

# Fixed Point theorems for multiplicative Contraction mappings on Multiplicative Cone - b metric spaces

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**Abstract:** The purpose of this work is to use some contraction mappings in the sense of multiplicative Cone - b metric space and prove some fixed point theorems in setting of complete multiplicative cone - b metric space.

**Keywords:** Multiplicative Cone b - metric space, Fixed point, Multiplicative kannan contraction, Multiplicative Chatterjea - contraction.

**Mathematics Subject classification:** Primary 47h10, Secondary 54h25.

## 1. Introduction

Metric Fixed point theory is an essential part of Mathematical Analysis because of its applications in different areas like variational and linear inequalities, improvement and approximation theory. The fixed point theorem in metric spaces plays a significant role to construct methods to solve the problem in Mathematics and Sciences. Although metric fixed point theory is vast field of study and is capable of solving many equations.

In this paper, we proved some fixed point theorems using some contractive conditions in Multiplicative Cone - b metric spaces. The study of fixed point of mappings satisfying certain contraction conditions has many applications and has been at the centre of various research activities.

## 2. Preliminaries

We recall some definitions Properties for b-metric spaces given by Czerwik [14], multiplicative cone -b metric spaces [16] and Consistent with Haung and Zhang[9] and Hussain and shah[10], will be needed the following definitions in this paper.

Suppose E be a real Banach space and P be a subset of E. Then P is called Cone iff:

- (i) P is closed, nonempty and  $P \neq \{0\}$ .
- (ii)  $cx + dy \in P$  for all  $x, y \in P$  and non - negative real numbers c, d.
- (iii)  $P \cap (-P) = \{0\}$ .

**Definition 2.1[9]** Suppose P be a Cone in real Banach space E, define a partial ordering  $\leq$  with respect to P by  $a \leq b \Leftrightarrow b - a \in P$ . We can write  $a < b$  to show that  $a \leq b$  and  $a \neq b$  while  $a \ll b$  will stand for  $b - a \in \text{int } P$ , where  $\text{int } P$  denotes interior of P.

**Definition 2.2[3]** Let X be a non-empty set. Let the mapping  $d: X \times X \rightarrow R$  satisfies:

- (i)  $1 \leq d(x, y)$  for all  $x, y \in Y$  with  $x \neq y$ ;
- (ii)  $d(x, y) = 1$  if and only if  $x = y$ ;
- (iii)  $d(x, y) = d(y, x)$  for all  $x, y \in X$ ;
- (iv)  $d(x, y) \leq d(x, z).d(z, y)$  for all  $x, y, z \in X$ .

Then the function d is said to be a multiplicative metric on X and (X,d) is called a cone multiplicative metric space.

**Definition 2.3[10]** Suppose that X be a non-empty set and  $s \geq 1$  be a given positive real number. A function  $d: X \times X \rightarrow R$  is said to be cone b- metric if and only if  $\forall x, y, z \in X$ , the following conditions are satisfied:

- (i)  $0 \leq d(x, y)$  for all  $x, y \in Y$  with  $x \neq y$ ;
- (ii)  $d(x, y) = 0$  if and only if  $x = y$ ;
- (iii)  $d(x, y) = d(y, x)$  for all  $x, y \in X$ ;
- (iv)  $d(x, y) \leq s[d(x, z) + d(z, y)]$  for all  $x, y, z \in X$ .

Then the function d is said to be a cone b- metric on X and (X,d) is called a cone b- metric space.

**Definition 2.4 [16]:** Let X be a non-empty set and  $s \geq 1$  be a given positive real number. A mapping  $d: X \times X \rightarrow R$  satisfies:

- (i)  $1 \leq d(x, y)$  for all  $x, y \in Y$  with  $x \neq y$ ;
- (ii)  $d(x, y) = 1$  if and only if  $x = y$ ;
- (iii)  $d(x, y) = d(y, x)$  for all  $x, y \in X$ ;

(iv)  $d(x, y) \leq [d(x, z).d(z, y)]^s$  for all  $x, y, z \in X$ .

Then the function  $d$  is said to be a multiplicative cone b- metric on  $X$  and  $(X, d)$  is called a multiplicative cone b- metric space.

**Example 2.5[14]:** Let  $d(x, y) = a^{(\sum_{i=1}^{\infty} |x_i - y_i|^p)^{\frac{1}{p}}}$  for all  $x, y \in X; P > 1$ .

We show that  $d(x, y)$  is multiplicative cone b - metric space i.e.,  $(m_{cb}, d)$ , but not multiplicative cone metric space i.e.,  $(m_c, d)$

**Definition 2.6[12](Multiplicative Convergence)**

Let  $(X, d)$  be a multiplicative metric space,  $(x_n)$  be a sequence in  $X$  and  $x \in X$ . If for every multiplicative open ball  $B_e(x)$ , there exists a natural number  $N$  such that  $n \geq N \Rightarrow x_n \in B_e(x)$ , then the sequence  $(x_n)$  is said to be multiplicative convergent to  $x$ , denoted by  $x_n \rightarrow_* x (n \rightarrow \infty)$ .

**Lemma 2.7:[12]** Let  $(X, d)$  be a multiplicative metric space,  $(x_n)$  be a sequence in  $X$ . If the sequence  $(x_n)$  is multiplicative convergent, then the multiplicative limit point is unique.

**Theorem 2.8:[6,7]** Let  $(X, d)$  be a Complete metric spaces and  $T: X \rightarrow X$  be a Kannan Contraction mapping (ie)

$$d(Tx, Ty) \leq K[d(x, Tx) + d(x, Ty)]$$

for all  $x, y \in X$ , where  $K \in [0, \frac{1}{2})$ . Then  $T$  has a unique fixed point.

**Theorem 2.9 : [6,7]** Let  $(X, d)$  be a Complete metric spaces and  $T: X \rightarrow X$  be a Chatterjea- Contraction mapping (ie)

$$d(Tx, Ty) \leq K[d(x, Ty) + d(y, Tx)]$$

for all  $x, y \in X$ , where  $K \in [0, \frac{1}{2})$ . Then  $T$  has a unique fixed point.

**Definition 2.10 :[12]** Let  $(X, d)$  be a multiplicative metric space. A self mapping  $f$  is said to be multiplicative Kannan contraction if

$$d(fx, fy) \leq (d(fx, x).d(fy, y))^{\lambda},$$

for all  $x, y \in X$ , where  $\lambda \in [0, \frac{1}{2})$ .

**Definition 2.11 [12]** Let  $(X, d)$  be a multiplicative metric space. A self mapping  $f$  is said to be multiplicative Chatterjea contraction if

$$d(fx, fy) \leq (d(fx, y).d(fy, x))^{\lambda},$$

for all  $x, y \in X$ , where  $\lambda \in [0, \frac{1}{2})$ .

**Lemma 2.12 :[16]** Suppose that  $\{g_m\}$  be a sequence in real Banach Space  $E$  and  $P$  be a cone. If  $c \in \text{int } P$  and  $g_m \rightarrow 1$  as  $(m \rightarrow \infty)$ , then there exist  $N$  such that for all  $m > N$ , we have  $g_m \ll c$ .

**Lemma 2.13:[16]** Suppose  $P$  be a Cone and  $g \ll c$  for all  $c \in \text{int } P$ , then  $g = 1$ .

**Lemma 2.14:[16]** Suppose  $P$  be a Cone .If  $g \in P$  and  $g \leq g^k$  for some  $k \in [0, 1)$ , then  $g = 1$ .

**Theorem 2.15:[16]** Let  $(X, d)$  be a complete multiplicative cone b-metric space with power  $s \geq 1$ . Suppose the mapping  $T: X \rightarrow X$  satisfies the following contractive condition,

$$d(Ta, Tb) \leq d(a, b)^{\lambda} \text{ for all } a, b \in X$$

where  $0 \leq \lambda < 1$  is a constant. Then  $T$  has a unique fixed point in  $X$ .

**Theorem 2.16:[15]** Let  $X$  be a Complete metric space with metric  $d$  and let  $T: X \rightarrow X$  be a function with the property

$$d(T(x), T(y)) \leq ad(x, T(x)) + bd(y, T(y)) + cd(x, y)$$

for all  $x, y \in X$  where  $a, b, c$  are non-negative and satisfy  $a+b+c < 1$ . Then  $T$  has a unique fixed point.

**Theorem 2.17:[15]** Let  $X$  be a Complete b - metric space with metric  $d$  and let  $T: X \rightarrow X$  be a function with the following condition

$$d(T(x), T(y)) \leq ad(x, T(x)) + bd(y, T(y)) + cd(x, y)$$

for all  $x, y \in X$  where  $a, b, c$  are non-negative and satisfy  $a+s(b+c) < 1$  for  $s \geq 1$ . Then  $T$  has a unique fixed point.

**3. Main Results**

In this section, we prove some fixed theorems using Multiplicative Kannan contraction and multiplicative Chatterjea - contraction and one more contractive conditions in multiplicative cone b - metric spaces. Our main result is follow as:

**Theorem 3.1** Let  $(X, d)$  be a complete multiplicative cone b- metric space with power  $s \geq 1$ . Suppose the mapping  $T: X \rightarrow X$  satisfies the following Kannan contractive condition,

$$d(Tx, Ty) \leq (d(Tx, x).d(Ty, y))^{\lambda},$$

for all  $x, y \in X$ , where  $0 \leq \lambda < \frac{1}{2}$  is a constant.

Then  $T$  has a unique fixed point in  $X$  and for any  $x \in X$ , iterative sequence  $(T^n x)$  converges to the fixed point.

**Proof.** Choose  $x_0 \in X$ . We construct the iterative sequence  $\{x_n\}$ , where  $x_1 = Tx_0, x_2 = Tx_1 = T^2x_0, \dots, x_n = Tx_{n-1}, x_{n+1} = Tx_n = T^{n+1}x_0, n \geq 1$ , we have

$$d(x_{n+1}, x_n) = d(Tx_n, Tx_{n-1})$$

$$\begin{aligned} &\leq (d(Tx_n, x_n) \cdot d(Tx_{n-1}, x_{n-1}))^\lambda \\ &\leq \dots \leq (d(x_n, x_{n-1}))^{\frac{\lambda}{1-\lambda}} \\ &\leq d(x_1, x_0)^h \end{aligned}$$

for all  $n \in \mathbb{N}$ , where  $h = \frac{\lambda}{1-\lambda} < 1$ . For  $n > m$ .

$$\begin{aligned} &d(x_n, x_m) \\ &\leq (d(x_n, x_{n-1}) \cdot d(x_{n-1}, x_{n-2}) \dots d(x_{m+1}, x_m))^s \\ &\leq (d(x_1, x_0))^{h^{n-1} + h^{n-2} + \dots + h^m} \\ &\leq (d(x_1, x_0))^{\frac{h^m}{1-h}} \\ &\leq d(x_1, x_0)^{\frac{sh^m}{1-sh}} \end{aligned}$$

This implies  $d(x_n, x_m) \rightarrow 1 (n, m \rightarrow \infty)$ .

(i.e)  $d(x_1, x_0)^{\frac{sh^m}{1-sh}} \rightarrow 1$  as  $m \rightarrow \infty$  for any  $s \geq 1$ . Hence  $\{x_n\}$  is a Cauchy sequence. Since  $(X, d)$  is a Complete multiplicative Cone b-metric space, there exist  $z$  in  $X$  such that  $x_n \rightarrow z (n \rightarrow \infty)$ . for all  $n > n_0$ .

$$\begin{aligned} d(Tz, z) &\leq (d(Tz, Tx_n) \cdot d(Tx_n, z))^s \\ &\leq d(Tz, Tx_n)^s \cdot d(Tx_n, z)^s \\ &\leq (d(Tz, z) \cdot d(z, Tx_n))^{s\lambda} \cdot d(x_{n+1}, z)^s \\ &\leq (d(Tz, z))^{s\lambda} \cdot d(x_{n+1}, z)^{s\lambda+s} \\ &\leq d(x_{n+1}, z)^{\frac{s(\lambda+1)}{1-s\lambda}} \ll c, \text{ for each } n > n_0. \end{aligned}$$

we have

By use of lemma 2.12, we deduce that  $d(Tz, z) = 1$ . This implies  $Tz = z$ . So  $z$  is a fixed point of  $T$ . For proving uniqueness, if there is another fixed point  $z_1$ , then by the given assertion,

$$\begin{aligned} d(z, z_1) &= d(Tz, Tz_1) \\ &\leq (d(Tz, z) \cdot d(Tz_1, z_1))^\lambda \\ &\leq d(Tz, z)^\lambda \cdot d(Tz_1, z_1)^\lambda \end{aligned}$$

By lemma 2.12,  $z = z_1$ . This Completes the proof.

**Theorem 3.2** Let  $(X, d)$  be a complete multiplicative cone b-metric space with power  $s \geq 1$ . Suppose the mapping  $T: X \rightarrow X$  holds the contractive condition,

$$d(Tx, Ty) \leq (d(Tx, y) \cdot d(Ty, x))^\lambda,$$

for all  $x, y \in X$ , where  $\lambda \in [0, \frac{1}{2})$  is a constant. Then  $T$  has a unique fixed point in  $X$ . And for any  $x \in X$ , iterative sequence  $(T^n x)$  converges to the fixed point.

**Proof.** Choose  $x_0 \in X$ . Set  $x_1 = Tx_0, x_2 = Tx_1 = T^2x_0, \dots, x_{n+1} = Tx_n = T^{n+1}x_0, \dots$

we have

$$\begin{aligned} d(x_{n+1}, x_n) &= d(Tx_n, Tx_{n-1}) \\ &\leq d(Tx_n, x_{n-1}) \cdot d(Tx_{n-1}, x_n)^\lambda \\ &\leq d(x_n, x_{n-1}) \cdot d(x_n, x_n)^\lambda \\ &\leq d(x_n, x_{n-1})^\lambda \\ &\dots \dots \dots \\ &\leq d(x_1, x_0)^{\lambda^n} \end{aligned}$$

For  $n > m$ ,  
 $d(x_n, x_m)$

$$\begin{aligned} &\leq (d(x_n, x_{n-1}) \cdot d(x_{n-1}, x_{n-2}) \dots d(x_{m+1}, x_m))^s \\ &\leq (d(x_1, x_0))^{\lambda^{n-1} + \lambda^{n-2} + \dots + \lambda^m} \\ &\leq (d(x_1, x_0))^{\frac{\lambda^m}{1-\lambda}} \\ &\leq d(x_1, x_0)^{\frac{s\lambda^m}{1-s\lambda}} \end{aligned}$$

This implies  $d(x_n, x_m) \rightarrow 1 (n, m \rightarrow \infty)$ .

(i.e)  $d(x_1, x_0)^{\frac{s\lambda^m}{1-s\lambda}} \rightarrow 1$  as  $m \rightarrow \infty$  for any  $s \geq 1$ . Hence  $\{x_n\}$  is a Cauchy sequence. Since  $(X, d)$  is a Complete multiplicative Cone b-metric space, there is  $z \in X$  such that  $x_n \rightarrow z (n \rightarrow \infty)$  for all  $n > n_0$ .

$$\begin{aligned} d(Tz, z) &\leq (d(Tz, Tx_n) \cdot d(Tx_n, z))^s \\ &\leq (d(Tz, x_n) \cdot d(Tx_n, z))^{s\lambda} \cdot d(x_{n+1}, z)^s \\ &\leq (d(Tz, z) \cdot d(x_n, z))^{s\lambda} \cdot d(x_{n+1}, z)^s \\ &\leq d(x_n, z)^{s^2\lambda} \cdot d(x_{n+1}, z)^{s\lambda} \cdot d(x_{n+1}, z)^s \end{aligned}$$

we have,  $d(Tz, z) \leq d(x_{n+1}, z)^{\frac{s}{1-s\lambda}} \ll c$ , for each  $n > n_0$ , By use of lemma 2.14, we deduce that  $d(Tz, z) = 1$ . This implies  $Tz = z$ . So  $z$  is a fixed point of  $T$ . For proving uniqueness, if there is another fixed point  $z_1$ , then by the given assertion,

$$\begin{aligned} d(z, z_1) &= d(Tz, Tz_1) \\ &\leq (d(Tz, z_1) \cdot d(Tz_1, z))^\lambda \\ &\leq d(Tz, z_1)^\lambda \cdot d(Tz_1, z)^\lambda \end{aligned}$$

By lemma 2.14,  $z = z_1$ , This Completes the proof.

**Theorem 3.3** Let  $(X, d)$  be a complete cone b-metric space with metric  $d$  and let  $T: X \rightarrow X$  be a function with the following condition,

$$d(Tx, Ty) \leq d(x, Tx)^p \cdot d(y, Ty)^q \cdot d(x, y)^r,$$

for all  $x, y \in X$ , where  $p, q, r$  are non-negative real numbers and satisfy  $p + (q + r)^s < 1$  for  $s \geq 1$ . Then  $T$  has a unique fixed point.

**Proof.** Let  $x_0 \in X$  and  $\{x_n\}$  be a sequence in  $X$  such that  $x_n = Tx_{n-1} = T^n x_0$ .

$$\begin{aligned} \text{Now, } d(x_{n+1}, x_n) &= d(Tx_n, Tx_{n-1}) \\ &\leq d(x_n, Tx_n)^p \cdot d(x_{n-1}, Tx_{n-1})^q \cdot d(x_n, Tx_{n-1})^r \\ &\leq (d(x_n, x_{n+1}) \cdot d(x_{n+1}, Tx_n))^{sp} \cdot d(x_{n-1}, Tx_{n-1})^q \cdot d(x_n, Tx_{n-1})^r \\ &\leq d(x_n, x_{n+1})^{sp} \cdot d(x_{n-1}, x_n)^{q+r} \\ &\leq d(x_n, x_{n+1})^{1-sp} \leq d(x_{n-1}, x_n)^{q+r} \end{aligned} \quad \text{-----} \rightarrow (1)$$

secondly,

$$\begin{aligned} d(x_{n+1}, x_n) &= d(Tx_n, Tx_{n-1}) = d(Tx_{n-1}, Tx_n) \\ &\leq d(x_{n-1}, Tx_{n-1})^p \cdot d(x_n, Tx_n)^q \cdot d(x_{n-1}, Tx_n)^r \\ &\leq d(x_{n-1}, Tx_{n-1})^p [d(x_n, x_{n+1}) \cdot d(x_{n+1}, Tx_n)]^{sq} d(x_{n-1}, Tx_n)^r \\ &\leq d(x_{n-1}, x_n)^p d(x_n, x_{n+1})^{sq} d(x_{n-1}, Tx_n)^r \\ &\leq d(x_{n+1}, x_n)^{1-sq} \leq d(x_{n-1}, x_n)^{p+r} \end{aligned} \quad \text{-----} \rightarrow (2)$$

Multiplying (1) & (2)

$$\begin{aligned} d(x_n, x_{n+1})^{1-sp} d(x_n, x_{n+1})^{1-sq} &\leq d(x_{n-1}, x_n)^{q+r} \cdot d(x_{n-1}, x_n)^{p+r} \\ d(x_n, x_{n+1})^{2-s(p+q)} &\leq d(x_{n-1}, x_n)^{p+q+2r} \\ d(x_n, x_{n+1}) &\leq d(x_{n-1}, x_n)^{\frac{p+q+2r}{2-s(p+q)}} \end{aligned}$$

Put  $\lambda = \frac{p+q+2r}{2-s(p+q)}$ , it is easy to see that  $\lambda \in [0,1)$ . Thus,

$$d(x_n, x_{n+1}) \leq d(x_n, x_{n-1})^\lambda \dots \leq d(x_1, x_0)^{\lambda^n}$$

we can follow the same argument that is given in Theorem 3.1 [16], there exist  $x^* \in X$  such that  $x_n \rightarrow x^*$ . Let  $c >> 1$  be arbitrary. Since  $x_n \rightarrow x^*$ , there exist  $N$  such that

$$d(x_n, x^*) \leq c$$

Next we show that  $x^*$  is fixed point of  $T$ , we have

$$\begin{aligned} d(Tx^*, x^*) &= [d(Tx^*, Tx_n) \cdot d(Tx_n, x^*)]^s \\ &\leq d(Tx^*, Tx_n)^s \cdot d(x_{n+1}, x^*)^s \\ &\leq [d(x^*, Tx^*)^p \cdot d(x_n, Tx_n)^q \cdot d(x^*, x_n)^r]^s \end{aligned}$$

$$\begin{aligned} d(x_{n+1}, x^*)^s &\leq [d(x^*, Tx^*)^{sp} \cdot [d(x_n, x^*) \cdot d(x^*, Tx_n)]^{s^2q} \cdot d(x^*, x_n)^{sr} d(x_{n+1}, x^*)^s]^s \\ &\leq [d(x^*, Tx^*)^{sp} \cdot d(x_n, x^*)^{s^2q} \cdot d(x^*, x_{n+1})^{s^2q} \cdot d(x^*, x_n)^{sr} d(x_{n+1}, x^*)^s]^s \\ &= d(x_n, x^*)^{sr+s^2q} \cdot d(x^*, x_{n+1})^{s^2p+rs} \\ d(x^*, Tx^*)^{1-sp} &\leq d(x^*, Tx^*)^{sr+s^2q} \cdot d(x^*, x_{n+1})^{s^2q+s} \dots (3) \end{aligned}$$

On the Other hand ,

$$\begin{aligned} d(x^*, Tx^*) &= [d(x^*, Tx_n) \cdot d(Tx_n, Tx^*)]^s = d(x^*, x_{n+1})^s \cdot d(Tx_n, Tx^*)^s \\ &\leq d(x^*, x_{n+1})^s \cdot [d(x_n, Tx_n)^p \cdot d(x^*, Tx^*)^q \cdot d(x_n, x^*)^r]^s \\ &\leq d(x^*, x_{n+1})^s \cdot [d(x_n, x^*) \cdot d(x^*, Tx_n)]^{s^2p} \cdot d(x_n, x^*)^{rs} \\ &\leq d(x^*, x_{n+1})^s \cdot d(x_n, x^*)^{s^2p} \cdot d(x^*, x_{n+1})^{s^2p} \cdot d(x^*, Tx^*)^{rs} \end{aligned}$$

$$\begin{aligned} d(x^*, Tx^*) \cdot d(x^*, Tx^*)^{-sq} &= d(x^*, x_{n+1})^{s+s^2p} \cdot d(x_n, x^*)^{s^2p+rs} \\ d(x^*, Tx^*)^{1-sq} &\leq d(x^*, x_{n+1})^{s+s^2p} \cdot d(x_n, x^*)^{s^2p+rs} \end{aligned} \quad \text{-----} \rightarrow (4)$$

Multiplying (3) & (4) gives ,

$$\begin{aligned} d(x^*, Tx^*)^{2-sp-sq} &\leq d(x^*, x_{n+1})^{s+s^2p+s+s^2q} \cdot d(x_n, x^*)^{s^2p+rs+sr+s^2q} \\ &\leq d(x^*, x_{n+1})^{2s+s^2(p+q)} \cdot d(x_n, x^*)^{2rs+s^2(p+q)} \\ d(x^*, Tx^*) &\leq c \end{aligned}$$

we can see from lemma 2.14, that  $d(x^*, Tx^*) = 1$ . (ie) the mapping  $T$  has a fixed point  $x^*$ . At last, for uniqueness, if there is another fixed point  $y^*$  then

$$\begin{aligned} d(x^*, y^*) &= d(Tx^*, Ty^*) \leq d(x^*, Tx^*)^p \\ d(y^*, Ty^*)^q d(x^*, y^*)^r &\Rightarrow d(x, y) \leq d(x^*, y^*)^r \end{aligned}$$

Owing to  $0 \leq r < 1$ , we deduce from lemma 2.14 that  $x^* = y^*$ . This completes the Proof.

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