On Intuitionistic Fuzzy γ* Generalized Closed Mappings

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Abstract: *In this paper, we have introduced the notion of intuitionistic fuzzy γ* generalized closed mappings, intuitionistic fuzzy γ* generalized open mappings and intuitionistic fuzzy M γ* generalized closed mappings. Furthermore we have provided some properties of intuitionistic fuzzy γ* generalized closed mappings and discussed some fascinating theorems.*

Keywords*:* *Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy γ* generalized closed sets, intuitionistic fuzzy γ* generalized closed mappings, intuitionistic fuzzy γ* generalized open mappings.*

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I. INTRODUCTION

Atanassov [1] introduced the idea of intuitionistic fuzzy sets using the notion of fuzzy sets by Zadeh. Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Later this was followed by the introduction of intuitionistic fuzzy γ* generalized closed sets by Riya, V. M and Jayanthi, D [7] in 2017 which was simultaneously followed by the introduction of intuitionistic fuzzy γ* generalized continuous mappings [8] by the same authors. We have now extended our idea towards intuitionistic fuzzy γ* generalized closed mappings and discussed some of their properties.

2. PRELIMINARIES

Definition 2.1*:** [1] An ***intuitionistic fuzzy set (IFS for short) A is an object having the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$

where the functions $\mu_A: X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x , \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$

and

$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$.

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle: x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i: i \in J\} \in \tau$.

In this case the pair (X, τ) is called an ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 2.4: [11] Two IFSs A and B are said to be ***q-coincident*** ($A_q B$ in short) if and only if there exits an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5: [11] Two IFSs A and B are said to be ***not q-coincident*** ($A_{\bar{q}}$ B in short) if and only if $A \subseteq B^c$.

Definition 2.6: [3] An ***intuitionistic fuzzy point*** (IFP for short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.7: [4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy γ closed set (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$

(ii) intuitionistic fuzzy γ open set (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$

Definition 2.8: [4] Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$\gamma int(A) = \cup\{G \mid G$ is an IFγOS in X and $G \subseteq A\}$

$\gamma cl(A) = \cap\{K \mid K$ is an IFγCS in X and $A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A)^c = (\gamma cl(A))^c$.

Corollary 2.9: [3] Let A, $A_i (i \in J)$ be intuitionistic fuzzy sets in X and B, $B_j (j \in K)$ be intuitionistic fuzzy sets in Y and $f: X \to Y$ be a function. Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]

e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

g) $f^{-1}(0_\sim) = 0_\sim$

h) $f^{-1}(1_\sim) = 1_\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Definition 2.10: [7] An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy γ* generalized closed set (briefly IFγ*GCS) if $cl(int(A)) \cap int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.11: [8] A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy γ* generalized continuous (IFγ*G continuous for short) mapping if $f^{-1}(V)$ is an IFγ*GCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.12: **[7]** If every IFγ*GCS in (X, τ) is an IFγCS in (X, τ), then the space can be called as an intuitionistic fuzzy γ* $T_{1/2}$ (IFγ*$T_{1/2}$ in short) space.

Definition 2.13: **[7]** If every IFγ*GCS in (X, τ) is an IFCS in (X, τ), then the space can be called as an intuitionistic fuzzy γ*c $T_{1/2}$ (IFγ*c$T_{1/2}$ in short) space.

III. INTUITIONISTIC FUZZY γ* GENERALIZED CLOSED MAPPINGS AND INTUITIONISTIC FUZZY γ* GENERALIZED OPEN MAPPINGS

In this section we have introduced intuitionistic fuzzy γ* generalized closed mappings, intuitionistic fuzzy γ* generalized open mappings, intuitionistic fuzzy M γ* generalized closed mappings and study some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy*** γ* ***generalized closed mapping*** (IFγ*G closed mapping for short) if f (V) IFγ*GCS in Y for every IFCS V of X.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩, G_2 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then,

Now G_1^c = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFCS in X. Then f(G_1^c) = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ is an IFγ*GCS in Y as cl(int(f(G_1^c))) ∩ int(cl(f(G_1^c))) = $0_\sim$ ∩ G_2 = $0_\sim$ ⊆ G_2 where f(G_1^c) ⊆ G_2. Therefore f is an IFγ*G closed mapping.

Theorem 3.3: Every IF closed mapping is an IFγ*G closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF closed mapping [4]. Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since every IFCS is an IFγ*GCS [6], f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Example 3.4: In Example 3.2, f is an IFγ*G closed mapping but not an IF closed mapping, since G_1^c = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFCS in X, but f(G_1^c) = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ is not an IFCS in Y, since cl(f(G_1^c)) = G_2^c ≠ f(G_1^c).

Theorem 3.5: Every IFα closed mapping is an IFγ*G closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα closed mapping [6]. Let A be an IFCS in X. Then f(A) is an IFαCS in Y, by hypothesis. Since every IFαCS is an IFγ*GCS [7], f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Example 3.6: In example 3.2, f is an IFγ*G closed mapping but not an IFα closed mapping, since G_1^c = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFCS in X, but f(G_1^c) = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ is not an IFαCS in Y, since cl(int(cl(f(G_1^c)))) = G_2^c ⊈ f(G_1^c).

Theorem 3.7: Every IF semi closed mapping is an IFγ*G closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semi closed mapping [6]. Let A be an IFCS in X. Then f(A) is an IFSCS in Y, by hypothesis. Since every IFSCS is an IFγ*GCS [7], f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Example 3.8: In Example 3.2, f is an IFγ*G closed mapping but not an IF semi closed mapping, since G_1^c = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFCS in X, but f(G_1^c) = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ is not an IFSCS in Y, as int(cl(f(G_1^c))) = G_2 ⊈ f(G_1^c).

Theorem 3.9: Every IF pre closed mapping is an IFγ*G closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF pre closed mapping [6]. Let A be an IFCS in X. Then f(A) is an IFPCS in Y, by hypothesis. Since every IFPCS is an IFγ*GCS [7], f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Example 3.10: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are

IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then,

Now G_1^c = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩ is an IFCS in X. Therefore f(G_1^c) = ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩ ⊆ $1_\sim$ and int(cl(f (G_1^c))) ∩ cl(int(f (G_1^c))) = $1_\sim$ ⊆ $1_\sim$. Hence f (G_1^c) is an IFγ*GCS in Y. Thus f is an IFγ*G closed mapping.

We have G_1^c = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩ is an IFCS in X. But f(G_1^c) = ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩ is not an IFPCS in Y, since cl(int(f (G_1^c))) = cl(G_2) = $1_\sim$ ⊈ f (G_1^c). Hence f (G_1^c) is not an IFPCS in Y. Thus f is not an IF pre closed mapping.

Theorem 3.11: Every IF generalized closed mapping is an IFγ*G closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF generalized closed mapping [10]. Let A be an IFCS in X. Then f(A) is an IFGCS in Y, by hypothesis. Since every IFGCS is an IFγ*GCS [7], f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Example 3.12: In Example 3.2, f is an IFγ*G closed mapping but not an IF generalized closed mapping as cl(f (G_1^c)) = G_2^c ⊈ G_2, but f (G_1^c) ⊆ G_2.

Definition 3.13: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy M γ* generalized closed mapping*** (IFMγ*G closed mapping for short) if f(A) is an IFγ*GCS in Y for every IFγ*GCS A in X.

Example 3.14: Let X = {a, b} and Y= {u, v}. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively, where G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩ and G_2 = ⟨y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)⟩. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

IFγ*GC(X) ={$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0, 1]$ / $0 \le \mu_a + \nu_a \le 1$, $0 \le \mu_b + \nu_b \le 1$}

IFγ*GC(Y) ={$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0, 1]$ / $0 \le \mu_u + \nu_u \le 1$, $0 \le \mu_v + \nu_v \le 1$}

We have every IFγ*GCS in X is an IFγ*GCS in Y. Therefore f is an IFMγ*G closed mapping.

Theorem 3.15: Every IFMγ*G closed mapping is an IFγ*G closed mapping but not conversely in general.

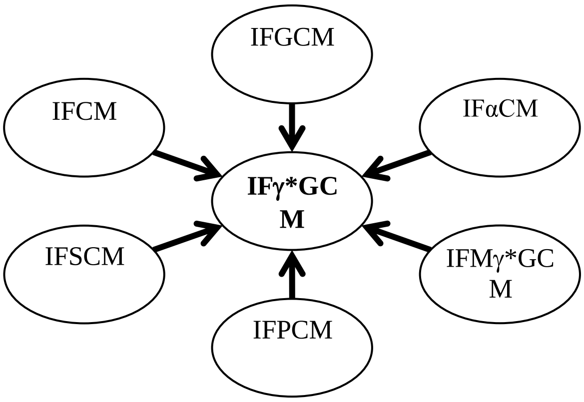
Proof: Let f: (X, τ) → (Y, σ) be an IFMγ*G closed mapping. Let A be an IFCS in X. Then A is an IFγ*GCS in X. By hypothesis f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Example 3.16: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 = ⟨y, (0.6_u, 0.8_v), (0.2_u, 0.1_v)⟩ and G_3 = ⟨y, (0.3_u, 0.3_v), (0.2_u, 0.2_v)⟩, Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Now G_1^c = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩ is an IFCS in X. We have f (G_1^c) = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩ is an IFγ*GCS, since f (G_1^c) ⊆ G_2 and int(cl(f (G_1^c))) ∩ cl(int(f (G_1^c))) = $1_\sim$ ∩ $0_\sim$ = $0_\sim$ ⊆ G_2 , Hence f is an IFγ*G closed mapping.

Now consider, A = ⟨x, (0.3_a, 0.3_b), (0.2_a, 0.2_b)⟩ in X. Then A ⊆ $1_\sim$ and int(cl(A)) ∩ cl(int(A)) = $1_\sim$ ∩ $0_\sim$ = $0_\sim$ ⊆ $1_\sim$. Hence A is an IFγ*GCS in X. But it is not an IFγ*GCS in Y, since f(A) ⊆ G_1, G_2 but int(cl(f (A))) ∩ cl(int(f(A))) = $1_\sim$ ⊈ G_1, G_2. Hence f is not an IFMγ*G closed mapping.

The relation between various types of intuitionistic fuzzy closed mappings is given in the following diagram.



The reverse implications are not true in general in the above diagram.

Theorem 3.17: Let f: X → Y be a bijective mapping. Then the following are equivalent if Y is an IFγ*$T_{1/2}$ space:

(i) f is an IFγ*G closed mapping

(ii)γcl(f(A)) ⊆ f(cl(A)) for each IFS A of X

(iii)f $^{-1}$(γcl(B)) ⊆ cl(f $^{-1}$(B)) for every IFS B of Y

Proof: (i) ⇒ (ii) Let A be an IFS in X. Then cl(A) is an IFCS in X. (i) implies that f(cl(A)) is an IFγ*GCS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(cl(A)) is an IFγCS in Y. Therefore γcl(f(cl(A))) = f(cl(A)). Now γcl(f(A)) ⊆ γcl(f(cl(A))) = f(cl(A)). Hence γcl(f(A)) ⊆ f(cl(A)) for each IFS A of X.

(ii) ⇒ (i) Let A be any IFCS in X. Then cl(A) = A. (ii) implies that γcl(f(A)) ⊆ f(cl(A)) = f(A). But f(A) ⊆ γcl(f(A)). Therefore γcl(f(A)) = f(A). This implies f(A) is an IFγCS in Y. Since every IFγCS is an IFγ*GCS, f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

(ii) ⇒ (iii) Let B be an IFS in Y. Then f $^{-1}$(B) is an IFS in X. Since f is onto, γcl(B) = γcl(f(f $^{-1}$(B))) and (ii) implies γcl(f(f $^{-1}$(B))) ⊆ f(cl(f $^{-1}$(B))). Therefore γcl(B) ⊆ f(cl(f $^{-1}$(B))). Now f $^{-1}$(γcl(B)) ⊆ f $^{-1}$(f(cl(f $^{-1}$(B)))) = cl(f $^{-1}$(B)), since f is one to one.

(iii) ⇒ (ii) Let A be any IFS of X. Then f(A) is an IFS of Y. Since f is one to one, (iii) implies that f $^{-1}$(γcl(f(A))) ⊆ cl(f $^{-1}$(f(A))) = cl(A). Therefore f(f $^{-1}$(γcl(f(A)))) ⊆ f(cl(A)). Since f is onto γcl(f(A)) = f(f $^{-1}$(γcl(f(A)))) ⊆ f(cl(A)).

Theorem 3.18: Let f: X → Y be an IFγ*G closed mapping. Then for every IFS A of X, f(cl(A)) is an IFγ*GCS in Y.

Proof: Let A be any IFS in X. Then cl(A) is an IFCS in X. By hypothesis f(cl(A)) is an IFγ*GCS in Y.

Theorem 3.19: Let f: X → Y be an IFγ*G closed mapping where Y is an IFγ*$T_{1/2}$ space, then f is an IF closed mapping if every IFγCS is an IFCS in Y.

Proof: Let f be an IFγ*G closed mapping. Then for every IFCS A in X, f(A) is an IFγ*GCS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(A) is an IFγCS in Y and by hypothesis f(A) is an IFCS in Y. Hence f is an IF closed mapping.

Theorem 3.20: If every IFS is an IFCS in X, then an IFγ*G closed mapping f: X → Y is an IFγ*G continuous mapping.

Proof: Let A be an IFCS in Y. Then f $^{-1}$(A) is an IFS in X. Therefore f $^{-1}$(A) is an IFCS in X. Since every IFCS is an IFγ*GCS [7], f $^{-1}$(A) is an IFγ*GCS in X. This implies that f is an IFγ*G continuous mapping.

Theorem 3.21: A bijective mapping f: X → Y is an IFγ*G closed mapping if and only if for every IFS B of Y and for every IFOS U containing f $^{-1}$(B), there is an IFγ*GOS A of Y such that B ⊆ A and f $^{-1}$(A) ⊆ U.

Proof: Necessity: Let B be any IFS in Y. Let U be an IFOS in X such that f $^{-1}$ (B) ⊆ U, then U^c is an IFCS in X. By hypothesis f(U^c) is an IFγ*GCS in Y. Let A = $(f(U^c))^c$, then A is an IFγ*GOS in Y and B ⊆ A. Now f $^{-1}$(A) = f $^{-1}$ $(f(U^c))^c$ = (f $^{-1}$$(f(U^c)))^c$ ⊆ U.

Sufficiency: Let A be any IFCS in X, then A^c is an IFOS in X and $f^{-1}(f(A^c)) \subseteq A^c$. By hypothesis there exists an IFγ*GOS B in Y such that $f(A^c) \subseteq B$ and $f^{-1}(B) \subseteq A^c$. Therefore $A \subseteq (f^{-1}(B))^c$. Hence $B^c \subseteq f(A) \subseteq f(f^{-1}(B))^c \subseteq B^c$. This implies that $f(A) = B^c$. Since B^c is an IFγ*GCS in Y, f(A) is an IFγ*GCS in Y. Hence f is an IFγ*G closed mapping.

Theorem 3.22: If f: (X, τ) → (Y, σ) is an IF closed mapping and g: (Y, σ) → (Z, δ) is an IFγ*G closed mapping, then g o f : (X, τ) → (Z, δ) is an IFγ*G closed mapping.

Proof: Let A be an IFCS in X, then f(A) is an IFCS in Y, since f is an IF closed mapping. Since g is an IFγ*G closed mapping, g(f(A)) is an IFγ*GCS in Z. Therefore g o f is an IFγ*G closed mapping.

Theorem 3.23: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ*c$T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγ*G closed mapping

(ii)f(B) is an IFγ*GOS in Y for every IFOS B in X

(iii)$f(int(B)) \subseteq cl(int(f(B)))$ for every IFS B in X

Proof: (i) ⇒ (ii) is obvious as $f(A^c) = (f(A))^c$ for a bijection mapping.

(ii) ⇒ (iii) Let B be an IFS in X, then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFγ*GOS in Y. Since Y is an IFγ*c$T_{1/2}$ space, f(int(B)) is an IFOS in Y. Therefore $f(int(B)) = int(f(int(B))) \subseteq cl(int(f(int(B)))) \subseteq cl(int(f(B)))$.

(iii) ⇒ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(int(A^c)) = f(A^c) \subseteq cl(int(f(A^c))))$. That is $int(cl(f(A))) \subseteq f(A)$. This implies f(A) is an IFSCS in Y and hence an IFγ*GCS in Y [7]. Therefore f is an IFγ*G closed mapping.

Theorem 3.24: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ*c$T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγ*G closed mapping

(ii)f(B) is an IFγ*GCS in Y for every IFCS B in X

(iii)$f(cl(B)) \supseteq int(cl(f(B)))$ for every IFS B in X

Proof: (i) ⇒ (ii) is obvious as $f(A^c) = (f(A))^c$ is a bijection mapping.

(ii) ⇒ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFγ*GCS in Y. Since Y is an IFγ*c$T_{1/2}$ space, f(cl(B)) is an IFCS in Y. Therefore $f(cl(B)) = cl(f(cl(B))) \supseteq int(cl(f(cl(B)))) \supseteq int(cl(f(B)))$.

(iii) ⇒ (i) Let A be an IFCS in X. By hypothesis, $f(cl(A)) = f(A) \supseteq int(cl(f(A)))$. This implies f(A) is an IFSCS in Y and hence an IFγ*GCS in Y. Therefore f is an IFγ*G closed mapping.

Definition 3.25: A mapping f: X→ Y is said to be an ***intuitionistic fuzzy γ* generalized open mapping*** (IFγ*G open mappinhg for short) if f(A) is an IFγ*GOS in Y for each IFOS A in X.

Theorem 3.26: Let f: (X, τ) → (Y, σ) be a mapping. Then the following are equivalent if Y is an IFγ*$T_{1/2}$ space:

(i)f is an IFγ*G open mapping

(ii)$f(int(A)) \subseteq \gamma int(f(A))$ for each IFS A of X

(iii)$int(f^{-1}(B)) \subseteq f^{-1}(\gamma int(B))$ for every IFS B of Y

Proof: (i) ⇒ (ii) Let f be an IFγ*G open mapping. Let A be any IFS in X. Then int(A) is an IFOS in X. (i) implies that f(int(A)) is an IFγ*GOS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(int(A)) is an IFγOS in Y. Therefore $f(int(A)) = \gamma int(f(int(A))) \subseteq \gamma int(f(A))$.

(ii) ⇒ (iii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. (ii) implies that $f(int(f^{-1}(B))) \subseteq \gamma int(f(f^{-1}(B))) \subseteq \gamma int(B)$. Now $int(f^{-1}(B)) \subseteq f^{-1}(f(int(f^{-1}(B)))) \subseteq f^{-1}(\gamma int(B))$.

(iii) ⇒ (i) Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. (iii) implies that $int(f^{-1}(f(A))) \subseteq f^{-1}(\gamma int(f(A)))$. Now $A = int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(\gamma int(f(A)))$. Therefore $f(A) \subseteq f(f^{-1}(\gamma int(f(A)))) \subseteq \gamma int(f(A)) \subseteq f(A)$. This implies $\gamma int(f(A)) = f(A)$. Hence f(A) is an IFγOS in Y. Since every IFγOS is an IFγ*GOS, f(A) is an IFγ*GOS in Y. Thus f is an IFγ*G open mapping.

Theorem 3.27: A mapping f: (X, τ) → (Y, σ) is an IFγ*G open mapping if $f(\gamma int(A)) \subseteq \gamma int(f(A))$ for every A ∈ X.

Proof: Let A be an IFOS in X. Then int(A) = A. Now $f(A) = f(int(A)) \subseteq f(\gamma int(A)) \subseteq \gamma int(f(A))$, by hypothesis. But $\gamma int(f(A)) \subseteq f(A)$. Therefore f(A) is an IFγOS in X. That is f(A) is an IFγ*GOS in X. Hence f is an IFγ*G open mapping.

Theorem 3.28: A mapping f: (X, τ) → (Y, σ) is an IFγ*G open mapping if and only if $int(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for every B ∈ Y, where Y is an IFγ*c$T_{1/2}$ space.

Proof: Necessity: Let B ∈ Y. Then $f^{-1}(B) \subseteq X$ and $int(f^{-1}(B))$ is an IFOS in X. By hypothesis, $f(int(f^{-1}(B)))$ is an IFγ*GOS in Y. Since Y is an IFγ*c$T_{1/2}$ space, $f(int(f^{-1}(B))$ is an IFOS in Y. Therefore $f(int(f^{-1}(B))) = int(f(int(f^{-1}(B)))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. This implies $int(f^{-1}(B)) \subseteq f^{-1}(f(int(f^{-1}(B)))) \subseteq f^{-1}(int(B))$.

Sufficiency: Let A be an IFOS in X. Therefore int(A) = A. Then $f(A) \subseteq Y$. By hypothesis $int(f^{-1}(f(A))) \subseteq f^{-1}(int(f(A)))$. That is $int(A) \subseteq int(f^{-1}(f(A))) \subseteq f^{-1}(int(f(A)))$. Therefore $A \subseteq f^{-1}(int(f(A)))$. This implies $f(A) \subseteq f(f^{-1}(int(f(A)))) \subseteq int(f(A)) \subseteq f(A)$. Hence f(A) is an IFOS in Y and hence an IFγ*GOS in Y. Thus f is an IFγ*G open mapping.

Theorem 3.29: Let (X, τ) be an IFTS where X is an IFγ*c$T_{1/2}$ space. An IFS A is an IFγ*GOS in X if and only if A is an IFN [9] of $p_{(\alpha, \beta)}$ for each $p_{(\alpha, \beta)} \in A$.

Proof: Necessity: Let $p_{(\alpha, \beta)} \in A$. Let A bean IFγ*GOS in X. Since X is an IFγ*c$T_{1/2}$ space, A is an IFOS in X. Then clearly A is an IFN [9] of $p_{(\alpha, \beta)}$ as $p_{(\alpha, \beta)} \in A \subseteq A$.

Sufficiency: Let $p_{(\alpha, \beta)} \in A$. Since A is an IFN of $p_{(\alpha, \beta)}$, there is an IFOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$. Now $A = \bigcup_{p(\alpha,\beta)\in A} p(\alpha,\beta) \subseteq \bigcup_{p(\alpha,\beta)\in A} B \subseteq A$. This implies $A = \bigcup_{p(\alpha,\beta)\in A} B$. Since each B is an IFOS, A is an IFOS and hence A is an IFγ*GOS in X.

Theorem 3.30: For any IFS A in an IFTS (X, τ) where X is an IFγ*c$T_{1/2}$ space, A ∈ IFγ* GO(X) if and only if for every IFP $p_{(\alpha, \beta)} \in A$, there exists an IFγ*GOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Proof: Necessity: If A ∈ IFγ*GO(X), then we can take B = A so that $p_{(\alpha, \beta)} \in B \subseteq A$ for every IFP $p_{(\alpha, \beta)} \in A$.

Sufficiency: Let A be an IFS in X and assume that there exists B ∈ IFγ*GO(X) such that $p_{(\alpha, \beta)} \in B \subseteq A$. Since X is an IFγ*c$T_{1/2}$ space, B is an IFOS of X. Then $A = \bigcup_{p(\alpha,\beta)\in A} p(\alpha,\beta) \subseteq \bigcup_{p(\alpha,\beta)\in A} B \subseteq A$. Therefore $A = \bigcup_{p(\alpha,\beta)\in A} B$ is an IFOS and hence A is an IFγ*GOS [7], in X. Thus A ∈ IFγ*GO(X).

Theorem 3.31: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ*c$T_{1/2}$ space. Then f is an IFγ*GOM if and only if for any IFP $p_{(\alpha, \beta)} \in Y$ and for any IFN of $f^{-1}(p_{(\alpha,\beta)})$, there is an IFN A of $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$.

Proof: Necessity: Let $p_{(\alpha,\beta)} \in Y$ and B be an IFN of $f^{-1}(p_{(\alpha,\beta)})$. Then there is an IFOS C in X such that $f^{-1}(p_{(\alpha,\beta)}) \in C \subseteq B$. Since f is an IFγ*G open mapping, f(C) is an IFγ*GOS in Y. Since Y is an IFγ*c$T_{1/2}$ space, f(C) is an IFOS in Y and $p_{(\alpha,\beta)} \in f(f^{-1}(p_{(\alpha,\beta)})) \subseteq f(C) \subseteq f(B)$. Put A = f(C). Then A is an IFN of $p_{(\alpha,\beta)}$ and $p_{(\alpha,\beta)} \in A \subseteq f(B)$. Thus $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$. That is $f^{-1}(A) \subseteq B$.

Sufficiency: Let $B \in X$ be an IFOS. If $f(B) = 0_\sim$ then there is nothing to prove. Suppose that $p_{(\alpha,\beta)} \in f(B)$. This implies $f^{-1}(p_{(\alpha,\beta)}) \in B$. Then B is an IFN of $f^{-1}(p_{(\alpha,\beta)})$. By hypothesis there is an IFN A of $p_{(\alpha,\beta)}$ such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$. Therefore there is an IFOS C in Y such that $p_{(\alpha,\beta)} \in C \subseteq A = f(f^{-1}(A)) \subseteq f(B)$.

Hence $f(B) = \cup\{ p_{(\alpha,\beta)} / p_{(\alpha,\beta)} \in f(B)\} \subseteq \cup\{C / p_{(\alpha,\beta)} \in f(B)\} \subseteq f(B)$. Thus $f(B) = \cup\{C / p_{(\alpha,\beta)} \in f(B)\}$. Since each C is an IFOS, f(B) is also an IFOS and hence is an IFγ*GOS in Y. Therefore f is an IFγ*G open mapping.

Theorem 3.32: If f: (X, τ) → (Y, σ) is a bijective mapping, then the following are equivalent:

(i)f is an IFMγ*G closed mapping

(ii)f(A) is an IFγ*GCS in Y for every IFγ*GCS A in X

(iii)f(A) is an IFγ*GOS in Y for every IFγ*GOS A in X

Proof: (i) ⇔(ii) is obvious from the Definition 3.1.

(ii) ⇒ (iii) Let A be an IFγ*GOS in X. Then A^c is an IFγ*GCS in X. By hypothesis, $f(A^c)$ is an IFγ*GCS in Y. That is $f(A)^c$ is an IFγ*GCS in Y and hence f(A) is an IFγ*GOS in Y as f is a bijective mapping.

(iii) ⇒ (i) Let A be an IFγ*GCS in X. Then A^c is an IFγ*GOS in X. By hypothesis, $f(A^c)$ is an IFγ*GOS in Y. That is $f(A)^c$ is an IFγ*GOS in Y and hence f(A) is an IFγ*GOS in Y as $f(A^c) = (f(A))^c$. Hence f is an IFMγ*G closed mapping.

Theorem 3.33: If f: (X, τ) → (Y, σ) is a bijective mapping and Y is an IFγ*$T_{1/2}$ space then the following are equivalent:

(i) f is an IFMγ*G closed mapping

(ii)f(A) is an IFγ*GOS in Y for every IFγ*GOS A in X

(iii)for every IFP $p_{(\alpha,\beta)} \in Y$ and for every IFγ*GOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \subseteq B$, there exists an IFγ*GOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$

Proof: (i) ⇒ (ii) is obvious by Theorem 3.32.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in Y$ and let B be an IFγ*GOS in X such that $f^{-1}(p_{(\alpha,\beta)}) \subseteq B$. This implies $p_{(\alpha,\beta)} \in f(B)$. By hypothesis, f(B) is an IFγ*GOS in Y. Let A = f(B). Therefore $p_{(\alpha,\beta)} \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) ⇒ (i) Let B be an IFγ*GCS in X. Then B^c is an IFγ*GOS in X. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \subseteq B^c$. This implies $p_{(\alpha,\beta)} \in f(B^c)$. By hypothesis there exists an IFγ*GOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B^c$, then $A = f(f^{-1}(A)) \subseteq f(B^c)$. Therefore $p_{(\alpha,\beta)} \in f(B^c)$. Hence by [7], $f(B^c)$ is an IFγ*GOS in Y. As f is a bijective, $f(B^c) = (f(B))^c$. Therefore f(B) is an IFγ*GCS in Y. Thus f is an IFMγ*G closed mapping.

Theorem 3.34: If f: (X, τ) → (Y, σ) is a bijective mapping, Where X and Y are IFγ*$T_{1/2}$ spaces then the following are equivalent:

(i) f is an IFMγ*closed mapping

(ii)f(A) is an IFγ*GOS in Y for every IFγ*GOS A in X

(iii)f(γint(B)) ⊆ γint(f(B)) for every IFS B in X

(iv)γcl(f(B)) ⊆ f(γcl(B)) for every IFS B in X

Proof: (i) ⇒ (ii) is obvious.

(ii) ⇒ (iii) Let B be any IFS in X. Since γint(B) is an IFγOS, it is an IFγ*GOS in X. Then by hypothesis, f(γint(B)) is an IFγ*GOS in Y. Since Y is an IFγ*$T_{1/2}$ space, f(γint(B)) is an IFγOS in Y. Therefore f(γint(B)) = γint(f(γint(B))) ⊆ γint(f(B)).

(iii) ⇒ (iv) can easily proved by taking complement in (iii).

(iv) ⇒ (i) Let A be an IFγ*GCS in X. By hypothesis, γcl(f(A)) ⊆ f(γcl(A)). Since X is an IFγ*$T_{1/2}$ space, A is an IFγCS in X. Therefore, γcl(f(A)) ⊆ f(γcl(A)) = f(A) ⊆ γcl(f(A)). Hence f(A) is an IFγCS in Y and hence an IFγ*GCS in Y. Thus f is an IFMγ*G closed mapping.

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