# On Symmetric Reverse $\theta^{*}$-Bicentralizer in Semiprime *- Rings 

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#### Abstract

Let $R$ be $a{ }^{*}$ - ring and $\theta$ is a mapping on $R$. In this paper, we introduce the concepts of the symmetric reverse $\theta^{*}$ - Bicentralizers in $R$, also we look for its relationship with a symmetric Jordan $\theta^{*}$ - Bicentralizers . Further, we investigate some identities satisfy by symmetric reverse $\theta^{*}$ - Bicentralizer .


Keywords : *- ring, semiprime *- ring, involution, reverse $\theta^{*}$ - Bicentralizer, symmetric reverse $\theta^{*}$ Bicentralizer, symmetric Jordan $\theta^{*}$ - Bicentralizers .

## I. INTRODUCTION

Let R be $\mathrm{a}^{*}$ - ring and $\theta$ is a mapping on R . This paper consists of two sections. In section one, we recall some basic definitions and other concepts which will be used in our paper, we explain these concepts by examples and remarks. In section two, we introduce the notion of symmetric reverse $\theta^{*}$ - Bicentralizer in R, moreover, we look for the conditions under which every symmetric left (right) Jordan $\theta^{*}$ - Bicentralizers become a symmetric left (right) reverse $\theta^{*}$ - Bicentralizers. Also, we investigate some identities satisfied by a symmetric reverse $\theta^{*}$ - Bicentralizer .

## 11. BASIC CONCEPTS

Definition 2.1:[1] A ring $R$ is called a semiprime ring, if for any $a \in R, a R a=\{0\}$, implies that $a=0$.
Example 2.2:[1] Let $\mathrm{R}=\mathrm{z}_{6}$ be a ring. To show that the ring R is semiprime ring, let $\mathrm{a} \in \mathrm{R}$ such that $\mathrm{aRa}=0$, implies that $\mathrm{a}^{2}=0$, hence $\mathrm{a}=0$, therefore R is semiprime ring .

Definition 2.3:[1] A ring $R$ is said to be $n$-torsion free where $n \neq 0$ is an integer if whenever $n a=0$ with $a \in R$, then $\mathrm{a}=0$.
Definition 2.4:[1] An additive mapping $x \rightarrow x^{*}$ on a ring R is called an involution if for all $\mathrm{x}, \mathrm{y} \in \mathrm{R}$ we have $(x y)^{*}=y^{*} x^{*}$ and $x^{* *}=x$. A ring equipped with an involution is called ${ }^{*}$ - ring .

Definition 2.5:[2] Let R be a ring, a Lie product on R , denoted by [,] and defined by : $[\mathrm{x}, \mathrm{y}]=\mathrm{xy}-\mathrm{yx}$, for all $x, y \in R$.
Properties 2.6:[2] Let $R$ be a ring, then for any $x, y \in R$, we have :

1. $[\mathrm{x}, \mathrm{yz}]=[\mathrm{x}, \mathrm{y}] \mathrm{z}+\mathrm{y}[\mathrm{x}, \mathrm{z}]$
2. $[\mathrm{xz}, \mathrm{y}]=[\mathrm{x}, \mathrm{y}] \mathrm{z}+\mathrm{x}[\mathrm{z}, \mathrm{y}]$
3. $[\mathrm{x}+\mathrm{z}, \mathrm{y}]=[\mathrm{x}, \mathrm{y}]+[\mathrm{z}, \mathrm{y}]$
4. $[\mathrm{x}, \mathrm{y}+\mathrm{z}]=[\mathrm{x}, \mathrm{y}]+[\mathrm{x}, \mathrm{z}]$

Definition 2.7:[2] Let $R$ be a ring. A mapping $S: R x R \rightarrow R$ is symmetric if $S(x, y)=S(y, x)$ holds for all pairs $x, y \in R$.

Definition 2.8:[3] Let R be a *- ring. A symmetric biadditive mapping $\mathrm{T}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ is called a symmetric left (right ) *- Bicentralizer if : $T(x y, z)=T(x, z) y^{*}\left(\operatorname{resp} . T(x y, z)=x^{*} T(y, z)\right)$ is fulfilled for all $x, y, z \in R$.
A mapping T is called symmetric *- Bicentralizer of R if T is both left and right *- Bicentralizer .
Example 2.9:[3] Let S be a commutative ring, and
$\mathrm{R}=\left\{\left(\begin{array}{ll}a & 0 \\ 0 & b\end{array}\right), a, b \in S\right\}$.
It is easy to verify that R is a *- ring with respect to the usual operation of addition and multiplication of matrices, as well as the involution * on R defined by
$\left(\begin{array}{ll}a & 0 \\ 0 & b\end{array}\right)=\left(\begin{array}{ll}b & 0 \\ 0 & a\end{array}\right)$, for all $\left(\begin{array}{ll}a & 0 \\ 0 & b\end{array}\right) \in R$.
Let $T: R \times R \rightarrow R$ be a symmetric biadditive mapping defined as:
$\left(\left(\begin{array}{ll}a & 0 \\ 0 & b\end{array}\right),\left(\begin{array}{ll}c & 0 \\ 0 & d\end{array}\right)\right) \rightarrow\left(\begin{array}{cc}0 & 0 \\ 0 & a c\end{array}\right)$
Then T is symmetric *- Bicentralizer .
Definition 2.10:[3] ] Let R be a *- ring . A biadditive mapping $\mathrm{T}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ is called a symmetric left ( right) Jordan ${ }^{*}$ - Bicentralizer if :
$\mathrm{T}\left(\mathrm{x}^{2}, \mathrm{z}\right)=\mathrm{T}(\mathrm{x}, \mathrm{z}) \mathrm{x}^{*}\left(\right.$ resp. $\left.\mathrm{T}\left(\mathrm{x}^{2}, \mathrm{y}\right)=\mathrm{x}^{*} \mathrm{~T}(\mathrm{x}, \mathrm{y})\right)$ is fulfilled for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{R}$.
A symmetric Jordan ${ }^{*}$ - Bicentralizer of a ring R is both symmetric left and right Jordan $*$-Bicentralizer .
Remark 2.11:[3] Every symmetric *- Bicentralizer is symmetric Jordan *-Bicentralizer, but the converse in general is not true .

Definition 2.12:[4] Let R be a *- ring . A symmetric biadditive mapping $\mathrm{T}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ is called a symmetric left ( right) reverse *- Bicentralizer if :
$\mathrm{T}(\mathrm{xy}, \mathrm{z})=\mathrm{T}(\mathrm{y}, \mathrm{z}) \mathrm{x}^{*}\left(\operatorname{resp} . \mathrm{T}(\mathrm{xy}, \mathrm{z})=_{y^{*}} \mathrm{~T}(\mathrm{x}, \mathrm{z})\right)$ holds for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{R}$.
A symmetric reverse *-Bicentralizer of a ring R is both symmetric left and right reverse *-Bicentralizer .
Example 2.13:[4] Let F be a field. Then the set $\mathrm{R}=\mathrm{M}_{2}(\mathrm{~F})$ of all matrices of order 2 from an associative *- ring with respect the usual operation of addition and multiplication of matrices, and the transpose of matrix as an involution.
Let $\mathrm{T}_{1}, \mathrm{~T}_{2}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ are two biadditive mappings defined as :
$\mathrm{T}_{1}\left(\left(\begin{array}{ll}x & y \\ z & w\end{array}\right),\left(\begin{array}{ll}u & v \\ s & t\end{array}\right)\right)=\left(\begin{array}{cc}0 & 0 \\ y v & w t\end{array}\right)$
$\mathrm{T}_{2}\left(\left(\begin{array}{ll}x & y \\ z & w\end{array}\right) \cdot\left(\begin{array}{ll}u & v \\ s & t\end{array}\right)\right)=\left(\begin{array}{cc}0 & z s \\ 0 & w t\end{array}\right)$
Then $T_{1}$ is a symmetric left reverse $*$ - Bicentralizer, and $T_{2}$ is a symmetric right reverse $*$ - Bicentralizer.
Lemma 2.14 : $[4]$ Let $R$ be a semiprime ring, and $G, F: R \times R \rightarrow R$ be a biadditive mappings. If $G(x, y) w$ $F(x, y)=0$, for all $x, y, w \in R$, then $G(x, y) w F(u, v)=0$, for all $x, y, u, v, w \in R$.

Lemma 2.15 :[4] Let $R$ be a 2-torsion free semiprime ring and let $a, b \in R$, then the following conditions are equivalent :
i. $a x b=0$, for all $x \in R$.
ii. $b x a=0$, for all $x \in R$.
iii. $a x b+b x a=0$, for all $x \in R$

If one of these conditions is fulfilled then $a b=b a=0$.
Lemma 2.16 :[4] Let $R$ be a semiprime ring, and $a \in R$ some fixed element. If $a[x, y]=0$, for all $x, y$ $\in R$, then there exists an ideal $U$ of $R$ such that $a \in U \subset Z(R)$.

Lemma 2.17:[4] Let $R$ be *- ring. If $x \in Z(R)$, then $x^{*} \in Z(R)$.

## III . SYMMETRIC REVERSE 日 $^{*}$ - BICENTRALIZER

First we introduce the basic definition in this paper
Definition 3.1 : Let $R$ be a ${ }^{*}$ - ring and $\theta$ is a mapping on $R$. A symmetric biadditive mapping $T: R \times R \rightarrow R$ is called a symmetric left (right) reverse $\theta^{*}$ - Bicentralizer if :
$T(x y, z)=T(y, z) \theta\left(x^{*}\right)\left(\right.$ resp. $\left.T(x y, z)=\theta\left(y^{*}\right) T(x, z)\right)$ holds for all $x, y, z \in R$.
A symmetric reverse $\theta^{*}$ - Bicentralizer of a ring $R$ is both symmetric left and right reverse $\theta^{*}$ -
Bicentralizer.
We now explain this definition by the following example .

Example 3.2 : Let F be a field with an involution * , and $\mathrm{R}=\mathrm{D}_{2}(\mathrm{~F})$ the set of all diagonal matrices of order 2 . Then R is an associative ring with involution with respect to the usual operation of addition and multiplication of matrices, as wel as with an involution * on $\mathrm{D}_{2}(\mathrm{~F})$ defined by :
$\left(\begin{array}{ll}\mathrm{x} & 0 \\ 0 & y\end{array}\right)=\left(\begin{array}{cc}x^{*} & 0 \\ 0 & y^{*}\end{array}\right)$, for all $\left(\begin{array}{cc}x & 0 \\ 0 & y\end{array}\right) \in \mathrm{D}_{2}(\mathrm{~F})$.
Suppose $\mathrm{T}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ is a symmetric biadditive mapping defined as:
$\mathrm{T}\left(\left(\begin{array}{ll}x & 0 \\ 0 & y\end{array}\right) \cdot\left(\begin{array}{ll}u & 0 \\ 0 & v\end{array}\right)\right)=\left(\begin{array}{cc}x^{*} u^{*} & 0 \\ 0 & 0\end{array}\right)$ and $\theta: \mathrm{R} \rightarrow \mathrm{R}$ is defined by
$\theta\left(\begin{array}{ll}x & 0 \\ 0 & y\end{array}\right)=\left(\begin{array}{ll}x & 0 \\ 0 & y\end{array}\right)$
Then $T$ is a symmetric reverse $\theta^{*}$ - Bicentralizer .
Also, we introduce the following notion
Definition 3.3 : Let R be $\mathrm{a}^{*}$ - ring and $\theta$ is a mapping on R . A symmetric biadditive mapping $\mathrm{T}: \mathrm{R} \times \mathrm{R}$ $\rightarrow \mathrm{R}$ is called a symmetric left (right) Jordan $\theta^{*}$ - Bicentralizer if :
$T\left(x^{2}, z\right)=T(x, z) \theta\left(x^{*}\right)\left(\operatorname{resp} . T\left(x^{2}, y\right)=\theta\left(x^{*}\right) T(x, y)\right)$ holds for all $x, y, z \in R$.
A symmetric Jordan $\theta^{*}$ - Bicentralizer of a ring $R$ is both symmetric left and right Jordan $\theta^{*}$ - Bicentralizer .
Remark 3.4 : Every symmetric reverse $\theta^{*}$ - Bicentralizer is symmetric Jordan $\theta^{*}$ - Bicentralizer but the converse in general is not true , as we show in the following example .

Example 3.5 : Let F be a field, and let
$\mathrm{R}=\left\{\left(\begin{array}{cccc}0 & a & b & c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right), a, b, c \in F\right\}$
Then R is a ring with respect to the usual operation of addition and multiplication of matrices .
Define an involution $*: \mathrm{R} \rightarrow \mathrm{R}$ such that
$\left(\begin{array}{cccc}0 & a & b & c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right)=\left(\begin{array}{cccc}0 & a & b & -c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right)$, for all $\left(\begin{array}{cccc}0 & a & b & c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right) \in \mathrm{R}$.
Let $\mathrm{T}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ be a symmetric biadditive mapping defined by:
$\left(\left(\begin{array}{cccc}0 & a & b & c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right) \cdot\left(\begin{array}{cccc}0 & s & t & k \\ 0 & 0 & 0 & t \\ 0 & 0 & 0 & -s \\ 0 & 0 & 0 & 0\end{array}\right)\right) \rightarrow\left(\begin{array}{cccc}0 & 0 & 0 & c k \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right)$

And $\theta: R \rightarrow R$ is defined by
$\theta\left(\begin{array}{cccc}0 & a & b & c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right)=\left(\begin{array}{cccc}0 & a & b & c \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & -a \\ 0 & 0 & 0 & 0\end{array}\right)$
Then T is a symmetric Jordan $\theta^{*}$ - Bicentralizer, but not symmetric reverse $\theta^{*}$ - Bicentralizer .

We precede the first main results in this section, by the following lemma.
Lemma 3.6: Let $R$ be a 2-torsion free semiprime *- ring, and $T: R \times R \rightarrow R$ be a Symmetric left

Jordan $\theta^{*}$ - Bicentralizer, then $\beta_{y}(v, w)=0$, for all $v \in Z(R), w, y \in R$, where $\beta_{y}(x, w)=$ $\mathrm{T}(\mathrm{xw}, \mathrm{y})-\mathrm{T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right)$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{R}$. Where $\theta$ is an automorphism on R .

Proof: For any $x, y \in R$, we have :
$T\left(x^{2}, y\right)=T(x, y) \theta\left(x^{*}\right)$
The linearization of (3.1) with respect to $x$ gives
$\mathrm{T}(\mathrm{xw}+\mathrm{wx}, \mathrm{y})=\mathrm{T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right)+\mathrm{T}(\mathrm{w}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, \mathrm{u}, w \in \mathrm{R}$.
Putting $x w+w x$ instead of $w$ in (3.2) and using (3.2), we obtain :
$T\left(x^{2} w+w x^{2}, y\right)+2 T(x w x, y)=T(x, y) \theta\left(x^{*}\right) \theta\left(w^{*}\right)+2 T(x, y) \theta\left(w^{*}\right) \theta\left(x^{*}\right)+$
$\mathrm{T}(\mathrm{w}, \mathrm{y}) \theta\left(\mathrm{x}^{* 2}\right)$, for all $\mathrm{x}, \mathrm{y}, w \in \mathrm{R}$.
On the other hand, the substitution $x^{2}$ for $x$ in (3.2) gives:
$T\left(x^{2} w+w x^{2}, y\right)=T(x, y) \theta\left(x^{*}\right) \theta\left(w^{*}\right)+T(w, y) \theta\left(x^{* 2}\right), \quad$ for all $x, y, w \in R$.
Combining relations (3.3) and (3.4) implies that:
$\mathrm{T}(\mathrm{xwx}, \mathrm{y})=\mathrm{T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right) \theta\left(\mathrm{x}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, \mathrm{w} \in \mathrm{R}$.
Again, the linearization of (3.5) with respect to $x$ leads to :
$\mathrm{T}(\mathrm{xwz}+\mathrm{zwx}, \mathrm{y})=\mathrm{T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right) \theta\left(\mathrm{z}^{*}\right)+\mathrm{T}(\mathrm{z}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right) \theta\left(\mathrm{x}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{w} \in \mathrm{R}$.
Now, consider
$\mathrm{T}(\mathrm{xvw}+\mathrm{wvx}, \mathrm{y})=\mathrm{T}((\mathrm{xv}) \mathrm{w}+\mathrm{w}(\mathrm{xv}), \mathrm{y})$
We shall compute (3.7) by using (3.6) and (3.2), we get
$\mathrm{T}(\mathrm{xvw}+\mathrm{wvx}, \mathrm{y})=\mathrm{T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right) \theta\left(\mathrm{v}^{*}\right)+\mathrm{T}(\mathrm{v}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right) \theta\left(\mathrm{x}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, v, w \in \mathrm{R}$.
$\mathrm{T}((\mathrm{xv}) \mathrm{w}+\mathrm{w}(\mathrm{xv}), \mathrm{y})=\mathrm{T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{v}^{*}\right) \theta\left(\mathrm{w}^{*}\right)+\mathrm{T}(\mathrm{vw}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right), \quad$ for all $\mathrm{x}, \mathrm{y}, v, w \in \mathrm{R}$.
Comparing the relations (3.8) and (3.9), we arrive at :
$\beta_{y}(x, v) R \beta_{y}(v, x)=0$, for all $v \in Z(R), x, y \in R$.
Using the semiprimeness of $R$, we obtain $\beta_{y}(x, v)=0$.
Theorem 3.7 : Let R be a 2-torsion free semiprime ${ }^{*}$ - ring, then every symmetric left Jordan $\theta^{*}$ Bicentralizer of $R$ is a symmetric left reverse $\theta^{*}$ - Bicentralizer. Where $\theta$ is an automorphism on $R$.

Proof: We consider
$\mathrm{T}(\mathrm{xwzwx}+\mathrm{wxzxw}, \mathrm{y})=\mathrm{T}((\mathrm{xw}) \mathrm{zwx}+(\mathrm{wx}) \mathrm{zxw}, \mathrm{y})$
In view relations (3.5) and (3.6) , using similar arguments as used to get (3.10) from (3.7), the above relation becomes:
$\delta_{y}(w, x) \theta\left(z^{*}\right) \theta\left(w^{*}\right) \theta\left(x^{*}\right)+\delta_{y}(x, w) \theta\left(z^{*}\right) \theta\left(x^{*}\right) \theta\left(w^{*}\right)=0$,
for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{w} \in \mathrm{R}$.
Where $\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w})=\mathrm{T}(\mathrm{xw}, \mathrm{y})-\mathrm{T}(\mathrm{w}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, \mathrm{w} \in \mathrm{R}$, according to this assumption the relation (3.2) has the form :
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w})=-\delta_{\mathrm{y}}(\mathrm{w}, \mathrm{x})$, for all $\mathrm{x}, \mathrm{y}, w \in \mathrm{R}$.
According to the above relation, the relation (3.11) is reduced to :
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w}) \theta\left(\mathrm{z}^{*}\right)\left[\theta\left(\mathrm{x}^{*}\right), \theta\left(\mathrm{w}^{*}\right)\right]=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{w} \in \mathrm{R}$.
Using Lemma (2.14) implies that:
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w}) \theta\left(\mathrm{z}^{*}\right)[\theta(\mathrm{u}), \theta(\mathrm{v})]=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{u}, v, w \in \mathrm{R}$.
An application of Lemma (2.15) leads to:
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w})[\theta(\mathrm{u}), \theta(\mathrm{v})]=0$, for all $\mathrm{x}, \mathrm{y}, u, v, w \in \mathrm{R}$.
By Lemma (2.16), we conclude $\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w}) \in \mathrm{Z}(\mathrm{R})$
Now, for $\theta(v) \in Z(R)$, let us consider
$\delta_{y}(\mathrm{x}, \mathrm{w}) \theta\left(\mathrm{v}^{*}\right)=\mathrm{T}(\mathrm{xw}, \mathrm{y}) \theta\left(\mathrm{v}^{*}\right)-\mathrm{T}(\mathrm{w}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right) \theta\left(\mathrm{v}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, w \in \mathrm{R}$.
Using the above relation gather together the Lemma (2.17) and (3.6), we have :

$$
\begin{align*}
\delta_{y}(\mathrm{x}, \mathrm{w}) \theta\left(\mathrm{v}^{*}\right) & =\mathrm{T}(\mathrm{xwv}, \mathrm{y})-\mathrm{T}(\mathrm{w}, \mathrm{y}) \theta\left(\mathrm{v}^{*}\right) \theta\left(\mathrm{x}^{*}\right)  \tag{3.15}\\
& =\mathrm{T}(\mathrm{xwv}, \mathrm{y})-\mathrm{T}(\mathrm{wv}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right) \\
& =\mathrm{T}(\mathrm{xwv}, \mathrm{y})-\mathrm{T}(\mathrm{vw}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right) \\
& =\mathrm{T}(\mathrm{xwv}, \mathrm{y})-\mathrm{T}(\mathrm{v}, \mathrm{y}) \theta\left(\mathrm{w}^{*}\right) \theta\left(\mathrm{x}^{*}\right) \\
& =\mathrm{T}(\mathrm{xwv}, \mathrm{y})-\mathrm{T}(\mathrm{v}, \mathrm{y}) \theta\left((\mathrm{xw})^{*}\right) \\
& =\mathrm{T}(\mathrm{xwv}, \mathrm{y})-\mathrm{T}(\mathrm{xwv}, \mathrm{y})=0 \tag{3.16}
\end{align*}
$$

Therefore
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w}) \theta\left(\mathrm{v}^{*}\right)=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{w} \in \mathrm{R}$ and $\theta(\mathrm{v}) \in \mathrm{Z}(\mathrm{R})$.
According to Lemma (2.17) we have $\theta\left(\mathrm{v}^{*}\right) \in \mathrm{Z}(\mathrm{R})$, so by taking $\theta\left(\mathrm{v}^{*}\right)=\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w})$ in (3.16), we arrive at :
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w}) \delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w})=0$, for all $\mathrm{x}, \mathrm{y}, w \in \mathrm{R}$.
Right multiplication of (3.17) by $\theta(\mathrm{z})$, we obtain :
$\delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w}) \theta(\mathrm{z}) \delta_{\mathrm{y}}(\mathrm{x}, \mathrm{w})=0$, for all $\mathrm{x}, \mathrm{y}, z, w \in \mathrm{R}$.

Using the semiprimeness of $R$, we arrive at:
$\mathrm{T}(\mathrm{xw}, \mathrm{y})=\mathrm{T}(\mathrm{w}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right)$, for all $\mathrm{x}, \mathrm{y}, \mathrm{w} \in \mathrm{R}$.
In similar manner we can obtain the following theorem .
Theorem 3.8 : Let R be a 2-torsion free semiprime *- ring , then every symmetric right Jordan $\theta^{*}$ -
Bicentralizer of R is a symmetric right reverse $\theta^{*}$ - Bicentralizer. Where $\theta$ is an automorphism on R
Lemma 3.9: Let R be a *- ring with identity. Then a symmetric biadditive mapping $\mathrm{T}: \mathrm{R} \times \mathrm{R} \rightarrow \mathrm{R}$ is a symmetric left (right ) reverse $\theta^{*}$ - Bicentralizer if and only if $T$ is of the form $T(x, y)=a \theta\left(y^{*}\right) \theta\left(x^{*}\right)$ $\left(T(x, y)=\theta\left(x^{*}\right) \theta\left(y^{*}\right) a\right)$ for some fixed element $a \in R$. Where $\theta$ is an automorphism on $R$.

Proof: Suppose T is a symmetric left reverse $\theta^{*}$ - Bicentralizer, then :
$T(x z, y)=T(z, y) \theta\left(x^{*}\right)=T(z .1, y) \theta\left(x^{*}\right)=T(1, y) \theta\left(z^{*}\right) \theta\left(x^{*}\right)$
$=T(1, y .1) \theta\left(z^{*}\right) \theta\left(x^{*}\right)=a \theta\left(y^{*}\right) \theta\left(z^{*}\right) \theta\left(x^{*}\right)$, where a stands for $T(1,1)$
Hence $\left.T(x z, y)=a \operatorname{l} y^{*}\right) \theta\left(z^{*}\right) \theta\left(x^{*}\right)$, for all $x, y, z \in R$.
So taking $\mathrm{z}=1$ leads to:
$T(x, y)=a\left(y^{*}\right) \theta\left(x^{*}\right)$, for all $x, y \in R$.
Conversely, suppose $T(x, y)=a \theta\left(y^{*}\right) \theta\left(x^{*}\right)$, for all $x, y \in R$, then :
$T(x z, y)=\mathrm{a} \theta\left(y^{*}\right) \theta\left(z^{*}\right) \theta\left(x^{*}\right)=\left(a \theta\left(y^{*}\right) \theta\left(z^{*}\right)\right) \theta\left(x^{*}\right)=T(z, y) \theta\left(x^{*}\right)$
Hence $T$ is a symmetric left reverse $\theta^{*}$ - Bicentralizer.
In similar arguments as above, we can prove that T is a symmetric right reverse $\theta^{*}$ - Bicentralizer if and only if $T(x, y)=\theta\left(x^{*}\right) \theta\left(y^{*}\right) a$.

Theorem 3.10 : Let R be a 2-torsion free semiprime ${ }^{*}$ - ring with identity. If a symmetric biadditive mapping $T: R x R \rightarrow R$ satisfies that $T\left(x^{3}, y\right)=\theta\left(x^{*}\right) T(x, y) \theta\left(x^{*}\right)$, for all $x, y \in R$, then $T$ is a symmetric reverse $\theta^{*}$ - Bicentralizer. Where $\theta$ is an automorphism on $R$.

Proof: Suppose that for any $x, y \in R$, we have
$T\left(x^{3}, y\right)=\theta\left(x^{*}\right) T(x, y) \theta\left(x^{*}\right)$
Putting $\mathrm{x}+1$ for x in (3.19), where 1 is the identity element, we get
$3 T\left(x^{2}, y\right)+2 T(x, y)=T(x, y) \theta\left(x^{*}\right)+\theta\left(x^{*}\right) T(x, y)+\theta\left(x^{*}\right) T(1, y)+T(1, y) \theta\left(x^{*}\right)+\theta\left(x^{*}\right) T(1, y) \theta\left(x^{*}\right)$, for all $x, y \in R$.
Replacing $x$ by -x in (3.20) gives :
$3 T\left(x^{2}, y\right)-2 T(x, y)=T(x, y) \theta\left(x^{*}\right)+\theta\left(x^{*}\right) T(x, y)-\theta\left(x^{*}\right) T(1, y)-T(1, y) \theta\left(x^{*}\right)+\theta\left(x^{*}\right) T(1, y) \theta\left(x^{*}\right)$
Combining (3.20) with the above relation, we arrive at :
$6 \mathrm{~T}\left(\mathrm{x}^{2}, \mathrm{y}\right)=2 \mathrm{~T}(\mathrm{x}, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right)+2 \theta\left(\mathrm{x}^{*}\right) \mathrm{T}(\mathrm{x}, \mathrm{y})+2 \theta\left(\mathrm{x}^{*}\right) \mathrm{T}(1, \mathrm{y}) \theta\left(\mathrm{x}^{*}\right)$,
for all $x, y \in R$.
Also , comparing (3.20) with (3.21) implies that:
$2 T(x, y)=T(1, y) \theta\left(x^{*}\right)+\theta\left(x^{*}\right) T(1, y)$, for all $x, y \in R$.
The substitution $x^{2}$ for $x$ in (3.22) leads to :
$2 \mathrm{~T}\left(\mathrm{x}^{2}, \mathrm{y}\right)=\mathrm{T}(1, \mathrm{y}) \theta\left(\mathrm{x}^{* 2}\right)+\theta\left(\mathrm{x}^{* 2}\right) \mathrm{T}(1, \mathrm{y})$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{R}$. (3.23)
In view of (3.22), (3.23) and the fact R a 2-torsion free ring, the relation (3.21) is reduced to :
$T(1, y) \theta\left(x^{* 2}\right)+\theta\left(x^{* 2}\right) T(1, y)-2 \theta\left(x^{*}\right) T(1, y) \theta\left(x^{*}\right)=0$, for all $x, y \in R$.
This relation can be written as :
$\left[\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right], \theta\left(\mathrm{x}^{*}\right)\right]=0$, for all $\mathrm{x}, \mathrm{y} \in \mathrm{R}$.
The linearization of the above relation with respect to $x$ gives :
$\left[\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right], \theta\left(\mathrm{z}^{*}\right)\right]+\left[\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{z}^{*}\right)\right], \theta\left(\mathrm{x}^{*}\right)\right]=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{R}$.
Putting zx instead of z in (3.24) leads to:
$\left[\left[\mathrm{T}(1, y), \theta\left(x^{*}\right)\right], \theta\left(x^{*} z^{*}\right)\right]+\left[\left[T(1, y), \theta\left(x^{*} z^{*}\right)\right], \theta\left(x^{*}\right)\right]=0$
$\theta\left(\mathrm{x}^{*}\right)\left[\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right], \theta\left(\mathrm{z}^{*}\right)\right]+\left[\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right] \theta\left(\mathrm{z}^{*}\right)+\theta\left(\mathrm{x}^{*}\right)\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{z}^{*}\right)\right], \theta\left(\mathrm{x}^{*}\right)\right]=0$.
$\theta\left(x^{*}\right)\left[\left[T(1, y), \theta\left(x^{*}\right)\right], \theta\left(z^{*}\right)\right]+\left[\left[T(1, y), \theta\left(x^{*}\right)\right] \theta\left(z^{*}\right), \theta\left(x^{*}\right)\right]+\left[\theta\left(x^{*}\right)\left[T(1, y), \theta\left(z^{*}\right)\right], \theta\left(x^{*}\right)\right]=0$
$\theta\left(x^{*}\right)\left[\left[T(1, y), \theta\left(x^{*}\right)\right], \theta\left(z^{*}\right)\right]+\left[T(1, y), \theta\left(x^{*}\right)\right]\left[\theta\left(z^{*}\right), \theta\left(x^{*}\right)\right]+\theta\left(x^{*}\right)\left[\left[T(1, y), \theta\left(z^{*}\right)\right], \theta\left(x^{*}\right)\right]=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{R}$.
According to (3.24), the above relation reduces to :
$\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right]\left[\theta\left(\mathrm{z}^{*}\right), \theta\left(\mathrm{x}^{*}\right)\right]=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{R}$.
The substitution $\theta\left(z^{*}\right) T(1, y)$ for $\theta\left(z^{*}\right)$ in the above relation gives:
$\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right]^{\theta}\left(\mathrm{z}^{*}\right)\left[\mathrm{T}(1, \mathrm{y}), \theta\left(\mathrm{x}^{*}\right)\right]=0$, for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{R}$.

Using the semiprimeness property of R implies that:
$\left[T(1, y), \theta\left(x^{*}\right)\right]=0$, for all $x, y \in R$.
Replacing $\theta(x)$ by $\theta\left(x^{*}\right)$ in the above relation, we get:
$[T(1, y), \theta(x)]=0$, for all $x, y \in R$.
That is $T(1, y) \in Z(R)$ for all $y \in R$, hence the relation (3.22) is reduced to :
$T(x, y)=T(1, y) \theta\left(x^{*}\right)=\theta\left(x^{*}\right) T(1, y)$, for all $x, y \in R$.
On the other hand, in view of (3.19) and the symmetry of T we have:
$T\left(x, y^{3}\right)=\theta\left(y^{*}\right) T(x, y) \theta\left(y^{*}\right)$, for all $x, y \in R$.
Now, using similar techniques as used on (3.19) to get the relation (3.25) we arrive at :
$T(x, y)=T(x, 1) \theta\left(y^{*}\right)=\theta\left(y^{*}\right) T(x, 1)$, for all $x, y \in R$.
Setting $\mathrm{x}=1$ in (3.26), we get:
$\mathrm{T}(1, \mathrm{y})=\mathrm{a} \theta\left(\mathrm{y}^{*}\right)=\theta\left(\mathrm{y}^{*}\right) \mathrm{a}$, where a stands for $\mathrm{T}(1,1)$.
Combining the relations (3.25) and (3.27) leads to :
$T(x, y)=a \theta\left(y^{*}\right) \theta\left(x^{*}\right)=\theta\left(x^{*}\right) \theta\left(y^{*}\right) a$, for all $x, y \in R$.
Using Lemma (3.9) we obtain the required results .

## IV.CONCLUSIONS

In this paper, we present certain conditions which make every symmetric Jordan $\theta^{*}$ - Bicentralizer is a symmetric reverse $\theta^{*}$ - Bicentralizer. Further, we investigate some identities that force a biadditive mapping to be an symmetric reverse $\theta^{*}$ - Bicentralizer

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