Primary and Secondary Velocity Profiles for Different Hartmann Number (H_a)

S. Sreedhar

Asst. Professor, Dept. of Applied Mathematics, GITAM Institute of Science,
GITAM University, Visakhapatnam. (A.P), India.

Abstract *- The advent of technology that involves the MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting fluids. The use of liquid metals as heat transfer agents and as a working fluid in MHD power generator has created a growing interest in the behavior of liquid metal flows and in particular the nature of interaction with magnetic field. The interaction between the conducting fluid and the magnetic field radically modifies the flow, with attendant effects on such important flow properties as pressure drop and heat transfer, the detailed nature of which is strongly dependent on the orientation of the magnetic field relative to the field. It is assumed that the fluids in the two regions are incompressible, immiscible and electrically conducting, having different viscosities, electrical conductivities. With these assumptions and considering that the magnetic Reynolds number is small the basic equations of motion, current, the no-slip boundary conditions at the walls and interface conditions between the two-fluid regions have been formulated. The resulting governing linear differential equations are solved analytically, using the prescribed boundary and interface conditions to obtain the exact solutions for velocity distributions such as primary and secondary distributions in both regions. Also, their corresponding numerical results for various sets of values of the governing parameters are obtained to represent them graphically and are discussed in detail.*

Key Words*: Hartmann Number, MHD, Non Conducting Fluids, Two Phases, Viscous.*

Basic governing equations with boundary and interface conditions and mathematical analysis of the problem:

The fundamental equations to be solved are the equations of motion and current for the steady state two-fluid flow of neutral fully–ionized gas valid under assumptions given below and simplified as:

(i) The ionization is in equilibrium which is not affected by the applied electric and magnetic fields.
(ii) The effect of space charge is neglected.
(iii) The flow is fully developed and stationary, that is $\partial/\partial t = 0$ And $\partial/\partial x = 0$ except $\partial p/\partial x \neq 0$.
(iv) The magnetic Reynolds number is small [so that the externally applied magnetic field is undisturbed by the fluid, namely the induced magnetic field is small compared with the applied field [Shercliff (1965)]. Therefore components in the conductivity tensor are expressed in terms of B_0.
(v) The flow is two-dimensional, namely $\partial/\partial z = 0$.

With these assumptions, the governing equations of motion and current can be formulated as follows for the two-dimensional steady state problem of study in two regions.

Region – I

$$-\left[1-s\left(1-\frac{\sigma_{11}}{\sigma_{01}}\right)\right]\frac{\partial p}{\partial x}+\rho_1\nu_1\frac{d^2u_1}{dy^2}+B_0[-\sigma_{11}(E_z+u_1B_0)+\sigma_{21}(E_x-w_1B_0)]=0,$$

$$\left(s\,\frac{\sigma_{21}}{\sigma_{01}}\right)\frac{\partial p}{\partial x}+\rho_1\nu_1\frac{d^2w_1}{dy^2}+B_0[\sigma_{11}(E_x-w_1B_0)+\sigma_{21}(E_z+u_1B_0)]=0,$$

Region – II

$$-\left[1-s(1-\frac{\sigma_{12}}{\sigma_{02}})\right]\frac{\partial p}{\partial x}+\rho_2\nu_2\frac{d^2u_2}{dy^2}+B_0[-\sigma_{12}(E_z+u_2B_0)+\sigma_{22}(E_x-w_2B_0)]=0,$$

$$(s\ \frac{\sigma_{22}}{\sigma_{02}})\frac{\partial p}{\partial x}+\rho_2\nu_2\frac{d^2w_2}{dy^2}+B_0[\sigma_{12}(E_x-w_2B_0)+\sigma_{22}(E_z+u_2B_0)]=0$$

In the above equations, the subscripts 1 and 2 refer to the quantities for region -I and II respectively, such as u_1, u_2 and w_1, w_2 known as primary and secondary velocity distributions in the two regions respectively. E_x and E_z, also J_x and J_z, are x- and z- components of electric field, also current densities respectively, s = p_e/p is the ratio of the electron pressure to the total pressure. The value of s is 1/2 for neutral fully–ionized plasma and approximately zero for a weakly–ionized gas.

While, σ_{11}, σ_{12} and σ_{21}, σ_{22} are the modified conductivities parallel and normal to the direction of electric field respectively

The boundary condition on velocity requires the no slip condition. In addition, the fluid velocity and sheer stress must be continuous across the interface y=0. The boundary and interface conditions for u_1, w_1 and u_2, w_2 are:

$$u_1(h_1)=0,\ w_1(h_1)=0,\ u_2(-h_2)=0,\ w_2(-h_2)=0,\ \ u_1(0)=u_2(0), w_1(0)=w_2(0),$$

$$\mu_1\frac{du_1}{dy}=\mu_2\frac{du_2}{dy}\quad \text{And}\quad \mu_1\frac{dw_1}{dy}=\mu_2\frac{dw_2}{dy}\quad at\quad y=0$$

To make equations dimensionless, we use the following non-dimensional variables:

$u^{\bullet}_1=\frac{u_1}{u_p}$, $u^{\bullet}_2=\frac{u_2}{u_p}$, $y^{\bullet}_i=\left(\frac{y_i}{h_i}\right)(i=1,2)$, $w^{\bullet}_1=\frac{w_1}{u_p}$, $w^{\bullet}_2=\frac{w_2}{u_p}$, $u_p=\left(-\frac{\partial p}{\partial x}\right)\frac{h_1^2}{\rho_1\nu_1}$, $k_1=1-s(1-\frac{\sigma_{11}}{\sigma_{01}})$, $k_2=-s\frac{\sigma_{21}}{\sigma_{01}}$, $m_x=\frac{E_x}{B_0u_p}$, $m_z=\frac{E_z}{B_0u_p}$, $I_x=\frac{J_x}{\sigma_{0i}B_0u_p}$, $I_z=\frac{J_z}{\sigma_{0i}B_0u_p}(i=1,2)$, H_a^2 (Hartmann number) $=B_0^2h_1^2\left(\frac{\sigma_{01}}{\rho_1\upsilon_1}\right)$, α (ratio of the viscosities) = $\frac{\mu_1}{\mu_2}$, h(ratio of the heights)=$\frac{h_2}{h_1}$, σ_0 (Ratio of the electrical conductivities) $=\frac{\sigma_{01}}{\sigma_{02}}, \sigma_1=\left(\frac{\sigma_{12}}{\sigma_{11}}\right)$ $\sigma_2=\left(\frac{\sigma_{22}}{\sigma_{21}}\right)$; $\frac{1}{1+m^2}=\frac{\sigma_{11}}{\sigma_{01}}$, $\frac{m}{1+m^2}=\frac{\sigma_{21}}{\sigma_{01}}$, m(Hall parameter) $=\left(\frac{w_e}{\frac{1}{\tau}+\frac{1}{\tau_e}}\right)$.

In which ω_e is the gyration frequency of electron, τ and τ_e are the mean collision time between electron and ion, electron and neutral particles respectively. Also, the above expression for Hall parameter 'm' which is valid in the case of partially–ionized gas agrees with that of fully–ionized gas when τ_e approaches infinity.

Solutions of the problem

Exact solutions of the governing differential equations with the help of boundary and interface conditions for the primary and secondary velocities u_1, u_2 and w_1, w_2 respectively. The numerical values of the expressions given at equations and computed for different sets of values of the governing parameters involved in the study and these results are presented graphically from figures 1and 2, also discussed in detail.

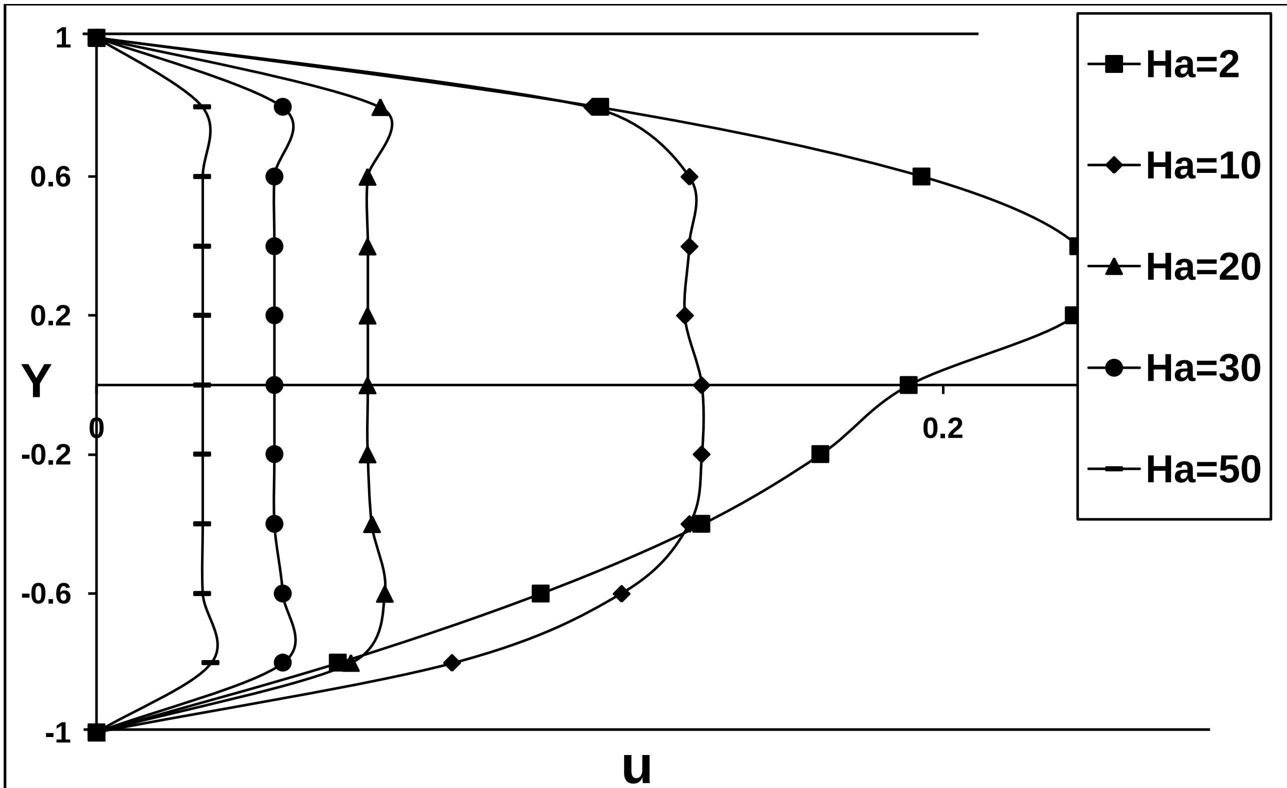

Fig.1 Primary Velocity profiles for different H_a and α=0.333, $\sigma_0$2, σ_1=1.2, σ_2=1.5, ρ=1, h=0.8, m=2.

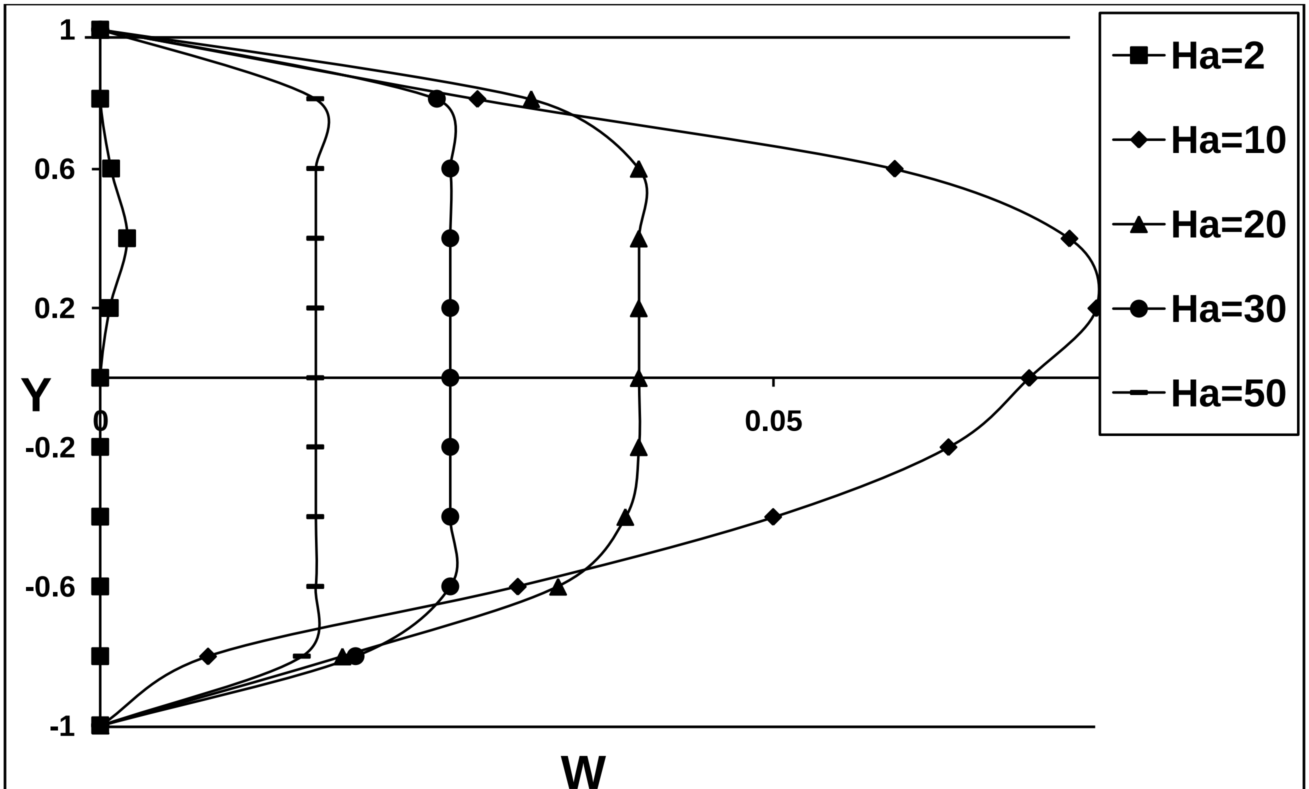

Fig.2 Secondary Velocity profiles for differet H_a and α=0.333, σ_0=2,σ_1=1.2,σ_2=1.5, h=0.8, m=2.

The effect of varying the Hartmann number H_a on velocity distribution in the two regions is shown in figures 1 and 2 respectively. From fig.1, it is observed in both the regions that, an increase in Hartmann number diminishes the primary velocity distribution, while it enhances the secondary velocity distribution for lower Hartmann numbers, say up to 10 and diminishes beyond this number (fig.2). Also, the maximum velocity in the channel tends to move above the channel centerline towards region–I for Hartmann number H_a =2 when all the remaining parameters are fixed in the case of primary velocity. This type of effect can also be observed in case of secondary velocity distribution at H_a=10.

References

Chan, C.K. (1979) Nuclear Engg., and Design, **51**, p.253.

Chandrasekhar, S. (1961) 'Hydrodynamic and Hydro-magnetic stability' (Oxford Univ. Press).

Chang,C.C. and Yen,J.T. (1959) Phys.Fluids, **2**, p.393.

Chao, J. Mikic, B.B. and Todreas, N.E. (1979) Nuclear Technology, **42**, p.22.

Charles, M. E., Govier, G.W. and Hodgson, G.W. (1961) Can. J. chem. Engg., **39**, p.27.

Charles, M. E. and Lilleleht, L.U. (1965) Can. J. chem. Engg., **43**, p.110.

Charles, M. E. and Redberger, P.J. (1962) Can. J. chem. Engg., **40**, p.70.

Chen, T.S, Tien, H.C and Armaly, B.F (1986) Int. J. Heat Mass Transfer, **29,** No.10, pp.1465.

Cowling, T.G. (1957) 'Magnetohydrodynamics', Interscience, New York.

Cramer, K.R. (1959) J. Aerospace Sci., **26**, p.121.