String Cosmological Model in a New Scalar Tensor Theory of Gravitation

R.N.Patra
P.G. Department of Mathematics, Berhampur University, Odisha (India)

Abstract
Explicit field equation of a new scalar tensor theory of gravitation proposed by the Sen-Dunn theory are obtained with the help of a four dimensional spherically symmetric metric in the context of cosmic string. The solution of the various field equations are obtained for geometric strings $(\lambda = \rho)$. *Some physical and geometric properties of the solutions are also discussed.*

Key words: *New scalar tensor theory, geometric string model, spherically symmetric.*

1. Introduction:

It is well known that a gravitational scalar field, besides the metric of the space time, must exist in the frame work of the present unified theories. Hence there has been much interest in scalar tensor theories of gravitation. Several theories are proposed as alternatives to Einstein's theory to reveal the nature of the universe in the early stage of evolution. The most important among them were scalar tensor theories proposed by Lyra[1], Branspicke[2], Reddy and Rao[3], Mohanty and Mohanta[4], Pradhan[5]. Sen and Dunn[6] have proposed a new scalar tensor theory of gravitation in which both the scalar and tensor fields have intrinsic geometrical significance. The scalar field in this theory is characterized by the function $\phi = \phi(x^i)$ where 'x^i' are co-ordinates in four-dimensional Lyra manifold and tensor field is identified with the metric tensor 'g_{ij}' of the manifold. The field equations given by Sen and Dunn [6] for the combined scalar and tensor field are

$$R_{ij} - \frac{1}{2}Rg_{ij} = \omega\phi^{-2}\left(\phi_{;i}\phi_{;j} - \frac{1}{2}g_{ij}\,\phi_{;k}\phi^{;k}\right) - \phi^{-2}T_{ij} \quad (1)$$

Where 'ω' is dimension less constant, 'R_{ij}' and 'R' are the usual Ricci-tensor and Riemann curvature scalar respectively.

The study of string theory is important in the early stages of the evolution of the universe before the creation of the particles. Cosmic strings have received considerable attention in cosmology as they are believed to give rise density perturbations leading to the formation of galaxies. The general relativistic treatment of strings was initiated by Letelier [7], Pradhan and Mathur [8] and others who have studied about string cosmologies in the theory of relativity, Recently Patra and Sethi [9] have investigated on Chplygin gas in Lyra geometry on bianchi type-III models in bulk viscous fluid.

The string that form the cloud and massive strings instead of geometric string. Each massive string is formed by a geometric string with particles attached along its extension. In this paper we have discussed the various physical and geometrical properties of a geometric string model, found as a result of the application of Sen-Dunn theory of gravitation in a four-dimensional spherically symmetric space-time in presence of a cloud massive string. The last section contains the concluding remarks.

2. Metric and field equation:

Here we consider the four-dimensional spherically symmetric metric of the form

$$dS^2 = -e^{\lambda}dr^2 - r^2d\theta^2 - r^2sin^2\theta d\phi^2 + e^{\mu}dt^2 \quad (2)$$

where $\lambda(t)$ and $\mu(t)$ are cosmic scale factors. The energy momentum tensor for cloud of massive string can be written as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \quad (3)$$

Where 'ρ' is the rest energy density of the cloud of strings with particles attached to them. 'λ' is the tensor density of the strings and $\rho = \rho_p + \lambda$, ρ_p is the energy density of the particles. The velocity 'u^i' describes the four-velocity, which has components $\left(0,0,0,e^{\frac{\mu}{2}}\right)$ for a cloud of particles and $x^i = \left(-e^{\frac{\lambda}{2}},0,0,0\right)$ represents the direction of strings 'x_i' which will satisfy

$$u^i u_j = -x^i x_j = 1 \;\; and \;\; u^i x_i = 0 \qquad (4)$$

$$T_{11(String)} = \lambda, T_{44\,(String)} = \rho \; and \;\; T_{22} = T_{33} = 0 \qquad (5)$$

The field equations for metric (2) using equation (1) and (5) can be written as

$$-\frac{1}{4}e^{\lambda-\mu}\lambda_4^2 - \frac{e^{\lambda-\mu}}{8}\lambda_4\mu_4 = \frac{e^{\lambda}\phi_4^2\omega}{2\phi^2} - \frac{\lambda}{\phi^2} \qquad (6)$$

$$-\frac{\lambda_4}{r} = 0 \Rightarrow \lambda_4 = 0 \;\; or \lambda = k_1 = constant \qquad (7)$$

$$\frac{r^2}{2}e^{-\mu}\left(\lambda_{44} + \lambda_4^2 - \frac{\lambda_4\mu_4}{4}\right) = \frac{\omega}{\phi^2}\left(\frac{r^2}{2}\phi_4^2\right) \qquad (8)$$

and $$-\frac{\lambda_4\mu_4}{8} + \frac{e^{\mu-\lambda}}{r^2} - \frac{e^{\mu}}{r^2} = \frac{\omega\phi_4^2}{2\phi^2} - \frac{\rho}{\phi^2}, \qquad (9)$$

where the subscript '4' represents the differentiation with respect to 't'.

3. Solution to field equations:

Using equation (7) in equation (8) we get,

$$\frac{\omega r^2\phi_4^2}{2\phi^2} = 0 \qquad (10)$$

$$\Rightarrow \phi_4^2 = 0 \Rightarrow \phi = constant = k_2 \qquad (11)$$

Using equation (10) and (7) in equation (9) we get,

$$\frac{e^{\mu}}{2}(e^{-k_1} - 1) = -\frac{\rho}{\phi^2} \qquad (12)$$

$$\Rightarrow \mu = ln\left[\frac{r^2\rho}{k_2^2}(e^{-k_1} - 1)\right] \qquad (13)$$

The simplest relation between 'ρ' and 'λ' is $\rho = \beta\lambda$ where 'β' is a proportionality constant which gives rise to the following two cases.

a. For $\boldsymbol{\beta = 1}$, we get geometric strings or Nambu string.
b. For $\boldsymbol{\beta = -1}$, we get massive strings.

3.1. Case-I: Geometric string($\rho = \lambda$)

From equation (7), $\lambda = k_1 \Rightarrow \lambda = \rho = k_1$

Using the value of 'ρ' in equation (13), we get

$$\mu = ln\left[\frac{r^2k_1}{k_2^2}(e^{-k_1} - 1)\right] \qquad (14)$$

Using equation (7) and (14) in the metric (2) takes the form

$$dS^2 = -e^{k_1}dr^2 - r^2d\phi^2 - r^2sin^2\theta d\phi^2 + e^{ln\left[\frac{r^2k_1}{k_2^2}(e^{-k_1}-1)\right]}dt^2 \qquad (15)$$

3.2. Case-II:

Massive strings ($\rho + \lambda = 0$)

Using equation (10) and (7) in equation (6), we get

$$0 = \frac{\lambda}{\phi^2} \qquad (16)$$

Adding equation (12) and (16) ,we get

$$\frac{e^{\mu}}{r^2}(e^{-k_1} - 1) = -\frac{1}{\phi^2}(\lambda + \rho) = 0 \qquad (17)$$

$$\Rightarrow e^{\mu} = 0. \qquad (18)$$

Putting equation (7) and (17), the metric (2) takes the form

$$dS^2 = -e^{k_1}dr^2 - r^2d\theta^2 - r^2sin^2\theta d\phi^2 \quad (19)$$

That means the model reduces to three dimensional space without cosmic time which is meaningless. Here we see that the cosmic massive string do not co-exist with the scalar field in this theory.

4. Physical and geometrical properties:

1. We found that $\boldsymbol{\phi = k_2 = constant}$. This suggests that the scalar field is not a function of cosmic time.

2. The kinematical parameters '$\boldsymbol{\rho}$' and '$\boldsymbol{\lambda}$' are constant ($\boldsymbol{\rho = \lambda = k_1}$) and are independent of time. Also '$\boldsymbol{\mu}$' is found to be independent of '$\boldsymbol{t}$'.

3. Volume $(\boldsymbol{V}) = (-g)^{\frac{1}{2}}$

$$= e^{\frac{1}{2}\left[k_1 + ln\frac{r^2c}{k_2^2}(e^{-k_1}-1)\right]}r^2sin\theta$$

i.e. The volume expansion is independent of cosmic time but depend on parameter '$\boldsymbol{r}$'.

Taking $\boldsymbol{k_1 = k_2 = 1, \theta = 90^0}$ the $\boldsymbol{V \sim r}$ graph has been plotted.

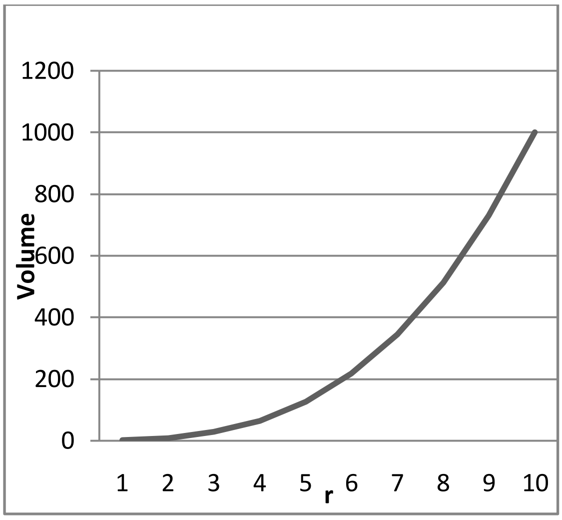

Fig.1 variation of volume expansion verses

4. Hubble parameter $(H) = \frac{1}{2}(H_1 + H_2) .=$ $\frac{1}{2}\left(\frac{\lambda_4}{\lambda} + \frac{\mu_4}{\mu}\right) = 0$

5. The expansion scalar in commoving co-ordinates is,$(\theta) = 2H = 0$.

6. The means the universe becomes isotropic throughout its evolution.

7. Shear scalar (σ^2) is defined as

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{2} H_i^2 - 2\,H^2\right] = AH^2 = 0$$

$$\Rightarrow \frac{\sigma}{\theta} = 0$$

Which confirms that the model is isotropic in nature.

8. Deceleration parameter (q) is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 0$$

Which shows, neither the model is accelerating as $q \nless 0$ nor decelerating as $q \ngtr 0$ but the volume expansion is independent of time

5. Conclusion:

In this paper we have considered a four-dimensional spherically symmetric space time in Sen-Dunn theory of gravitation in context of cosmic strings. The solution of the field equations is obtained for geometric strings $(\lambda = \rho)$ only. It is observed that the scalar field (ϕ) and the kinematical parameters ρ, λandμ are not the functions of cosmic time 't'. The model is isotropic and the volume expansion is independent of time.

6. References:

1. G. Lyra, Math. Z. 54, 52 (1951).
2. C. Brans and R. H. Dieke, Physics Rev. 124, 925 (1961).
3. D. R. K. Reddy and M. V. S. Rao, Astrophysics, Space Science 302, 157 (2006).
4. G. Mohanty and K. L. Mahanty, Astrophysics, Space Science 312, 301 (2007).
5. A. Pradhan, J. Math. Physics 50, 022501 (2009).
6. D. Math. Physics 12, 578 (1971).
7. P. S. Letelier, Physics Rev. 020, 1294 (1979).
8. A. Pradhan and P. Mathur. Astrophysics. Space Science 318, 255 (2008).
9. R.N. Patra and A.K Sethi, Int. J.Pure and Applied Physics 13(3),pp.289-299(2017)

R.N. Patra: At present he is serving as assistant professor in P.G. Dept. of Mathematics, Berhampur University. He holds Ph.D degree in mathematics from Berhampur University. His ares of research is General Relativity and Cosmology. He has published research papers in reputed international journal such as African Review of Phy., Int.J. Phy and Math. Sc.,I.J. Phy and Appl. etc and author of some books also. Besides this, his specialization subjects are Functional analysis and Algebra. He has guided several M.Phil and Ph.D students. Currently, he is a life member of IMS and OMS. Also he is a member of Board of studies of Mathematics in the university. Besides this, he is reviewer of International Journal of Theoretical Physics.