Solving Fuzzy Fractional Differential Equation with Fuzzy Laplace Transform Involving Sine function

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Abstract :*This paper deals with fuzzy Laplace transforms to obtain the solution of fuzzy fractional differential equation (FFDEs) under Riemann Liouville H-differentiability with fractional order ($0 < \beta < 1$). There is limited research devoted to the analytical method to solve the FFDEs under Riemann Liouville H-differentiability. An analytical solution is presented to confirm the capability of proposed method*

I .INTRODUCTION

Fractional calculus is a mathematical branch investigating the properties of derivatives and integrals of non- integer orders. It has been applied in modelling physical and chemical processes and in engineering [4, 6, 18] Podlubny and kilbas[10,12,] gave the idea of fractional calculus and consider Riemann Liouville differentiability to solve FFDEs. Agarwal [2] proposed the concept of solutions for fractional differential equations with uncertainty.

Laplace transform is used for solving differential equations. To solve fuzzy fractional differential equation, fuzzy initial and boundary value problems, we use fuzzy Laplace transform. The advantage of fuzzy Laplace transform is that it solves the problem directly without determining a general solution.

Here we have seen some basic definitions and Riemann Liouville H- differentiability in section 2. In section 3, fuzzy Laplace transforms are introduced and we discuss the properties .The solutions of FFDEs are determined by Fuzzy Laplace transform under Riemann Liouville H- differentiability and solve the example involving sine terms in section 4. In section 5, a conclusion is drawn.

II. PRELIMINARIES

Definition: 2.1[8]

Fuzzy number is a mapping u : R $\rightarrow$[0, 1] with the following properties:

1. u is upper semi continuous.

2. u is fuzzy convex.

i.e., $u(\lambda x + (1 - \lambda)y) \geq \min\{u(x), u(y)\}$ for all $x, y \in$R, $\lambda \in$[0, 1].

3. u is normal. i.e., $\exists\ x_0 \in$R for which $u(x_0) = 1$

4. Supp $u = \{x \in R / u(x) > 0\}$ is the support of the u, and its closure cl (supp u) is compact.

Definition: 2.2[13, 14]

A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r), 0 \leq r \leq 1$, which satisfy the following requirements:

1. $\underline{u}(r)$ is a bounded non-decreasing left continuous function in (0, 1],and right continuous at 0,

2. $\overline{u}(r)$ is a bounded non-increasing left continuous function in (0,1],and right continuous at 0,

3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

Theorem: 2.1[15]   Let f be fuzzy valued function on [a,∞) represented by $\left(\underline{f}(x;r), \overline{f}(x;r)\right)$. For any fixed $r \in$[0, 1], assume $\underline{f}(x;r)$ and $\overline{f}(x;r)$ are Riemann-integrable on [a, b] for every b≥ a, and assume there are two positive functions $\underline{M}$(r), $\overline{M}$(r) such that

$$\int_a^b \left|\underline{f}(x;r)\right| dx \leq \underline{M}(r) \text{ and } \int_a^b \left|\overline{f}(x;r)\right| dx \leq \overline{M}(r)$$

for every b≥ a.Then $f(x)$ is improper fuzzy Riemann integrable on [a, ∞)and the improper fuzzy integral is a fuzzy number. Furthermore, we have

$$\int_a^\infty f(x;r)\, dx = \left[\int_a^\infty \underline{f}(x;r)dx, \int_a^\infty \overline{f}(x;r)dx\right]$$

Definition: 2.3 Let $x, y \in E$. If there exists z$\in E$ such that $x = y + z$, then z is called the H- difference of x and y and it is denoted by $x \ominus y$.

Riemann Liouville H-differentiability: [7]

C^F[a,b] is the space of all continuous fuzzy valued functions on [a,b]. Also we denote the space of all Lebesgue integrable fuzzy valued functions on [a, b] by $L^F[a, b]$.

Definition: 2.4 Let $f \in C^F$[a,b]$\cap L^F[a,b], x_0$ in (a,b) and $\Phi(x) = \frac{1}{\Gamma(1-\beta)} \int_a^x \frac{f(t)dt}{(x-t)^\beta}$.We say that f is Riemann Liouville H- differentiable about order $0 < \beta < 1$ at x_0, if there exists an element$(^{RL}D_{a^+}^\beta f)(x_0) \in E$ such that for h > 0 sufficiently small

(i) $(^{RL}D_{a^+}^\beta f)(x_0)$ $= \lim\limits_{h \to 0^+} \frac{\Phi(x_0+h)\ominus\Phi(x_0)}{h}$

$= \lim\limits_{h \to 0^+} \frac{\Phi(x_0)\ominus\Phi(x_0-h)}{h}$ (or)

(ii) $({}^{RL}D_{a^+}^{\beta}f)(x_0) = \lim\limits_{h \to 0^+} \frac{\Phi(x_0) \ominus \Phi(x_0+h)}{-h}$

$= \lim\limits_{h \to 0^+} \frac{\Phi(x_0-h) \ominus \Phi(x_0)}{-h}$ (or)

(iii) $({}^{RL}D_{a^+}^{\beta}f)(x_0) = \lim\limits_{h \to 0^+} \frac{\Phi(x_0+h) \ominus \Phi(x_0)}{h}$

$= \lim\limits_{h \to 0^+} \frac{\Phi(x_0-h) \ominus \Phi(x_0)}{-h}$ (or)

(iv) $({}^{RL}D_{a^+}^{\beta}f)(x_0) = \lim\limits_{h \to 0^+} \frac{\Phi(x_0) \ominus \Phi(x_0+h)}{-h}$

$= = \lim\limits_{h \to 0^+} \frac{\Phi(x_0) \ominus \Phi(x_0-h)}{h}$

We say that the fuzzy valued function f is $({}^{RL}(i)-\beta)$ differentiable if it is differentiable as in the definition2.5 case (i), and f is $({}^{RL}(ii)-\beta)$ differentiable if it is differentiable as in the definition 2.5 of case (ii) and so on for other cases.

Theorem:2.2[17]

Let $f \in C^F[a, b] \cap L^F[a,b], x_0$ in (a,b) and $0 < \beta < 1$.

Then

(i) Let us consider f is $({}^{RL}(i)-\beta)$ differentiable fuzzy valued function, then

$({}^{RL}D_{a^+}^{\beta}f)(x_0;r) = \left[({}^{RL}D_{a^+}^{\beta}\underline{f})(x_0;r), ({}^{RL}D_{a^+}^{\beta}\overline{f})(x_0;r)\right]$

$, 0 \le r \le 1$

(ii) Let us consider f is $({}^{RL}(ii)-\beta)$ differentiable fuzzy valued function, then

$({}^{RL}D_{a^+}^{\beta}f)(x_0;r) = \left[({}^{RL}D_{a^+}^{\beta}\underline{f})(x_0;r), ({}^{RL}D_{a^+}^{\beta}\overline{f})(x_0;r)\right]$

$, 0 \le r \le 1$

Where

$({}^{RL}D_{a^+}^{\beta}\underline{f})(x_0;r) = \left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x \frac{\underline{f}(t;r)dt}{(x-t)^{\beta}}\right]_{x=x_0}$ (1)

$({}^{RL}D_{a^+}^{\beta}\overline{f})(x_0;r) = \left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x \frac{\overline{f}(t;r)dt}{(x-t)^{\beta}}\right]_{x=x_0}$ (2)

3. FUZZY LAPLACE TRANSFORMS

Definition: 3.1[16]

Let f be continuous fuzzy valued function. Suppose that $f(x) \odot e^{-px}$ is improper fuzzy Riemann integrable on $[0,\infty)$, then $\int_0^\infty f(x) \odot e^{-px}dx$ is called fuzzy Laplace transforms and denoted by

$L[f(x)] = \int_0^\infty f(x) \odot e^{-px}dx$ (3)

(p>0 and integer)

Using Theorem 2.1 we have $0 \le r \le 1$;

$\int_0^\infty f(x;r) \odot e^{-px}dx$
$= \left[\int_0^\infty \underline{f}(x;r) \odot e^{-px}dx, \int_0^\infty \overline{f}(x;r) \odot e^{-px}dx\right]$

Using the classical Laplace transforms,

$l\left[\underline{f}(x;r)\right] = \int_0^\infty \underline{f}(x;r)e^{-px}dx$ and

$l\left[\overline{f}(x;r)\right] = \int_0^\infty \overline{f}(x;r)e^{-px}dx$

Then we get

$L[f(x;r)] = \left[l\left[\underline{f}(x;r)\right], l\left[\overline{f}(x;r)\right]\right]$

Definition: 3.2

Hypergeom (n,d,z) is the generalized hyper geometric function $F(n,d,z)$, also known as Barnes extended hyper geometric function. For scalars a ,b and c, hypergeom ([a,b],c ,z) is a Gauss hyper geometric function $2F_1(a,b;c;z)$. The Gauss hypergeometric function $2F_1(a,b;c;z)$ is defined in the unit disc as the sum of the hypergeometric series

$2F_1(a,b;c;z) = \sum_{k=0}^\infty \frac{(a)_k (b)_k}{(c)_k}\frac{z^k}{k!}, |z| < 1$

Definition: 3.3

The pochhammer symbol $(a)_k$ is defined by

$(a)_0 = 1$,

$(a)_n = a(a+1)\dots\dots(a+n-1), n \in N$

Definition: 3.4

A two parameters function of Mittag -Leffler type is defined by the series expansion

$E_{\alpha,\beta}(z) = \sum_{r=0}^\infty \frac{z^r}{\Gamma(\alpha r+\beta)}$ (α, β>0)

An error function is defined by

$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt.$

Theorem: 3.1 [16]

Let f &g are continuous fuzzy valued functions.Suppose that c_1 &c_2 are constants.

$L[(c_1 \odot f(x)) \oplus (c_2 \odot g(x))] = (c_1 \odot L[f(x)]) \oplus (c_2 \odot L[g(x)])$

Lemma: 3.2 [16]

Let f be continuous fuzzy valued function on $[0,\infty)$ and $\lambda \in R$ then
$L[\lambda \odot f(x)] = \lambda \odot L[f(x)]$

Derivative theorem: 3.2

Suppose that $f \in C^F[0,\infty) \cap L^F[0,\infty).Then$

$L\left[({}^{RL}D_{a^+}^{\beta}f)(x)\right] = s^{\beta}L[f(t)] \ominus ({}^{RL}D_{a^+}^{\beta-1}f)(0)$ (4)

if f is $({}^{RL}(i)-\beta)$ differentiable, and

$L\left[({}^{RL}D_{a^+}^{\beta}f)(x)\right] = -({}^{RL}D_{a^+}^{\beta-1}f)(0) \ominus -(s^{\beta}L[f(t)])$

if f is $({}^{RL}(ii)-\beta)$ differentiable. (5)

4. FUZZY FRACTIONAL DIFFERENTIAL EQUATIONS UNDER RIEMANN LIOUVILLE H-DIFFERENTIABILITY:

Let $f \in C^F[a,b] \cap L^F[a,b]$ and consider the fuzzy fractional differential equation of order $0 < \beta < 1$ with the initial condition and $x_0 \in (a, b)$.

$$\begin{cases} (^{RL}D_{a^+}^{\beta} y)(x) = f[x, y(x)], \\ (^{RL}D_{a^+}^{\beta-1} y)(x_0) = (^{RL}y_0{}^{(\beta-1)}) \in E \end{cases} \quad (6)$$

Solutions:

Here we use fuzzy Laplace transform and its inverse to derive the solution .By taking Laplace transform on both sides, we get

$$L[(^{RL}D_{a^+}^{\beta} y)(x)] = L[f(x, y(x))], \quad (7)$$

Based on the Riemann Liouville H- differentiability, we have the following cases:

Case (i) Let us consider $y(x)$ is a $(^{RL}(i) - \beta)$ differentiable function then the equation (7) is extended based on the its lower and upper functions as follows

$$s^{\beta} l\left[\underline{y}(x;r)\right] - (^{RL}D_{a^+}^{\beta-1}\underline{y})(0;r) = l\left[\underline{f}(x, y(x);r)\right]$$
$$s^{\beta} l[\overline{y}(x;r)] - (^{RL}D_{a^+}^{\beta-1}\overline{y})(0;r) = l[\overline{f}(x, y(x);r)]$$
$$, 0 \leq r \leq 1 \quad (8)$$

Where

$$\underline{f}(x, y(x);r) = \min\{f(x, u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

$$\overline{f}(x, y(x);r) = \max\{f(x, u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

To solve the linear system (8), we assume that $H_1(p;r), k_1(p;r)$ are the solutions

$$l\left[\underline{y}(x;r)\right] = H_1(p;r)$$

$$l[\overline{y}(x;r)] = k_1(p;r)$$

By using inverse Laplace transform $\underline{y}(x;r)$ & $\overline{y}(x;r)$ are computed as follows,

$$\underline{y}(x;r) = l^{-1}[H_1(p;r)]$$

$$\overline{y}(x;r) = l^{-1}[k_1(p;r)] \quad (9)$$

Case (ii)

Let us consider $y(x)$ is a $(^{RL}(ii) - \beta)$ differentiable function then the equation (7) can be written as follows

$$\begin{cases} -(^{RL}D_{a^+}^{\beta-1}\underline{y})(0;r) - (-s^{\beta} l[\underline{y}(x;r)]) = l[\underline{f}(x, y(x);r)] \\ -(^{RL}D_{a^+}^{\beta-1}\overline{y})(0;r) - (-s^{\beta} l[\overline{y}(x;r)]) = l[\overline{f}(x, y(x);r)] \end{cases}$$
$$0 \leq r \leq 1 \quad (10)$$

Where

$$\underline{f}(x, y(x);r) = \min\{f(x, u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

$$\overline{f}(x, y(x);r) = \max\{f(x, u)/u \in \left[\underline{y}(x;r), \overline{y}(x;r)\right]\}$$

To solve the linear system (10), we assume that $H_2(p;r), k_2(p;r)$ are the solutions

$$l\left[\underline{y}(x;r)\right] = H_2(p;r)$$

$$l[\overline{y}(x;r)] = k_2(p;r)$$

By using inverse Laplace transform $\underline{y}(x;r)$ & $\overline{y}(x;r)$ are computed as follows,

$$\underline{y}(x;r) = l^{-1}[H_2(p;r)]$$

$$\overline{y}(x;r) = l^{-1}[k_2(p;r)] \quad (11)$$

Example: 1

Let us consider the following fuzzy fractional differential equation

$$\begin{cases} (^{RL}D_{0^+}^{\beta} y)(x) = \lambda \odot y(x) + \sin x, \quad 0 < \beta, \ x < 1 \\ (^{RL}D_{0^+}^{\beta-1} y)(0) = (^{RL}y_0{}^{(\beta-1)}) \in E \end{cases}$$
$$(12)$$

Solution:

Case (i): Suppose $\lambda \in R^+ = (0, +\infty)$, then applying Laplace transform on both sides

$$L[(^{RL}D_{0^+}^{\beta} y)(x)] = L[\lambda \odot y(x) + \sin x], \quad (13)$$

$$L[(^{RL}D_{0^+}^{\beta} y)(x)] = L[\lambda \odot y(x)] + L[\sin x],$$

Using $(^{RL}(i) - \beta)$ differentiability, we get

$$\begin{cases} s^{\beta} l\left[\underline{y}(x;r)\right] - (^{RL}D_{a^+}^{\beta-1}\underline{y})(0;r) = \lambda l\left[\underline{y}(x;r)\right] + \frac{1}{s^2+1} \\ s^{\beta} l[\overline{y}(x;r)] - (^{RL}D_{a^+}^{\beta-1}\overline{y})(0;r) = \lambda l[\overline{y}(x;r)] + \frac{1}{s^2+1} \end{cases}$$
$$(14)$$

$$\Rightarrow \quad (s^{\beta} - \lambda) l\left[\underline{y}(x;r)\right] = (^{RL}D_{a^+}^{\beta-1}\underline{y})(0;r) + \frac{1}{s^2+1}$$

$$(s^{\beta} - \lambda) l[\overline{y}(x;r)] = (^{RL}D_{a^+}^{\beta-1}\overline{y})(0;r) + \frac{1}{s^2+1}$$

$$l\left[\underline{y}(x;r)\right] = (^{RL}D_{a^+}^{\beta-1}\underline{y})(0;r)\frac{1}{(s^{\beta}-\lambda)} + \frac{1}{(s^2+1)(s^{\beta}-\lambda)}$$

$$l[\overline{y}(x;r)] = (^{RL}D_{a^+}^{\beta-1}\overline{y})(0;r)\frac{1}{(s^{\beta}-\lambda)} + \frac{1}{(s^2+1)(s^{\beta}-\lambda)}$$
$$(15)$$

Applying inverse transform on both sides,

$$\underline{y}(x;r) = (^{RL}D_{a^+}^{\beta-1}\underline{y})(0;r) l^{-1}[\frac{1}{(s^{\beta}-\lambda)}] + l^{-1}[\frac{1}{(s^2+1)(s^{\beta}-\lambda)}]$$

$$\overline{y}(x;r) = (^{RL}D_{a^+}^{\beta-1}\overline{y})(0;r) l^{-1}[\frac{1}{(s^{\beta}-\lambda)}] + l^{-1}[\frac{1}{(s^2+1)(s^{\beta}-\lambda)}]$$
$$(16)$$

$$\underline{y}(x;r) = (^{RL}\underline{y_0}{}^{(\beta-1)}) \odot x^{\beta-1} E_{\beta,\beta}(\lambda x^{\beta})$$
$$+ \int_0^x (x-t)^{(\beta-1)} E_{\beta,\beta}(\lambda (x-t)^{\beta}) \sin t \, dt$$

$$\overline{y}(x;r) = ({}^{RL}\overline{y}_0{}^{(\beta-1)}) \odot x^{\beta-1} E_{\beta,\beta}(\lambda x^\beta)$$
$$+ \int_0^x (x-t)^{(\beta-1)} E_{\beta,\beta}(\lambda(x-t)^\beta) \, sint \, dt \qquad (17)$$

Case (ii) Suppose $\lambda \in R^- = (-\infty, 0)$, then using $({}^{RL}(ii) - \beta)$ differentiability the solution will obtain similar to equation (17).

For the special case, let us consider $\beta=0.5$, $\lambda=1$ and $({}^{RL}D_{0^+}^{-0.5} y)(0;r) = [1+r, 3-r]$ in case (i)

$$\underline{y}(x;r) = [1+r, 3-r] \odot x^{-\frac{1}{2}} E_{\frac{1}{2}\frac{1}{2}}(x^{\frac{1}{2}})$$
$$+ \int_0^x (x-t)^{(-\frac{1}{2})} E_{\frac{1}{2}\frac{1}{2}}(x-t)^{\frac{1}{2}}) \, sint \, dt$$

$$\overline{y}(x;r) = [1+r, 3-r] \odot x^{-\frac{1}{2}} E_{\frac{1}{2}\frac{1}{2}}(x^{\frac{1}{2}})$$
$$+ \int_0^x (x-t)^{(-\frac{1}{2})} E_{\frac{1}{2}\frac{1}{2}}(x-t)^{\frac{1}{2}}) sint \, dt \qquad (18)$$

Now consider 1st term in equation (18)

$$x^{-\frac{1}{2}} E_{\frac{1}{2}\frac{1}{2}}(x^{\frac{1}{2}}) = x^{-\frac{1}{2}} \left[\sum_{k=0}^\infty \frac{(x^{\frac{1}{2}})^k}{\Gamma(\frac{k+1}{2})} \right]$$

$$= \frac{1}{\sqrt{x}} \left[\frac{1}{\Gamma(\frac{1}{2})} + \frac{x^{\frac{1}{2}}}{\Gamma(1)} + \frac{x^1}{\Gamma(\frac{3}{2})} + \cdots \right]$$

$$= \left[\frac{1}{\sqrt{\pi x}} + \left(\frac{x^0}{\Gamma(1)} + \frac{x^{\frac{1}{2}}}{\Gamma(\frac{3}{2})} + \cdots \right) \right]$$

$$= \frac{1}{\sqrt{\pi x}} + \sum_{k=0}^\infty \frac{x^{(\frac{k}{2})}}{\Gamma(\frac{k}{2}+1)}$$

$$= \frac{1}{\sqrt{\pi x}} + E_{\frac{1}{2},1}(x^{\frac{1}{2}})$$

$$= \frac{1}{\sqrt{\pi x}} + e^{(x^{\frac{1}{2}})^2} \, \text{erfc}(-x^{\frac{1}{2}})$$

$$= \frac{1}{\sqrt{\pi x}} + e^x \, \text{erfc}(-\sqrt{x})$$

2nd term in equation (18)

$$\int_0^x (x-t)^{(-\frac{1}{2})} E_{\frac{1}{2}\frac{1}{2}}(x-t)^{\frac{1}{2}}) \, sint \, dt$$

$$= \int_0^x (x-t)^{(-\frac{1}{2})} \sum_{k=0}^\infty \frac{(x-t)^{\frac{k}{2}}}{\Gamma(\frac{k+1}{2})} \, sint \, dt$$

$$= \int_0^x \sum_{k=0}^\infty \frac{(x-t)^{\frac{k-1}{2}}}{\Gamma(\frac{k+1}{2})} \, sint \, dt$$

$$= \int_0^x \frac{(x-t)^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})} sint \, dt + \int_0^x \frac{(x-t)^0}{\Gamma(1)} sint \, dt +$$

$$\int_0^x \frac{(x-t)^{\frac{1}{2}}}{\Gamma(\frac{3}{2})} sint \, dt + \int_0^x \frac{(x-t)^1}{\Gamma(2)} sint \, dt +$$

$$\int_0^x \frac{(x-t)^{\frac{3}{2}}}{\Gamma(\frac{5}{2})} sint \, dt + \int_0^x \frac{(x-t)^2}{\Gamma(3)} sint \, dt + \cdots$$

$$\text{--(*)}$$

Rearrange the terms split series

$$= \frac{x^{\frac{3}{2}}}{(\frac{3}{2})!} \left[\begin{matrix} hypergeom \\ even \end{matrix} \left(1, \frac{5}{2}, ix\right) \right] + \frac{x^{\frac{5}{2}}}{(\frac{5}{2})!} \left[\begin{matrix} hypergeom \\ even \end{matrix} \left(1, \frac{7}{2}, ix\right) \right]$$
$$+ \frac{x^{\frac{7}{2}}}{(\frac{7}{2})!} \left[\begin{matrix} hypergeom \\ even \end{matrix} \left(1, \frac{9}{2}, ix\right) \right] + \cdots + \cdots +$$
$$\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \frac{x^{10}}{10!} + \frac{x^{11}}{11!} + \cdots$$

$$= \sum_{n=0}^\infty \frac{x^{n+\frac{3}{2}}}{(n+\frac{3}{2})!} \left\{ \begin{matrix} hypergeom \\ even \end{matrix} (1, n+\frac{5}{2}, ix) \right\} +$$
$$\sum_{n=0}^\infty \frac{x^{4n+2}}{(4n+2)!} + \sum_{n=0}^\infty \frac{x^{4n+3}}{(4n+3)!}$$

$$(18) \Rightarrow \underline{y}(x;r) = [1+r]\frac{1}{\sqrt{\pi x}} + e^x \text{erfc}(-\sqrt{x}) +$$
$$\sum_{n=0}^\infty \frac{x^{n+\frac{3}{2}}}{(n+\frac{3}{2})!} \left\{ \begin{matrix} hypergeom \\ even \end{matrix} (1, n+\frac{5}{2}, ix) \right\} +$$
$$\sum_{n=0}^\infty \frac{x^{4n+2}}{(4n+2)!} + \sum_{n=0}^\infty \frac{x^{4n+3}}{(4n+3)!}$$

$$\overline{y}(x;r) = [3-r]\frac{1}{\sqrt{\pi x}} + e^x \text{erfc}(-\sqrt{x}) +$$
$$\sum_{n=0}^\infty \frac{x^{n+\frac{3}{2}}}{(n+\frac{3}{2})!} \left\{ \begin{matrix} hypergeom \\ even \end{matrix} (1, n+\frac{5}{2}, ix) \right\} +$$
$$\sum_{n=0}^\infty \frac{x^{4n+2}}{(4n+2)!} + \sum_{n=0}^\infty \frac{x^{4n+3}}{(4n+3)!}$$

5. CONCLUSION:

In this paper, solving FFDEs of order $0 < \beta < 1$ using fuzzy Laplace transforms under Riemann Liouville –H differentiability was discussed. As an example, we solved a problem involving sine term.

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