On the Construction of Weighing Matrices from Coherent Configuration

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Abstract: *In this paper we forward two methods of construction of conference matrices of order 6 by suitable combinations of adjacency matrices of suitable coherent configuration.*
Key words: *Coherent configuration, weighing matrix, conference matrices.*

1. Introduction: We begin with the following definition:

1.1. WEIGHING MATRICES:

A weighing matrix W of order n and weight w is an $n \times n$ matrix with entries $(0, \pm 1)$ such that $WW^T = wI_n$, where W^T is the transpose of W and I_n is the identity matrix of order n .A weighing matrix of order n and weight w is denoted by $W(n, w)$.

(i) A $W(n, n)$ is a hadamard matrix.

(ii) A $W(n, n-1)$, n even with zeros on the diagonal such that $WW^T = (n-1)I_n$ is conference matrix.

(iii) If $n \equiv 2(\bmod 4)$ such that $W = W^T$ is symmetric conference matrix..

(Vide:[1]and[5])

Example:

$$W(6,5) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

(Vide: [5])

1.2. PROPERTIES OF WEIGHING MATRICES:

If W is a $W(n, w)$ then:

(i).The rows of W are pairwise orthogonal. Similarly , the columns are pairwise orthogonal.

(ii). each row and each column of W has exactly w non –zero elements.

(iii). $W^T W = wI$,since the definition means that $W^{-1} = w^{-1}W^T$ where W^{-1} is the inverse of W .

(iv).If there is a $W(n, p)$ then there is a symmetric $W(n^2, p^2)$.

(v)For a weighing matrix $W(n, n-1)$ $WW^T = (n-1)I_n$ then $\det W \equiv W(n) = (n-1)^{\frac{n}{2}}$.(**Vide : [1])**

1.3. CONFERENCE MATRICES:

A conference matrix of order n is an $n \times n$ matrix M with diagonal entries 0 and other entries ± 1 which satisfies $MM^T = (n-1)I_n$.

Where : M^T is transpose of M and I_n is the identity matrix.

Examples: $$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

(vide : [3])

1. 4.SYMMETRIC CONFERENCE MATRICES:

A conference matrix M with entries $o,+1, and ,-1$ is called symmetric conference matrix if $MM^T = MM^T = nI_n$

Where: n is order of matrix, I_n is identity matrix and M^T is the transpose of M .

(vide :[3])

Examples:

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 1 & -1 & 0 \end{bmatrix} \quad and \quad M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

(vide:[3])

1.5. PROPERTIES OF SYMMETRIC CONFERENCE MATRICES AND CONFERENCE MATRICES:

Some important properties of Symmetric Conference matrices and conference matrices are given below:

1. The order of conference matrix is of the form 4t+2,

2. $n-1$ where n is the order of a conference matrix , must be the sum of two squares;

3. If there is a conference matrix of order n then there is a symmetric conference matrix of order n with zero diagonal .The two forms are equivalent as one can be transformed into the other by

(i) Interchanging rows (columns) or

(ii) multiplying rows (columns) by -1;

4. A conference matrix is said to be normalized if it has first rows and columns all plus ones.

5. $M^{-1} = nM^T$

(vide :[3])

1.6. SKEW-CONFERENCE MATRIX:

A conference matrix M with entries $0, and ,\pm 1$ is called skew symmetric matrix conference matrix if

$$M^T = -M$$

Where: T denotes the matrix transpose.

(vide : [11])

Example: $$M = \begin{bmatrix} 0 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 0 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 0 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 0 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 0 \end{bmatrix}$$

1.7. COHERENT CONFIGURATION (CC):

Let X be a finite set .A coherent configuration on X is a set $C = \{C_1, C_2, C_2, ..., C_m\}$ of binary relation on X (subsets of $X \times X$) satisfying the following four conditions:

(i) $'C'$ is a partition of $X \times X$ that is

$$\bigcup_{i=1}^{m} C_i = X \times X \quad ;$$

(ii) There exist a sub set C_o of C which is a partition of the diagonal $D = ((x, x) : x \in X)$

(iii) For every relation $C_i \in C$, its converse $C_i^{'} = \{(\beta, \alpha) : (\alpha, \beta) \in C^i\}$ is in C say $C_i^{'} = C_{i^*} \in C_k$

(iv) There exist integer P_{ij}^k for $1 \le i, j, k \le m$ such that for any $(\alpha, \beta) \in C_k$ the number of points $\gamma \in X$ such that $(\alpha, \gamma) \in C_i, and, (\gamma, \beta) \in C_j$ is equal to P_{ij}^k (and ,in particular, is independent of the choice of $(\alpha, \beta) \in C_k$.

That is we have

$$P_{ij}^k = \left|\{C_i(\alpha) \cap C_j^{'}(\beta)\}\right| for (\alpha, \beta) \in C_k$$

Where $C(\alpha) = \{\beta \in X : (\alpha, \beta) \in C\}$.

C.C. is also defined by adjacency matrices of classes of C. If A_1 A_2 ,.., A_m are adjacency matrices of C_1, C_2,..., C_m respectively then the axioms takes the following from

(i) $A_1 + A_2 + ... + A_m = J$

(i) There exist a sub set of $\{A_{1,} A_2, ..., A_m\}$ with sum I=identity matrix ;

(ii) Each elements of the set $\{A_1, ..., A_m\}$ is closed under transposition ;

(iii) $A_i \; A_j = \sum_{i=1}^{m} P_{ij}^k \; A_k$ where P_{ij}^k are non – negative integers.

(Vide: Singh and Manjhi [8]).

2. MAIN WORK:

In this paper we construct two conference matrices each of orders 6 by suitable linear combination of coherent configurations.

2.1. CONSTRUCTION OF SYMMETRIC CONFERENCE MATRIX OF ORDER 6:

Consider $X = \{1,2,3,4,5,6\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ of $X \times X$ *whrere*

$C_1 = \{(i,i) : i = 1\}$,
$C_2 = \{(1,i) : i = 2,3,4,5,6\}$,
$C_3 = \{(i,1) : i = 2,3,4,5,6\}$,
$C_4 = \{(i,i) : i = 2,3,4,5,6\}$,
$C_5 = \{(2,i) : i = 3,6\} \cup \{(3,i) : i = 2,4\} \cup \{(4,i) : i = 3,5\} \cup \{(5,i) : i = 4,6\} \cup \{(6,i) : i = 2,5\}$,
$C_6 = \{(2,i) : i = 4,5\} \cup \{(3,i) : i = 5,6\} \cup \{(4,i) : i = 2,6\} \cup \{(5,i) : i = 2,3\} \cup \{(6,i) : i = 3,4\}$.

Then adjacency matrices M_1, M_2, M_3, M_4, M_5 and M_6 of C_1, C_2, C_3, C_4, C_5 *and* C_6 respectively are given bellow:

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} M_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} M_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

We see that

1. $M_1 + M_2 + M_3 + M_4 + M_1 + M_1 = J_6$
2. $M_1 + M_4 = I_6$
3. $M_1^{'} = M_1, M^{'}{}_2 = M_2, M_3^{'} = M_2, M_4^{'} = M_4, M_5^{'} = M_5, M_6^{'} = M_6$
4. We see the following calculations:

(i) $M_1^2 = M_1, M_1M_2 = M_2, M_1M_3 = 0, M_1M_4 = 0, M_1M_5 = 0, M_1M_6 = 0$

(ii) $M_2^2 = 0, M_2M_3 = 5M_1, M_2M_4 = M_2, M_2M_5 = 2M_2, M_2M_6 = 2M_2$

(iii) $M_3^2 = 0, M_3M_4 = 0, M_3M_5 = 0, M_3M_6 = 0$

(iv) $M_4^2 = M_4, M_4M_5 = M_5, M_4M_6 = M_6$

(v) $M_5^2 = 2M_4 + M_6, M_5M_6 = M_5 + M_6$

(vi) $M_6^2 = 2M_4 + M_5$

Hence product of any two adjacency matrices is some linear combinations of adjacency matrices.

Thus the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ is a C.C.

Consider the matrix $M = 0.M_1 + 1.M_2 + 1.M_3 + 0.M_4 + 1.M_5 + (-1).M_6$

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

$$\because \; MM^T = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$= 5I_6 = (6-1)I_6$

$\Rightarrow MM^T = (6-1)I_6$

$$M^TM = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & -1 & -1 & 1 \\ 1 & 1 & 0 & 1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 1 & -1 \\ 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$= 5I_6 = (6-1)I_6$

$\Rightarrow M^TM = (6-1)I_6$

Thus $MM^T = M^TM = (6-1)I_6$

Which show that M is a symmetric conference matrix of order 6 .

2.2. Consider $X = \{1,2,3,4,5,6\}$ and a partition $C = \{C_1, C_2, C_3, C_3, C_4, C_5, C_{6\}}\}$ of $X \times X \quad where$

$C_1 = \{(i,i) : i = 1\}$,

$C_2 = \{(1,i) : i = 2,3,4,5,6\}$,

$C_3 = \{(i,1) : i = 2,3,4,5,6\}$,

$C_4 = \{(i,i) : i = 2,3,4,5,6\}$

$C_5 = \{(2,i) : i = 4,5\} \cup \{(3,i) : i = 5,6\} \cup \{(4,i) : i = 2,6\} \cup \{(5,i) : i = 2,3\} \cup \{(6,i) : i = 2,5\}$

$C_6 = \{(2,i) : i = 3,6\} \cup \{(3,i) : i = 2,4\} \cup \{(4,i) : i = 3,5\} \cup \{(5,i) : i = 4,6\} \cup \{(6,i) : i = 2,5\}$

Then adjacency matrices $M_1, M_2, M_3, M_4 M_5, and, M_6$ of $C_1, C_2, C_3 C_4, C_5 \; and \; C_6$ respectively are given bellow:

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} M_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} M_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We see that

$1. M_1 + M_2 + M_3 + M_4 + M_5 + M_6 = J_6$

$2. M_1 + M_4 = I_6$

$3. M_1^{'} = M_1, M_2^{'} = M_3, M_3^{'} = M_2, M_4^{'} = M_4, M_5^{'} = M_5, M_6^{'} = M_6$

We see the following calculations

$(i)\ \ M_1^2 = M_1., M_1M_2 = M_2, M_1M_3 = 0, M_1M_4 = 0, M_1M_5 = 0, M_1M_6 = 0$

$(ii)\ \ M_2^2 = 0, M_2M_3 = 5M_1, M_2M_4 = M_2, M_2M_5 = 2M_2, M_2M_6 = 2M_2$

$(iii)\ \ M_3^2 = 0, M_3M_4 = 0, M_3M_5 = 0, M_3M_6 = 0$

$(iv)\ \ M_4^2 = M_4, M_4M_5 = M_5, M_4M_6 = M_6$

$(v)\ \ M_5^2 = 2M_4 + M_6, M_5M_6 = M_5 + M_6$

$(vi)\ \ M_6^2 = 2M_4 + M_5$

Hence, product of any two adjacency matrices is some linear combinations of adjacency matrices .

Also, $M_1^{'} = M_1, M_2^{'} = M_3, M_3^{'} = M_2, M_4^{'} = M_4, M_5^{'} = M_5, M_6^{'} = M_6$

Thus the set $C = \{C_1, C_2, C_3, C_4 C_5, C_6\}$ is a C.C.

Consider the matrix $M = 0.M_1 + 1.M_2 + 1.M_3 + 0.M_4 + 1.M_5 + (-1).M_6$

$M = 0.M_1 + 1.M_2 + 1.M_3 + 0.M_4 + 1.M_5 + (-1).M_6$

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 1 & -1 & 0 \end{bmatrix}$$

$$\because MM^T = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$= 5I_6 = (6-1)I_6$

$\Rightarrow MM^T = (6-1)I_6$

$$M^T M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 & 1 \\ 1 & 1 & 1 & -1 & 0 & -1 \\ 1 & -1 & 1 & 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$= 5I_6 = (6-1)I_6$

$\Rightarrow M^T M = (6-1)I_6$

Thus $MM^T = M^T M = (6-1)I_6$

Which show that M is another symmetric conference matrix of order 6.

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