Y. Then Y-U is closed set in Y. By (i) $\mathrm{f}^{-1}(\mathrm{Y}-\mathrm{U})$ is RMG closed in X. Therefore $\mathrm{f}^{-1}(\mathrm{U})$ is RMG-open in X. This proves (i)⟹(ii). The implications (ii)⟹(iii) and (iii)⟹(i) obviously.

Theorem 3.35: If a map f: (X, τ)→(Y, σ) is a RMG-continuous, Then $\mathrm{f}(\mathrm{RMG}-\mathrm{cl}(\mathrm{A})) \subset \mathrm{cl}(\mathrm{f}(\mathrm{A}))$ for every subset A of X.

Proof: Let A be subset of (X, τ). Then cl(f(A)) is closed in Y. Since f is RMG-continuous, $\mathrm{f}^{-1}(\mathrm{cl}(\mathrm{f}(\mathrm{A})))$ is RMG-closed in Y and A$\subset \mathrm{f}^{-1}(\mathrm{f}(\mathrm{A})) \subset \mathrm{f}^{-1}(\mathrm{cl}(\mathrm{f}(\mathrm{A})))$ that is $\mathrm{f}^{-1}(\mathrm{cl}(\mathrm{f}(\mathrm{A})))$ is RMG-closed subset of X containing A. Therefore RMG-cl(A)$\subset$ $\mathrm{f}^{-1}(\mathrm{cl}(\mathrm{f}(\mathrm{A})))$ this implies f(RMG-cl(A))$\subset$ $\mathrm{ff}^{-1}(\mathrm{cl}(\mathrm{f}(\mathrm{A})))$ this implies f(RMG-cl(A))$\subset$ cl(f(A)).

The converse of the above theorem 3.33 is not true in general as seen from the flowing example.

Example 3.36: Let X=Y={a, b, c, d} with the topologies τ ={X, $\emptyset$, {a}, {b},{ a, b}, {a, b, c}} and σ ={Y, $\emptyset$, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Let a map f: (X, τ)→(Y, σ) defied by f(a) =d, f(b) =a, f(c) =c, f(d) =d for every subset of X, f(RMG-cl(A))$\subset$ cl(f(A)) holds. But f is not RMG-continuous. Since closed set F={d} in Y. $\mathrm{f}^{-1}(\mathrm{F})$ ={a, d}which is not RMG-closed set in X.

Theorem 3.37: Let f: (X, τ)→(Y, σ) be a function from topological space X into a topological space Y. Then the following statements are equivalent.

i) For each point x in X and each open set V in Y with f(x)∈V, there is a RMG-open set U in X such that x∈U and f(U)⊆V.

ii) For each subset A of X, f(RMG-cl(A))⊆ cl(f(A)).

iii) For each subset B of Y, RMG-cl(f^{-1}(B))⊆ f^{-1}(cl(B)).

iv) For each subset B of Y, f^{-1}(int(B))⊆ RMG-int(f^{-1}(B)).

Proof: (i)⟹(ii). Suppose that (i) holds and let y∈f(RMG-cl(A)) and let V be any open set of Y. Since y∈f(RMG-cl(A)) implies that there exists x∈RMG-cl(A) such that f(x)=y. Since f(x)∈V, then by (i) there exists a RMG-open set U in X such that x∈U and f(U) ⊆V. Since x∈f(RMG-cl(A)) then by the Theorem2.9 U∩A≠ $\emptyset$. $\emptyset$ ≠f(U∩A) ⊆f(U)∩f(V) ⊆V∩f(A), then V∩f(A)≠ $\emptyset$. Therefore we have y=f(x)∈cl(f(A). Hence f(RMG-cl(A)) ⊆cl(f(A)).

(ii)⟹(i). Let if (ii) holds and let x∈X and V be any open set in Y containing f(x). Let A= $\mathrm{f}^{-1}(\mathrm{V}^{c})$ this implies that x∉A. Since f(RMG-cl(A)) ⊆cl(f(A)) ⊆V^{c} this implies that RMG-cl(A) ⊆ $\mathrm{f}^{-1}(\mathrm{cl}(\mathrm{V}^{c}))$ =A. Since x∉A implies that x∉RMG-cl(A) and by Theorem2.9 there exists a RMG-open set U containing x such that U∩A= $\emptyset$, then U⊆A^{c} and hence f(U) ⊆f(A^{c}) ⊆ V.

(ii)⟹(iii). Suppose that (ii) holds and Let B be any subset of Y. Replacing A by f^{-1}(B) we get from (ii) f(RMG-cl(f^{-1}(B))⊆ cl(f(f^{-1}(B)))⊆cl(B). Hence RMG-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

(iii)⟹(ii). Suppose that (iii) holds. Let B=f(A) where A is subset of X. Then we get from (iii) RMG-cl(f^{-1}(f(A))⊆ f^{-1}(cl(f(A))) implies RMG-cl(A) ⊆ f^{-1}(cl(f(A))). Therefore f(RMG-cl(A)⊆(cl(f(A))).

(iii)⟹(iv) Suppose that (iii) holds. Let B⊆Y, then Y-B⊆Y. By (iii), RMG-cl(f^{-1}(Y-B))⊆ f^{-1}(cl(Y-B)) this implies X-RMG-int(f^{-1}(B)) ⊆ X-f^{-1}(int(B)).Therefore f^{-1}(int(B)) ⊆ RMG-int(f^{-1}(B)).

(iv)⟹(iii) Suppose that (iv) holds. Let B⊆Y, then Y-B⊆Y. By (iv), f^{-1}(int(Y-B)) ⊆RMG-int(f^{-1}(Y-B)) this implies that X- f^{-1}(cl(B))⊆X-RMG-cl(f^{-1}(B)).Therefore RMG-cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

Definition 3.38: Let (X, τ) be topological space and $\tau_{RMG} = \{V \subseteq X: RMG - cl(V^c) = V^c\}$. τ_{RMG} is topology on X.

Definition 3.39: A space (X, τ) is called τ_{RMG} space if every RMG-closed is closed.

Theorem 3.40: Let f: (X, τ)→(Y, σ) be a function. Let (X, τ) and (Y, σ) be any two spaces such that τ_{RMG} is topology on X . Then the following statements are equivalent.

i)for every subset A of X $f(RMG - cl(A)) \subseteq cl(f(A))$ holds,

ii) f: (X, τ_{RMG})→(Y, σ) is continuous.

Proof: Suppose (i) holds. Let A be a closed in Y. By hypothesis $f(RMG - cl(f^{-1}(A))) \subseteq cl(f(f^{-1}(A)))$ ⊆cl(A)=A. That is RMG-cl(f^{-1} (A))⊆ f^{-1}(A). Also f^{-1}(A) ⊆RMG-cl(f^{-1} (A)). Hence RMG-cl(f^{-1} (A))⊆ f^{-1}(A). This implies f^{-1}(A)∈ τ_{RMG} . Thus f^{-1}(A) is closed in: (X, τ_{RMG}) and so f is continuous. This proves (ii).

Suppose (ii) holds. For every subset A of X, cl(f(A)) is closed in Y. Since f: (X, τ_{RMG})→(Y, σ) is continuous, f^{-1}(cl(A)) is closed in (X, τ_{RMG}) that implies by Definition 3.36 RMG-cl(f^{-1}(cl(f(A))))= f^{-1}(cl(f(A))). Now we have, A⊆ f^{-1}(cl(f(A)) and by RMG-closure, RMG-cl(A)⊆RMG-cl(f^{-1}(cl(f(A)))= f^{-1}(cl(f(A)). Therefore f(RMG-cl(A)) ⊆cl(f(A)). This prove(i).

4. Regular Mildly Generalized Irresolute Functions

Definition 4.1: A map f: (X, τ)→(Y, σ) is called regular mildly generalized irresolute (briefly RMG-irresolute) map if the inverse image of every RMG-closed set in (Y, σ) is RMG-closed in (X, τ).

Theorem 4.2: If a map f: (X, τ)→(Y, σ) is RMG-irresolute, then it is RMG-continuous.

Proof: Let f: (X, τ)→(Y, σ) be RMG-irresolute and F be a closed set in (Y, σ). Then F is RMG-closed set in (Y, σ). Since f is RMG-irresolute, f^{-1}(F) is RMG-closed in (X, τ) and so f is RMG-continuous.

The converse of the theorem 4.2 need not be true as seen from the following example.

Example 4.3: Let X=Y={a, b, c, d}, τ ={X, ∅. {a}, {b}, {a, b}, {a, b, c}} and σ = {Y, ∅, {a}, {b, c}, {a, b, c}}. Let map f: (X, τ)→(Y, σ) defined by f(a)=b, f(b)=a, f(c)=d, f(d)=c, then f is RMG-continuous but not RMG-irresolute, as inverse image of RMG-closed set F= {b} in Y, the f^{-1}(F)={a} in X, which is not RMG-closed in X

Theorem4.4: A map f: (X, τ)→(Y, σ) is RMG-irresolute if and only if the inverse image of an RMG-open set in (Y, σ) is RMG-open in (X, τ).

Proof: Let f: (X, τ)→(Y, σ) be an RMG-irresolute and U be n RMG-open set in (Y, σ). Then U^c is RMG-closed in (Y, σ). Since f is RMG-irresolute, $f^{-1}(U^c)$ is RMG-closed in (X, τ). But $f^{-1}(U^c)$= $(f^{-1}(U))^c$ and so f^{-1}(U) is RMG-open in (X, τ)

Conversely, assume that f^{-1}(U) is RMG-open in (X, τ) for each RMG-open set U in (Y, σ). Let F be a RMG-close set in (Y, σ) and by assumption $f^{-1}(F^c)$ is RMG-open in (X, τ). Since $f^{-1}(F^c)$= $(f^{-1}(F))^c$, we have f^{-1}(F) is RMG-closed in (X, τ) and so f is RMG-irresolute.

Theorem4.5: Let f: (X, τ)→(Y, σ) and g: (Y, τ)→(Z, η) be any two functions. Then

(i) $g \circ f$: (X, τ)→(Z, η) is RMG-continuous if g is r-continuous and f is RMG-continuous.

(ii) $g \circ f$: (X, τ)→(Z, η) is RMG-irresolute if g is RMG-irresolute and f is RMG-irresolute.

(iii) $g \circ f$: (X, τ)→(Z, η) is RMG-continuous if g is RMG-continuous and f is RMG-irresolute.

Proof: (i) Let U be a open set in (Z, η). Since g is r-continuous, g^{-1}(U) is r-open set in (Y, σ). Since every r-open is RMG-open then g^{-1}(U) is RMG-open in Y. Since f is RMG-irresolute $f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ). Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ) and hence gof is RMG-continuous.

(ii) Let U be a RMG-open set in (Z, η). Since g is RMG-irresolute, g^{-1}(U) is RMG-open set in (Y, σ). Since f is RMG-irresolute, $f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ). Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ) and hence g∘f is RMG-continuous.

(iii) Let U be an open set in (Z, η). Since g is continuous, g^{-1}(U) is open set in (Y, σ). Since every open is RMG-open then g^{-1}(U) is RMG-open in(Y, σ). Since f is RMG-irresolute $f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ). Thus $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ) and Hence gof is RMG-continuous.

Theorem4.6: If a map f: (X, τ)→(Y, σ) is RMG-irresolute, then for every subset A of X. f(RMG-cl(A))⊆p-cl(f(A)).

Proof: If A⊆X then consider p-cl(f(A)) which is RMG-closed in Y. Since f is RMG-irresolute, f^{-1}(p-cl(f(A)) is RMG-closed in X. Furthermore A⊆ f^{-1}(f(A))⊆ f^{-1}(p-cl(f(A))). Therefore by RMG-closure, RMG-cl(A)⊆ f^{-1}(p-cl(f(A))), consequently, f(RMG-cl(A)) ⊆p-cl(f(A)).

Theorem4.7: Let (X, τ) be any topological space, (Y, σ) be a topological space where "every RMG-closed subset is closed" and f: (X, τ)→(Y, σ) be a map. Then the following are equivalent. (i) f is RMG-irresolute (ii) f is RMG-continuous.

Proof: (i)⇒(ii) Follows from Theorem 4.2.

(ii)⟹(i) Let F be a RMG-closed set in (Y, σ). Then F is closed set in (Y, σ) by hypothesis. Since f is RMG-continuous, f^{-1}(F) is a RMG-closed set in (X, τ). Therefore f is RMG-irresolute.

5. Strongly Regular Mildly Generalized Continuous Functions

Definition 5.1: A map f: (X, τ)→(Y, σ) is called strongly regular mildly generalized continuous (briefly Strongly RMG-continuous) if the inverse image of every RMG-open set in (Y, σ) is open in (X, τ).

Theorem 5.2: If f: (X, τ)→(Y, σ) is strongly RMG-continuous then it is continuous but not conversely.

Proof: Suppose f: (X, τ)→(Y, σ) is strongly RMG-continuous. Let U be an open set in Y. As every open set is RMG-open set, U is RMG-open in Y. Since f is strongly RMG-continuous. f^{-1}(U) is an open set in X. Then f is continuous.

Example 5.3: Let X=Y= {a, b, c, d} with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = b, f(b) = a, f(c) = d$ and $f(d) = c$.Then f is continuous function but not strongly RMG-continuous. However, since $\{c\}$ is RMG-open set in Y but $f^{-1}(\{c\})$ =$\{d\}$ is not open set in X i.e. f is not RMG-continuous.

Theorem 5.4: Let f: (X, τ)→(Y, σ) is strongly continuous then it is strongly RMG-continuous.

Proof: Assume that f: (X, τ)→(Y, σ) is strongly continuous. Let U be RMG-open in Y and also it is any subset of Y. Since f is strongly continuous, f^{-1}(U) is open (and also closed) in X. f^{-1}(U) is open in X, therefore f is strongly RMG-continuous.

Example 5.5: Let X=Y= {a, b, c, d} with topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$. Define a functionf $: (X, \tau) \to (Y, \sigma)$by $f(a) = b, f(b) = a, f(c)$ =d and $f(d) = c$. Then f is strongly RMG- continuous but not strongly continuous, since $\{a\}$ is RMG-open set in Y but $f^{-1}(\{a\})$ =$\{b\}$ is open in X but not closed set in X i.e. f is not strongly continuous.

Theorem 5.6: Let f: (X, τ)→(Y, σ) is strongly RMG-continuous then it is RMG-continuous.

Proof: Let U be open set in Y, every open is RMG-open, U is RMG-open in Y. Since f is strongly RMG-continuous, f^{-1}(U) is open in X. And therefore f^{-1}(U) is RMG-open in X. Hence f is RMG-continuous.

Example 5.7: Let X={1, 2, 3, 4},Y= {a, b, c, d}with topology $\tau = \{X, \emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 3\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(1) = a, f(2) = b, f(3) = c$ and $f(4) = a$.Then f is strongly RMG-continuous but not RMG-continuous. However, since $\{a\}$ is open set in Y but $f^{-1}(\{a\})$ =$\{1, 4\}$ is not RMG-open set in X i.e. f is not RMG-continuous.

Theorem 5.8: A map f: (X, τ)→(Y, σ) is a strongly RMG-continuous if and only if the inverse image of every RMG-closed set in (Y, σ) is closed in (X, τ).

Proof: Suppose a map f: (X, τ)→(Y, σ) is strongly RMG-continuous. Let F be a RMG-closed in Y. Then F^c is a RMG-open set in Y. Since f is strongly RMG-continuous, $f^{-1}(F^c)$ is an open set in X. But $f^{-1}(F^c)$=X−f^{-1}(F) and so f^{-1}(F) is closed in X.

Conversely, assume that the inverse image of every RMG-closed set in Y is closed set in X. Let U be a RMG-open set in Y. Then U^c is a RMG-closed in Y. By hypothesis $f^{-1}(U^c)$=X−f^{-1}(U) is closed in X. That is f^{-1}(U) is an open set in X. Thus f is strongly RMG-continuous.

Theorem 5.9: In discrete space a map f: (X, τ)→(Y, σ) is strongly RMG-continuous then it is strongly continuous.

Proof: Let F be any subset of Y in discrete space. Every subset F in Y is both open and closed. Closed (i) Let F is RMG-continuous in Y. Since f is strongly RMG-continuous, then f^{-1}(F) is closed in X. (ii) Let F is RMG-open in

Y. Since f is strongly RMG-continuous, then $f^{-1}(F)$ is open in X. Therefore $f^{-1}(F)$ is closed and open in X. Hence f is strongly continuous.

Theorem 5.8: Let f: $(X, \tau)\to(Y, \sigma)$ and g:$(Y, \sigma)\to(Z, \eta)$ be any two functions. Then

(i)g∘f: $(X, \tau)\to(Z, \eta)$ is strongly RMG-continuous if g is strongly RMG-continuous and f is strongly RMG-continuous.

(ii) g∘f: $(X, \tau)\to(Z, \eta)$ is strongly RMG-continuous if g is strongly RMG-continuous and f is continuous.

(iii) g∘f: $(X, \tau)\to(Z, \eta)$ is RMG-irresolute if g is strongly RMG-continuous and f is RMG-continuous.

(iv) g∘f: $(X, \tau)\to(Z, \eta)$ is continuous if g is RMG-continuous and f is strongly RMG-continuous.

Proof: (i) Let U be a RMG-open set in (Z, η). Since g is strongly RMG-continuous, $g^{-1}(U)$ is an open set in (Y, σ). As every open set is RMG-open, $g^{-1}(U)$ is RMG-open set in (Y, σ). Since f is strongly RMG-continuous $f^{-1}(g^{-1}(U))$ is open set in (X, τ). Thus $(g \circ f)^{-1} = f^{-1}(g^{-1}(U))$ is on open in (X, τ) and hence gof is strongly RMG-continuous.

(ii) Let U be a RMG-open set in (Z, η). Since g is strongly RMG-continuous, $g^{-1}(U)$ is an open set in (Y, σ). Since f is continuous, $f^{-1}(g^{-1}(U))$ is open set in (X, τ). Thus $(g \circ f)^{-1} = f^{-1}(g^{-1}(U))$ is open set in (X, τ) and hence gof is strongly RMG-continuous.

(iii) Let U be a RMG-open set in (Z, η). Since g is strongly RMG-continuous, $g^{-1}(U)$ is an open set in (Y, σ). Since f is RMG-continuous, $f^{-1}(g^{-1}(U))$ is RMG-open set in (X, τ). Thus $(g \circ f)^{-1} = f^{-1}(g^{-1}(U))$ is an RMG-open set in (X, τ) and hence g∘f is strongly RMG-irresolute.

(iv) Let U be a open set in (Z, η). Since g is RMG-continuous, $g^{-1}(U)$ is an RMG-open set in (Y, σ). Since f is strongly RMG-continuous, $f^{-1}(g^{-1}(U))$ is open set in (X, τ). Thus $(gof)^{-1} = f^{-1}(g^{-1}(U))$ is an open set in (X, τ) and hence gof is continuous.

Theorem 5.10: Let (X, τ) be any topological space and (Y, σ) be a τ_{RMG}-space and f: $(X, \tau)\to(Y, \sigma)$ be a map. Then the following are equivalent.

(i)f is strongly RMG-continuous.

(ii) f is continuous.

Proof: (i)⇒(ii) Let U be any open set in (Y, σ). Since every open set is RMG-open, U is RMG-open in (Y, σ). Then $f^{-1}(U)$ is open in (X, τ). Hence f is continuous.

(ii)⇒(i) Let U be any RMG-open set in (Y, σ). Since (Y, σ) is a τ_{RMG}-space, U is open in (Y, σ). Since f is continuous. Then $f^{-1}(U)$ is open in (X, τ). Hence f is strongly RMG-continuous.

Theorem5.11: Let (X, τ) be a discrete topological space, (Y, σ) be a RMG-space and f: $(X, \tau)\to(Y, \sigma)$ be a map. Then the following are equivalent.

(i)f is strongly continuous.

(ii) f is strongly RMG-continuous.

Proof: (i)⇒(ii) Follows from Theorem 5.4.

(ii)⇒(i) Let U be any subset in (Y, σ). Since (Y, σ) is a RMG-space, U is a RMG-open subset in (Y, σ) and by hypothesis $f^{-1}(U)$ is open in (X, τ). But (X, τ) is discrete topological space and so $f^{-1}(U)$ is also a closed set in (X, τ). That is both open and closed in (X, τ) and so f is strongly continuous.

Theorem5.12: Let f: $(X, \tau)\to(Y, \sigma)$ be a map. Both (X, τ) and (Y, σ) are τ_{RMG}-space. Then the following are equivalent.

(i) f is RMG-irresolute.

(ii) f is strongly RMG-continuous.

(iii) f is continuous.

(iv) f is RMG-continuous.

Proof: Straight forward.

From the above discussion and know results we have the following implications. In the following diagram, by

$A \longrightarrow B$ we mean A implies B but not conversely.

$$\begin{tikzcd}
\text{Strongly continuous} \arrow[r] & \text{Strongly RMG-continuous} \arrow[r] \arrow[d] & \text{RMG-continuous} \\
& \text{Continuous function} &
\end{tikzcd}$$

6. Perfectly Regular Mildly Generalized Continuous Functions

Definition 6.1: A map f: $(X, \tau) \to (Y, \sigma)$ is called perfectly regular mildly generalized continuous (briefly perfectly RMG-continuous) if the inverse image of every RMG-open set in (Y, σ) is both open and closed in (X, τ).

Theorem 6.2: If a function $f: (X, \tau) \to (Y, \sigma)$ is perfectly RMG-continuous function iff the inverse image of every RMG-closed set in (Y, σ) is both open and closed in (X, τ).

Proof: Suppose that f is perfectly RMG-continuous. Let F be any RMG-closed set in (Y, σ). Then F^c is RMG-open in (Y, σ), since f is perfectly RMG-continuous, $f^{-1}(F^c)$ is both open closed sets in (X, τ).

Conversely, assume that the inverse image of every RMG-closed set in (Y, σ) is both open and closed set in(X, τ). Let V be any RMG-open set in (Y, σ).Then $V^c$is RMG-closed in (Y, σ). By the assumption, $f^{-1}(V^c)$ is both open and closed in(X, τ).

Theorem 6.3: If a function $f: (X, \tau) \to (Y, \sigma)$ is strongly continuous function, then it is perfectly RMG-continuous function but converse is not true.

Proof: Since $f: (X, \tau) \to (Y, \sigma)$ is strongly continuous, $f^{-1}(U)$ is both open and closed in (X, τ), for every RMG-open set U in (Y, σ). Therefore is perfectly RMG-continuous.

Example 6.4: Let X= {a, b, c, d}, Y= {m, n, o} with topologies $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{m, n\}, Y\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$by $f(a) = o, f(b) = m$ and $f(c) = n$. Then f is perfectly RMG-continuous but not strongly-continuous, since for $\{m\}$ is any subset of (Y, σ), then $f^{-1}(m) = \{b\}$ which is not clopen in (X, τ) i.e. f is not strongly-continuous.

Theorem 6.5: If function $f: (X, \tau) \to (Y, \sigma)$ is perfectly RMG-continuous function then it is strongly RMG-continuous function but converse is not true.

Proof: Assume that f is perfectly RMG-continuous function. Let V be any RMG-open set in (Y, σ). Since f is perfectly RMG-continuous then $f^{-1}(V)$ is clopen in(X, τ). Therefore, f is strongly RMG-continuous function.

Example 6.6: Let X=Y= {a, b, c, d} with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) =$c and $f(d) = c$. Then f is strongly RMG-continuous but not perfectly RMG-continuous, since for any set $\{c\}$ in Y but $f^{-1}(\{c\}) = \{c, d)$ which is not clopen set in X i.e. f is not perfectly RMG-continuous.

Theorem 6.7: If a function $f: (X, \tau) \to (Y, \sigma)$ is perfectly RMG-continuous, then f is perfectly continuous.

Proof: Let V be any open set of (Y, σ), since every open set is RMG-open set. We get V is RMG-open in (Y, σ). By hypothesis, we have $f^{-1}(V)$ is clopen in(X, τ). Hence f is perfectly continuous.

Theorem 6.8: If a function $f: (X, \tau) \to (Y, \sigma)$ perfectly RMG-continuous, then f is strongly RMG-continuous but converse is not true.

Proof: Let V be any RMG-open set in (Y, σ). By hypothesis, $f^{-1}(V)$ is open and closed in(X, τ), implies that $f^{-1}(V)$ is closed and open in (X, τ). Hence f is strongly RMG-continuous.

Example 6.9: Let X=Y= {a, b, c, d} with topology $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $\sigma = \{Y, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = d, f(b) = c, f(c) = c$ and $f(d) = a$. Then f is strongly RMG-continuous but not perfectly RMG-continuous, since for the RMG-open set $\{b\}$ in Y but $f^{-1}(\{b\}) =$$\{c\}$ which is not clopen set in X i.e. f is not perfectly RMG-continuous.

Theorem 6.10: Let f: $(X, \tau) \to (Y, \sigma)$ and g:$(Y, \sigma) \to (Z, \eta)$ be any two functions. Then

(i) g∘f: $(X, \tau) \to (Z, \eta)$ is strongly RMG-continuous if g is perfectly RMG-continuous and f is continuous.

(ii) g∘f: $(X, \tau) \to (Z, \eta)$ is perfectly RMG-continuous if g is strongly RMG-continuous and f is perfectly RMG-continuous.

Proof: i) Let U be a RMG-open set in (Z, η). Since g is perfectly RMG-continuous, $g^{-1}(U)$ is clopen set in (Y, σ), $g^{-1}(U)$ is RMG-open set in (Y, σ). Since f is continuous $f^{-1}(g^{-1}(U))$ is open set in (X, τ). Thus $(g \circ f)^{-1} = f^{-1}(g^{-1}(U))$ is an open in (X, τ) and hence gof is strongly RMG-continuous.

ii) Let U be a RMG-open set in (Z, η). Since g is strongly RMG-continuous, $g^{-1}(U)$ is open set in (Y, σ), $g^{-1}(U)$ is RMG-open set in (Y, σ). Since f is perfectly RMG-continuous $f^{-1}(g^{-1}(U))$ is clopen set in (X, τ). Thus $(g \circ f)^{-1} = f^{-1}(g^{-1}(U))$ is an clopen set in (X, τ) and hence gof is perfectly RMG-continuous.

Theorem6.11: If functions $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ are perfectly RMG-continuous, then the composition gof: $(X, \tau) \to (Z, \eta)$ is also perfectly RMG-continuous function.

Proof: Let U is a RMG-open set in (Z, η). Since g be a perfectly RMG-continuous, we get $g^{-1}(U)$ is open and closed in (Y, σ). As any open set is RMG-open in (X, τ) and f is also strongly RMG-continuous, $f^{-1}\left(g^{-1}(U)\right) = (g \circ f)^{-1}(U)$ is both open and closed in(X, τ). Hence gof is perfectly RMG-continuous.

Theorem6.12: Let (X, τ) be discrete topological space and (Y, σ) be any topological space. Let $f: (X, \tau) \to (Y, \sigma)$ be a function, then following are equivalent;

i) f is strongly RMG-continuous.

ii) f is perfectly RMG-continuous.

Proof: (i) → (ii), Let U be any RMG-open set in (Y, σ). By hypothesis, $f^{-1}(U)$ is open in(X, τ). Since (X, τ) is discrete space, then $f^{-1}(U)$ is also closed in(X, τ). Therefore $f^{-1}(U)$ is both open and closed in(X, τ). Hence f is perfectly RMG-continuous.

(ii) → (i), Let U be any RMG-open set in(Y, σ), then $f^{-1}(U)$ is both open and closed in (X, τ). Hence f is strongly RMG-continuous.

From the above discussion and know results we have the following implications. In the following diagram, by A ⟶ B we mean A implies B but not conversely.

$$\begin{tikzcd}
\text{Strongly continuous} \arrow[r] & \text{Perfectly RMG-continuous} \arrow[r] \arrow[d] & \text{Perfectly continuous} \\
& \text{Strongly RMG-continuous} &
\end{tikzcd}$$

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