On α Generalized Continuous Mappings in Ideal Topological Spaces

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Abstract: *The aim of this paper is to investigate and discuss the properties of αIg-continuous function in generalized ideal topological spaces and also the relationship between αIg-continuous function and the related function in ideal topological spaces.*

Keywords: *αIg-closed set, *-continuous mapping, Ig-continuous mapping, Iω-continuous mapping, αIg-continuous mapping, αIg-irresolute, Graph function.*

1. Introduction

The notion of α-open sets was introduced and investigated by Njastad[5]. By using α-open sets, Mashhour et al.[1] defined and studied the concept of α-closed sets, α- closure of a set, α-continuity and α- closedness in topology. Ideals play an important role in topology. It has been considered in topology since 1930. The concept of ideals in topological spaces is treated in the classic text by Kuratowski[4] and Vaidyanathaswamy [9]. Jankovi and Hamlett[2] investigated further properties of ideal spaces. The notation of αIg-closed set in generalized ideal topological space are introduced by Jafari, Viswanathan and Jayasudha[10]. The purpose of this paper is to introduce and study the notion of continuous and irresolute function in ideal topological space. We study the notion of αIg-continuous function, αIg-irresolute function and graph function in ideal topological spaces.

2. Preliminaries

Definition 2.1[3]:
An ideal I on a topological space (X, τ) is a non empty collection of subsets of X which satisfies the following conditions

i. A∈I, and B ⊂ A implies B ∈ I.
ii. A∈I and B∈I implies A ∪ B ∈ I.

An Ideal topological space (X, τ) with an ideal I on X is denoted by (X, τ, I) .

Definition 2.2[3]:
Let (X, τ, I) be an ideal topological space and A ⊂ X, $A^*(I, \tau)$ = { x ∈ X : A ∩ U ∉ I for every U ∈ τ (X, x) } is called a local function of A with respect to I and τ. For every ideal topological space (X, τ, I), there exists a topology τ^* finer than τ defined as τ^* = { U ⊂ X : $cl^*(X – U)$ = X – U } generated by the base β (I, J) = { U – J : U ∈ τ and J ∈ I }. In general β (I, J) is not always a topology. Additionally $cl^*(A)$ = A ∪ A^*. We simply write A^* instead of $A^*(I, \tau)$. $cl^*(A)$ and $int^*(A)$ will denote the closure and interior of A in (X, τ^*) respectively.

Definition 2.3:
A subset A of an ideal topological space (X, τ, I) is called

i. *-closed, if A* ⊂ A [11].
ii. *-dense, if A ⊂ A* [11].
iii. *-Perfect, if A = A* [8].

Definition 2.4[7]:
A subset A of a topological space (X, τ) is said to be

i. Semi closed set if int (cl(A)) ⊂ A.
ii. α-closed set if cl(int(cl (A))) ⊂ A.

Definition 2.5[6]:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be ideal generalized closed set (briefly Ig-closed set) if A* ⊂ U whenever A ⊂ U and U is open.

Definition 2.6[10]:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be Iω-closed set if A* ⊂ U whenever A ⊂ U and U is semi open.

Definition 2.7[11]:
Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be α-Ideal generalized closed set (briefly αIg-closed set) if A* ⊂ U whenever A ⊂ U and U is α-open.

Definition 2.8 [10]:
A function f : (X, τ, I) → (Y ,σ) is said to be

i. *-continuous, if f^{-1} (A) is *-closed in X for every closed set A in Y.
ii. Ig-continuous, if f^{-1} (A) is Ig-closed in X for every closed set A in Y.
iii. Iω-continuous, if f^{-1} (A) is Iω-closed in X for every closed set A in Y.

3. α-IDEAL CONTINUOUS MAPPING

Definition 3.1:
A function f : (X, τ, I) → (Y, σ) is called αIg-continuous, if the inverse image of every closed set in Y is αIg-closed set in X.

Example 3.2:
Let X = Y = {a , b, c, d} and τ = σ = {ϕ, {a, b}, X} with I = {ϕ, {a}, {b}, {a, b}}. Define f : (X , τ , I)

→ (Y , σ , I) by $f(a) = d, f(b) = c, f(c) = b, f(d) = a$, then f is αIg-continuous .

Lemma 3.3[11]:
Let (X, τ, I) be a space with an ideal I on X and A is a subset of X. Then,

i. Every *-closed set is αIg-closed set.
ii. Every αIg closed set is Ig-closed set.

Proposition 3.4:
Let (X, τ, I) be a space with an ideal I on X and A is a subset of X. Then, every Iω-closed set is αIg-closed set.

Definition 3.5:
A function f : (X , τ, I) → (Y , σ, J) is said to be αIg-irresolute, if f^{-1}(A) is αIg-closed in (X, τ, I) for every αIg-closed set A in (Y, σ, J).

Example 3.6:
Let X = Y = {a, b, c} and τ = {ϕ, {c}, {a, c}, X} and σ = {ϕ, {b}, {c}, {b, c}, X} with I = {ϕ, {a}} and J = {ϕ}. Define f : (X , τ , I) → (Y , σ , J) by f (a) = b, f (b) = a, f (c) = c then, f is αIg-irresolute.

Theorem 3.7:
For a function f : (X, τ, I) → (Y, σ), every continuous function is αIg-continuous .

Proof : Let f be a continuous function and V be a closed set in (Y, σ). Then f^{-1}(V) is closed in (X, τ, I). Since, every closed set is *-closed and hence αIg-closed, f^{-1} (V) is αIg-closed in (X, τ, I). Therefore, f is αIg-continuous.

Remark 3.8:
Every αIg-continuous function is not continuous function.

Example 3.9:
Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X}, I = {ϕ, {c}} and σ = {ϕ, {a} , Y}. Define f : (X, τ, I) → (Y, σ) as $f(a) = b, f(b) = a, f(c) = c$. Then, the inverse image of every closed set in Y is αIg-closed in X. Therefore, f is αIg-continuous. But, f is not continuous because, the subset {b, c} is closed in Y, $f^{-1}(\{b,c\}) = \{a,c\}$ is not closed in X.

Theorem 3.10:
For a function f : (X, τ, I) → (Y, σ), every *-continuous function is αIg-continuous.

Proof : Let f be a *-contiuous function and V be a closed set in (Y, σ). Then, f^{-1}(V) is *-closed in (X, τ, I). Since, every *-closed set is αIg-closed, f^{-1}(V) is αIg-closed in (X, τ, I). Therefore, f is αIg-continuous.

Remark 3.11:
Every αIg-continuous function is not *-continuous function.

Example 3.12:
Let X = Y = {1, 2, 3} , τ = {ϕ, {1}, {2, 3}, X}, I = {ϕ} and σ = {ϕ, {2,3}, Y}. Define f : (X, τ, I) → (Y, σ) as $f(1) = 1, f(2) = 2, f(3) = 3$. Then, the inverse image of every closed set in Y is αIg-closed in X. Therefore, f is αIg-continuous. But, f is not *-continuous because, the subset {3} is closed in Y, $f^{-1}(\{3\}) = \{3\}$ is not *-closed in X.

Theorem 3.13:
For a function f : (X, τ, I) → (Y, σ), every αIg-continuous function is Ig-continuous.

Proof: Let V be a closed set in (Y, σ) and f be an αIg-continuous function. Then, f^{-1} (V) is αIg-closed in (X, τ, I). Since, every αIg-closed set is Ig-closed, f^{-1}(V) is Ig-closed in (X, τ, I). Therefore, f is Ig-continuous.

Remark 3.14:
Every Ig-continuous function is not αIg-continuous.

Example 3.15:
Let X = {a, b, c, d}, τ = {ϕ , {a, b}, X} and I = {ϕ, {a}, {b}, {a, b}}. Let Y = {1 , 2 , 3 , 4} , σ = {ϕ, {1 , 2 }, Y} and J = {ϕ, {3}, {4}, {3, 4}}. Define f : (X , τ, I) → (Y, σ, J), by f (a) = 1, f (b) = 3, f (c) = 2 and $f(d) = 4$. Then, the inverse image of every closed set in Y is Ig-closed in X. Therefore, f is Ig-continuous. But, f is not αIg-continuous because, the subset {3, 4}is closed in Y, $f^{-1}(\{3, 4\}) = \{b, d\}$ is not αIg-closed in X.

Theorem 3.16:
For a function f : (X, τ, I) → (Y, σ), every Iω-continuous function is αIg-continuous.

Proof: Let f be Iω-continuous. Then f^{-1}(V) is Iω-closed in (X, τ, I), for every closed set V in (Y, σ). Since, every Iω-closed is αIg-closed, f^{-1}(V) is αIg-closed in (X, τ, I). Therefore, f is an αIg-continuous function.

Theorem 3.17:
A function f : (X, τ, I) → (Y, σ) is αIg-continuous, if and only if f^{-1}(V) is αIg-open in (X ,τ ,I) for every open set V in (Y, σ).

Proof: Let V be an open set in (Y, σ) and f : (X, τ, I) → (Y, σ) be αIg-continuous. Then V^c is closed in

(Y, σ) and $f^{-1}(V^c)$ is αIg-closed in (X, τ, I). But $f^{-1}(V^c) = (f^{-1}(V))^c$ and so $f^{-1}(V)$ is αIg-open in (X, τ, I). Conversely, suppose that $f^{-1}(V)$ is αIg-open in (X, τ, I) for each open set V in (Y, σ). Let F be a closed set in (Y, σ). Then, F^c is open in (Y, σ) and by hypothesis $f^{-1}(F^c)$ is αIg-open in (X, τ, I). Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is αIg-closed in (X, τ, I) and so, f is αIg-continuous .

Remark 3.18:
The composition of two αIg-continuous maps need not be αIg-continuous as seen from the following examples.

Example 3.19:
Let X = Y = Z = {1 ,2, 3, 4}, τ = {ϕ, {1}, {1, 2, 3}, X} and σ ={ϕ, {3}, Y}, η = {ϕ, {2, 4}, Z}, I = {ϕ, {1}, {2}, {1, 2}}, J = {ϕ, {1}, {3}, {1,3}}. Let f : (X, τ, I) → (Y, σ, J) and g : (Y, σ, J) → (Z, η) be identity maps. Then, the maps f and g are αIg-continuous. Let A= {1, 3} is closed in Z. Then, $(gcf)^{-1}(A) = f^{-1}(g^{-1}(A)) = f^{-1}(g^{-1}\{1, 3\}) = f^{-1}(\{1, 3\})$ = {1, 3} which is not αIg-closed in X. Hence gcf is not αIg-continuous.

Theorem 3.20:
Let f : (X, τ, I) → (Y, σ, J) be αIg-continuous and g : (Y, σ, J) → (Z, η) be continuous. Then, f : (X, τ, I) → (Z, η) is αIg continuous.

Proof: Let V be a closed set in (Z, η). Since, g is continuous, $g^{-1}(V)$ is closed in (Y, σ, J). Since, f is αIg-continuous, $f^{-1}(g^{-1}(V))$ is αIg-closed in (X, τ, I) which implies $(gcf)^{-1}(V)$ is αIg-closed in (X, τ, I). Therefore, gcf is αIg-continuous.

Theorem 3.21:
If f : (X, τ, I) → (Y, σ, J) and g : (Y, σ, J) → (Z, η, K) are αIg-irresolute then gcf: (X, τ, I) → (Z, η, K) is αIg- irresolute.

Proof: Let g be an αIg-irresolute function and V be any αIg-closed set in (Z, η, K). Then $g^{-1}(V)$ is αIg closed set in (Y, σ, J). Since, f is αIg-irresolute, $f^{-1}(g^{-1}(V)) = (gcf)^{-1}(V)$ is αIg-closed in (X, τ, I). Hence gcf is αIg- irresolute.

Theorem 3.22:
If f : (X, τ, I) → (Y, σ, J) is αIg-irresolute and g : (Y, σ, J) → (Z, η) is *-continuous then gcf: (X, τ, I) → (Z, η) is αIg-continuous.

Proof: Let g be *-continuous and V be any closed set of (Z, η). Then, $g^{-1}(V)$ is *-closed in (Y, σ, J). Since, every *-closed set is αIg-closed in (Y, σ, J), $g^{-1}(V)$ is αIg-closed set in (Y, σ, J). Therefore, $f^{-1}(g^{-1}(V)) = (gcf)^{-1}(V)$ is αIg-closed in (X, τ, I). Hence gcf is αIg-continuous.

Theorem 3.23:
If f : (X, τ, I) → (Y, σ, J) is αIg-irresolute and g : (Y, σ, J) → (Z, η) is continuous then, gcf : (X, τ, I) → (Z, η) is αIg-continuous.

Proof: Let g be continuous and V be any closed set in (Z, η). Then $g^{-1}(V)$ is closed in (Y, σ, J). Since, every closed set is *-closed and hence αIg-closed, $g^{-1}(V)$ is αIg-closed in (Y, σ, J). Therefore, $f^{-1}(g^{-1}(V)) = (gcf)^{-1}(V)$ is αIg-closed in (X, τ, I). Hence gcf is αIg-continuous.

Theorem 3.24:
If f : (X, τ, I) → (Y, σ, J) is αIg-irresolute and g : (Y, σ, J) → (Z, η) is Iω-continuous then, gcf: (X, τ, I) → (Z, η) is αIg-continuous.

Proof: Let g be Iω-continuous and V be any closed set of (Z, η). Then $g^{-1}(V)$ is an Iω-closed set in (Y, σ, J). Since, every Iω-closed set is αIg-closed, $g^{-1}(V)$ is αIg-closed in (X, τ, I). Therefore, $f^{-1}(g^{-1}(V)) = (gcf)^{-1}(V)$ is αIg-closed in (X, τ, I). Hence gcf is αIg-continuous.

Theorem 3.25:
Let f: (X, τ, I) → (Y, σ, J) be a function. Then, the following statement are equivalent.

1. f is αIg-continuous.
2. The inverse image of every closed set in Y is αIg-closed in X.
3. The inverse image of each open set in Y is αIg-open in X.

Definition 3.26:
Let f : (X, τ, I) → (Y, σ) be a function then, the graph function g : X → X × Y of f is defined as g (x) = (x , f (x)) for each x ∈ X.

Theorem 3.27:
A function f: (X, τ, I) → (Y, σ) is αIg-continuous, if the graph function g: X → X × Y, defined by g (x) = (x , f (x)) for each x ∈ X is αIg-continuous.

Proof: Let V be an open set in Y containing f(x). Then, X × V is an open set in X × Y and by αIg-continuity of g, there exists an αIg-open set U in X containing x such that $g(U) \subset X \times V$. Therefore, we obtain $f(U) \subset V$. This shows that f is αIg-continuous.

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