Anisotropic Cosmological Model of Cosmic String with Bulk Viscosity in Lyra Geometry

R.N.Patra
P.G. Department of Mathematics, Berhampur University, Odisha(India)

Abstract
A four dimensional spherically symmetric cosmological model with cosmic string and bulk viscosity in Lyra Geometry has been considered. The solution for cloud string model $\rho + \lambda_s = 0$ *yields that, the spatial volume decreases with increase of time, i.e. the universe is contracting with a constant deceleration and shows anisotropy throughout the evolution. Themodel is non-rotating, not a recurrent space and also not a space of constant curvature in generalized curved space.*

Keywords: *Spherically Symmetric, Lyra Geometry, Cosmic String, Bulk Viscosity, Recurrent Space, Space of Constant Curvature, Deceleration Parameter.*

1. Introduction:

Einstein's idea of geometrizing gravitation in general theory of relativity motivated others to geometrize other physical fields. Lyra [1] proposed a modification of Riemannian geometry in which he introduced a gauge function to remove the non-integrability of lengh of a vector under paralled transport. Various Cosmological models in Lyra manifold has been constructed by Rahaman[2], singh [3], Mohantry [4] and Reddy [5].

The study of string theory is important in the early stage of evolution of the universe before the particle creation. Cosmic strings are considered as two lines of concentrated energy forming a tangled web permitting the entire universe with closed loops. Kibble [6] Vilenkin [7] believed that strings may be one source of density for large scale structures of the universe. The desirable thing for the string models is that, either the strings fade away at a certain epoch of cosmic evolution or it has a particle dominated future asymptote with barely visible strings. This fact attracted many researchers for further studies.

2. Field Equation and the Cosmological Model:

Here we have considered the four dimensional spherically symmetric metric in the form

$$ds^2 = -e^{\lambda}\, dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\mu}\, dt^2 \qquad (1)$$

Where λ and μ are functions of 't' only. Assuming the coordinates to be co-moving i.e. $u_1 = u_2 = u_3 = 0$ and $u_4^2 = g_{44}$ and the displacement vector in the form

$$\phi_i = (\beta, 0, 0, 0) \qquad (2)$$

(where β is a constant), the energy momentum tensor T_{ij} for cosmic string with bulk viscosity is

$$T_{ij} = \rho\, u_i u_j - \lambda_s x_i x_j - \xi\theta(u_i u_j - g_{ij}), \qquad (3)$$

where

$\rho \rightarrow$particle density

λ_s →String tenson density

ξ → Bulk viscous coefficient

u^i → Four velocity vector

g_{ij} → covariant fundamental tensor

x^i →direction of anisotropy of cosmic string

satisfying $u_i u^i = -x_i x^1 = 1$ and $u_i x^i = 0$. (4)

The Einstein's field equation based on Lyra's manifold is given by

$$R_{ij} - \frac{1}{2} Rg_{ij} - \frac{3}{2}\phi_i\phi_j - \frac{3}{4} g_{ij}\ \phi_m\phi^m = -\chi\ T_{ij} \quad , \tag{5}$$

where $R \rightarrow Rici$ Scalar.

Using equations (2),(3) and(4) the explicit form of field equation (5) for the line element (1) are obtained as

$$-\frac{1}{4}e^{\lambda-\mu}\lambda_4^2 - \frac{e^{\lambda-\mu}}{8}\lambda_4\mu_4 - \frac{3}{2}\beta^2 + \frac{3}{4}e^{\lambda}\beta^2 = -\chi\lambda_s + \chi\xi\theta\ e^{\lambda} \tag{6}$$

$$-\frac{\lambda_4}{r} = \frac{3}{2}\beta^2 \tag{7}$$

$$\frac{r^2}{2}\lambda_{44}e^{-\mu} + \frac{r^2}{2}\lambda_4^2 e^{-\mu} - \frac{r^2}{8}\lambda_4\mu_4 e^{-\mu} - \frac{3}{2}\beta^2 + \frac{3}{4}r^2\beta^2 = \chi\xi\theta r^2. \tag{8}$$

$$-\frac{3}{2}\beta^2 cosec^2\theta + \frac{3}{4}r^2\beta^2 = \chi\xi\theta r^2. \tag{9}$$

$$-\frac{1}{8}\lambda_4\mu_4 e^{-\mu} + \frac{e^{-\lambda}}{r^2} - \frac{1}{r^2} - \frac{9}{4}\beta^2 = -\chi\rho. \tag{10}$$

3. Cosmological Solution:

From equation (7), we get the value of 'λ' as

$$\lambda = \frac{-3}{2}\beta^2 rt + c \quad , \tag{11}$$

where 'c' is the constant of integration.

Adding equation (6) and (10), we get

$$-\frac{1}{4}\lambda_4^2 e^{\lambda-\mu} - \frac{e^{\lambda-\mu}}{8}\lambda_4\mu_4 - \frac{1}{8}\lambda_4\mu_4 e^{-\mu} + \frac{e^{-\lambda}}{r^2} - \frac{15}{4}\beta^2 - \frac{1}{r^2} + \frac{3}{4}\beta^2 e^{\lambda} = -\chi(\lambda_s + \rho) + \chi\xi\theta e^{\lambda} \tag{12}$$

Special Case:

Cloud String Model $(\rho + \lambda_s = 0)$:

Using this condition in equation (12) and taking 'e^{λ}' common from both the sides, we get

$$-\frac{1}{4}\lambda_4^2 e^{-\mu} - \frac{1}{8}\lambda_4\mu_4 e^{-\mu} - \frac{1}{8}\lambda_4\mu_4 e^{-\mu-\lambda} + \frac{e^{-2\lambda}}{r^2} - \left(\frac{15}{4}\beta^2 + \frac{1}{r^2}\right)e^{-\lambda} + \frac{3}{4}\beta^2 = \chi\xi\theta \qquad (13)$$

Multiplying equation (12) with 'r^2' and equating its LHS of equation (9) and using

'$\lambda_{44} = 0$' from equation (11), we get

$$\frac{3}{4}\lambda_4^2 e^{-\mu} + \frac{r^2}{8}\lambda_4\mu_4 e^{-\mu-\lambda} = -\left(\frac{15}{4}r^2\beta^2 + 1\right)e^{-\lambda} + e^{-2\lambda} + \frac{3}{2}\beta^2. \qquad (14)$$

Putting the value of 'λ' from equation (12) in equation (14), we get

$$-\frac{27}{16}r^2\beta^4 e^{-\mu} - \frac{3}{16}r^3\beta^2\mu_4 e^{\left(-\mu+\frac{3}{2}\beta^2 r\, t-c\right)} = -\left(\frac{15\, r^2\beta^2+4}{4}\right)e^{\left(\frac{3}{2}\beta^2 r\, t-c\right)} + e^{(3\beta^2 r\, t-2c)} + \frac{3}{2}\beta^2 \qquad (15)$$

To avoid the complicacy in finding the solution,

let us take, $\mu_4 = 0$ or $\mu = K$(constant) (16)

for a particular case $t = \left[\frac{\ln\left(\frac{27}{16}r^2\beta^4 e^{-K}\right)-\frac{3}{2}\beta^2}{\left(\frac{-15\, r^2\beta^2+4}{4}\right)} + c\right]\frac{2}{3r\,\beta^2}$.

Using equation (12) and (16) the metric (1) takes the form

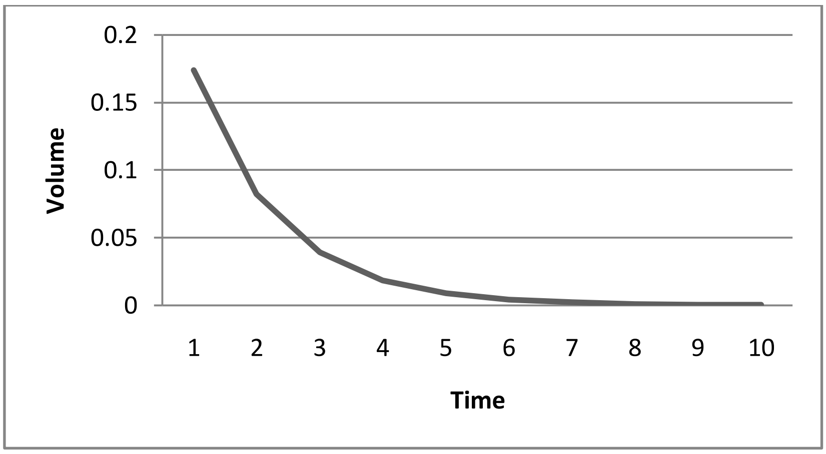
$$ds^2 = -e^{\left(-\frac{3}{2}r^2\beta\, t+c\right)}dr^2 - r^2\, d\theta^2 - r^2\sin^2\theta\, d\phi^2 + e^K\, dt^2. \qquad (17)$$

4. Physical and Geometrical Properties:

1. Taking β = constant, at initial epoch $(t = 0)$, the metric (11) becomes flat. As 't' increases, the first dimension contracts, but the others don't change.

2. Volume$(V) = (-g)^{\frac{1}{2}} = e^{\left(-\frac{3}{4}\beta^2 r\, t+\frac{K'}{2}\right)}r^2\, \sin\theta$, where $K' = c + K$ =constant.

Here we observe that, the spatial volume of the universe decreases with the increase ofcosmic time and may collapse shortly which has shown in the $(V \sim t)$graph.



3. The mean anisotropy parameter$(A) = \frac{1}{2}\sum_{i=1}^{2}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{1}{2}$ i.e.$A \neq 0$. Hence the model show anisotropy through out the evolution.

4. The model of the universe is non rotating since the verticity tensor $\left(w_{ij}\right)=0$

5. It is found that $R_{ij} \neq \frac{1}{4} Rg_{ij}$. Hence the space of the string model is not a Einstein's space.

6. the space of constant curvature is given by

$$R_{hijk} = \bar{k}\left(g_{hj}\ g_{ik} - g_{hk}\ g_{ij}\right) \tag{18}$$

where

$$R_{hijk} = \frac{1}{2}\left[\frac{r^2 g_{hk}}{rx^i\ rx^j} + \frac{\partial^2\ g_{ij}}{\partial x^h\ rx^k} - \frac{r^2 g_{hj}}{\partial x^2 \partial x^k} - \frac{\partial^2 g_{ik}}{rx^j rx^j}\right]$$

$$+ g_{ab}\left[\Gamma^a_{ij}\ \Gamma^b_{hk} - g_{ab}\ \Gamma^a_{ik}\ \Gamma^b_{hj}\right]$$

= Riemannian curvature tensor.

$\overline{K}$ is a constant known as the curvature of Riemannian manifold.

We found that, the value of $\overline{K}$ after comparing the existing values of L.H.S. and R.H.S. of equation (18) is not an unique constant. So our string space time in Lyra's manifold is not a space of constant curvature.

7. Recurrent Space:

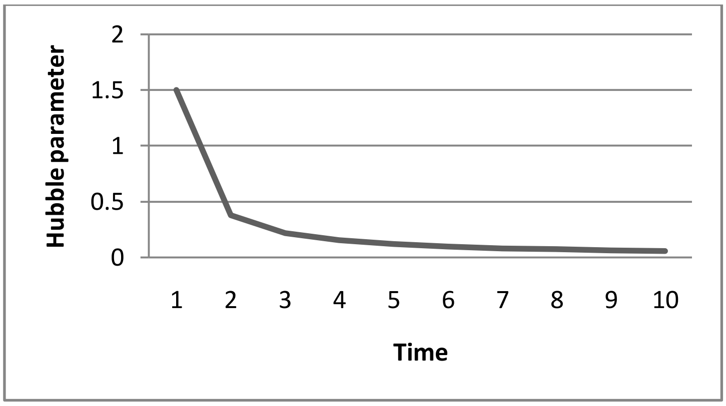
$$R_{hijk}, l = \overline{K}_e\ R_{hijk}$$

For our matric, the covariant derivatives of the existing components of Riemannian curvature tensor vanishes. Which implies that, ultimately the corresponding values of $\overline{K}_l$ are also zero,

$$K_l = \left(k_1, k_2, k_3, k_4\right) = \left(0,0,0,0\right).$$

Hence the matrics is not a recurrent space.

8.The Hubbleparameter (H) $= \frac{1}{2}(H_1 + H_2) = \frac{1}{2}\left(\frac{\lambda_4}{\lambda} + \frac{\mu_4}{\mu}\right) = \frac{\beta^2 r}{2\left(\beta^2 r\ t - \frac{2}{3}c\right)}$, which is a function of 't'. The $(H \sim t)$ graph shows the necessary variation.



9. The scalar expansion $(\theta) = 2H = \frac{\beta^2 r}{\beta^2 r\, t - \frac{2}{3}c}$, which means that the volume expansion depend on the cosmic time.

10. The deceleration parameter 'q' is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 1, \text{ as } 1 > 0,$$

the model of the universe is a decelerating one, where the spatial volume decreases with the increase in cosmic time.

5.Conclusion:-We havestudied various physical and geometrical properties of a fourdimensional cosmological model of cosmic string with bulk viscosity in Lyra geometry. We found the exact cosmological solution for the value of 'λ' but using cloud string model 'μ' is found to be independent of cosmic time. The verticity tensor, $w_{ij} = 0$ indicates the non-rotating nature of the universe. The deceleration parameter (q) is grater than zero shows that the spatial volume decreases uniformly with 't' and ultimately the universe may collapse at some time. The mean anisotropy parameter 'A' is not equal to zero shows the anisotropy nature of the model. Also it is observed that the universe is not a space of constant curvature, neither it is an Einstein's space nor a recurrent space.

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