EDGE – Even Graceful Labeling on Circulant Graphs

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Abstract

Let G = (V,E) be a simple ,finite undirected and connected graph. A graph G = (V,E) be a graph with order p and size q. G admits an Edge – even graceful labeling if there exists a bijection f from E to {2,4,6… 2q-2} so that the induced mapping f^+ from V to {0,1,2…2q-1} given by

$f^+(X) = \sum f\{XY / xy \in E\}(\text{mod}|2q|)$

In this paper we have constructed an Edge – even graceful labeling on circulant graphs with generating sets (1,2) (1,2,3) and (1,2,3,4) for odd n , nϵ I.

Keywords

Labeling ,Graceful labeling , Even graceful labeling ,Edge graceful labeling, Edge- even graceful labeling, Circulant graph.

I. Introduction

The study of graceful graphs and graceful labeling methods were introduced by Rosa in1967 . S.Lo introduced edge – graceful labeling and A.Solairaju and K.Chitra introduced a new type of labeling of a graph G with q edges called an Edge – odd graceful labeling Gayathri introduced Even - edge graceful labeling if there is a bijection f from the edge of the graph to the set {2,4,6…. 2q-2} such that vertex is assigned the sum of all the edges incident to it (mod2q) and the resulting vertex are distinct reference [5] and [8].

In this paper, we considered the problem of labeling the edges and the vertices in such a way that the labeling of the edges and the vertices should be distinct integers.

We have constructed an Edge – even graceful labeling for circulant graphs .

II Main Results :

Edge – Even Graceful Labeling on Circulant Graphs.

Theorem 2.1:

For odd n ≥ 5, the circulant graph,G = Cn(1,2) admits edge –even graceful labeling

Here (1,2) are the generators of G .

Proof :

Let G= C_n (1,2) be the 4- regular graph with n ≥ 5 .

Let V(G) = {V_i / i=0,1…n-1}.Here q=2n .

Define the mapping as follows f : E(G) → {2,4,…2q-1}by

$$f(V_iV_{i+1}) = \begin{cases} 2i+2 & \text{for } i = 0,2\ldots n-1 \\ n+2i-3 & \text{for } i = 1,3,\ldots n-2 \end{cases}$$

$f(V_iV_{i+2})$ = n+2i+7 for i = 0,1,…n-1.

It can be verified that the edge label under the labeling f is a bijection from the set E (C_n(1,2)) onto the set {2,4,6,…(2n)-1}.

For every vertex v∈ V(G) , the vertex-weight $f^+(v)$ of C_n (1,2) are defined as follows.

Case(i) : For i = 0,1

$\sum_{e \in N(v_0)} f(e)$ = 2i+2+n+2i+7+n+2i+7+n+2i-3.

= 2+n+7+n+2(n)+7+n+2(n-3)-3.

= 7n+7.

$f(v_0)$ = 42.

For i = 1

$\sum_{e \in N(v_1)} f(e)$ = n+2i-3+n+2i+7+n+2i+7+2i+2.

= n+2-3+n+2+7+n+2(n-4)+7+2(n-2)+2.

= 7n+5 .

$f(v_1)$ = 40.

Case(ii) For i = 2,4,6…

$\sum_{e \in N(v_i)} f(e)$ = 2i+2+n+2i+7+n+2i+7+n+2i-3

= 2i+2+n+2i+7+n+2(i)+7+n+2(i-3)-3.

= 3n+8i+7.

Case(iii) For i = 3,5,…

$\sum_{e \in N(v_i)} f(e)$ = n+2i-3+n+2i+7+n+2i+7+2i+2.

= n+2i-3+n+2i+7+n+2i+7+2(i-3)+2.

= 3n+8i+7.

Thus , we obtained that the sum of the values assigned to all the edges incident to a given vertex v∈V(G) are all distinct integers.

Thus, the vertex – weight induced f^+ from V to {0,1,2…2n-1} admits an edge – even graceful labeling .

Theorem 2.2:

For odd n ≥ 7 , the circulant graph G = Cn (1,2,3) admits an edge – even graceful labeling .

Here (1,2,3) are the generators of G .

Proof :

Let G = C_n (1,2,3) be the 6- regular graph with n ≥ 7 .

Let V(G) = {V_i / i = 0,1,….n-1}.Here q = 3n .

Define the function .

$$f(V_iV_{i+1}) = \begin{cases} 2i+2 & \text{for } i = 0,2,\dots n-1 \\ n-5+2i & \text{for } i = 1,3,\dots.n-2 \end{cases}$$

$$f(V_iV_{i+2}) = \begin{cases} 3n-5+2i & \text{for } i = 0,1,2\dots n-1 \\ 4n+i+14 & \text{for } i = 0 \end{cases}$$

$$f(V_iV_{i+3}) = 4n+2i \quad \text{for } i = 1,2,\dots n-1$$

It can be verified that the edge label under the labeling f is a bijection from the set E(C_n(1,2,3)) onto the set {2,4,6…(3n)-1}.

For every vertex v∈ V(G) the vertex –weight f^+(v) of C_n (1,2,3) are defined as follows

Case(i) For i = 0,1,2,3

For i = 0

$\sum_{e \in N(v_0)} f(e)$ = 2i+2+3n-5+2i+4n+i+14+4n+i+14+3n-5+2i+n-5+2i.

= 2+3n-5+4n+14+4n+(n-5)+14+3n-5+2(n-4)+n-5+2(n-3).

= 20n-4.

$f(v_0)$ = 136.

For i = 1

$\sum_{e \in N(v_1)} f(e)$ = n-5+2i+3n-5+2i+4n+2i+4n+i+14+3n-5+2i+2i+2

= n-5+2+3n-5+2+4n+2+4n+(n+1)+14+3n-5+ 2(n)+2(-n)+2.

= 16n +8.

$f(v_1)$ = 120.

For i = 2

$\sum_{e \in N(v_2)} f(e)$ = 2i+2+3n-5+2i+4n+2i+4n+2i+3n-5+2i+2i+2.

= 4+2+3n-5+4+4n+4+4n+2(n-2)+3n-5+2(n-3)+2(-2)+2.

= 18n-8.

$f(v_2)$ = 118.

For i = 3

$\sum_{e \in N(v_3)} f(e)$ = n-5+2i+3n-5+2i+4n+2i+4n+2i+3n-5+2i+2i+2

= n-5+6+3n-5+6+4n+6+4n+2(n-3)+3n-5+2(n-4)+6+2.

= 19n-3.

$f(v_3)$ = 130.

Case(ii) For i = 4,6…

$\sum_{e \in N(v_i)} f(e)$ = 2i+2+3n-5+2i+4n+2i+4n+2i+3n-5+2i+n-5+2i

= 2i+2+3n-5+2i+4n+2i+4n+i+2(i-1)+3n-5+

2(i-2)+n-5+2(i-3).

= 15n+12i-25.

Case(iii) For i= 5,7…

$\sum_{e \in N(v_i)} f(e)$ = n-5+2i+3n-5+2i+4n+2i+4n+2i+3n-5+2i+2i+2

= n-5+2i+3n-5+2i+4n+2i+4n+2(i-1)+3n-5+2(i-2)+

2(i-3)+2.

= 15n+12i-25.

Thus , we obtained that the sum of the values – assigned to all the edges incident to a given vertex v∈V(G) are all distinct integers.

Thus, the vertex – weight induced f^+ from V to {0,1,2…3n-1} admits an edge – even graceful labeling .

Theorem 2.3:

For odd n ≥ 9, the circulant graph G = C_n (1,2,3,4) admits an edge – even graceful labeling .

Here (1,2,3,4) are the generators of G .

Proof :

Let G = C_n(1,2,3,4) be the 8- regular graph with n ≥ 9 .

Let V(G) = {V_i / i = 0,1,….n-1}.Here q = 4n .Define the function f as follows:

$$f(V_iV_{i+1}) = \begin{cases} 2i+2 & \text{for } i = 0,2,\ldots,n-1 \\ n+2i-7 & \text{for } i = 1,3,5\ldots,n-2 \end{cases}$$

$f(V_iV_{i+2})$ = n+2i+11 for i = 0,1,2….n-1

$f(V_iV_{i+3})$ = 3n+2i+11 for i = 0,1,2…n-1

$f(V_iV_{i+4})$ = 5n+2i+11 for i = 0,1,2….n-1

It can be verified that the edge label under the labeling f is a bijection from the set E(C_n(1,2,3,4)) onto the set {2,4,6…(4n)-1}.

For every vertex v∈ V(G) the vertex –weight $f^+(v)$ of C_n (1,2,3,4) is defined as follows

Case(i) For i = 0,1,2,3

For i = 0

$\sum_{e \in N(v_0)} f(e)$ = 2i+2+n+2i+11+3n+2i+11+5n+2i+11+5n+2i+11+3n +2i +11+n+2i+11+n+2i-7.

= 2+n+11+3n+11+5n+11+5n+2(n-2)+11+3n+2(n-3)+11

+n+2(n-4)+11+n+2(n)-7.

= 27n+41.

$f(v_0)$ = 284.

For i = 1

$\sum_{e \in N(v_1)} f(e)$ = n+2i-7+n+2i+11+3n+2i+11+5n+2i+11+5n+2i+11+3n+

2i+11+n+2i+11+2i+2.

= n+2- 7+n+2+11+3n+2+11+5n+2+11+5n+

2(n-6)+11+3n+2(n-5)+11+n+2(n-4)+11+2(n)+2.

= 27n+39.

$f(v_1)$ = 282.

For i = 2

$\sum_{e \in N(v_2)} f(e)$ = 2i+2+n+2i+11+3n+2i+11+5n+2i+11+5n+

2i+11+3n+2i+11+n+2i+11+n+2i-7.

= 4+2+n+4+11+3n+4+11+5n+4+11+5n+2(n-7)+11+

3n+2(n-6)+11+n+2(n-5)+11+n+2(n-2)-7.

= 27n+37.

$f(v_2)$ = 280.

For i = 3

$\sum_{e \in N(v_3)} f(e)$ = n+2i-7+n+2i+11+3n+2i+11+5n+2i+11+5n+2i+11+3n

+2i+11+n+2i+11+2i+2.

= n+6- 7+n+6+11+3n+6+11+5n+6+11+5n+2(n-9)

+11+3n+2(n-8)+11+n+2(n-7)+11+2(n-1)+2.

= 27n+35.

$f(v_3)$ = 278.

Case(ii) : For i = 4,6…n-2

$\sum_{e \in N(v_i)} f(e)$ = 2i+2+n+2i+11+3n+2i+11+5n+2i+11+5n+2i+11+3n

+2i+11+n+2i+11+n+2i-7.

= 2i+2+n+2i+11+3n+2i+11+5n+2i+11+5n+2(i-4)+11+3n +2(i-3)+11+n+2(i-2)+11+n+2(i-1)-7.

= 19n+16i+41.

Case(iii) :For i = 3,5…

$\sum_{e \in N(v_i)} f(e)$ = n+2i-7+n+2i+11+3n+2i+11+5n+2i+11+3n+2i+11+n +2i+11+2i+2.

= n+2i-7+n+2i+11+3n+2i+11+5n+2i+11+5n+2(i-4) +11+3n+2(i-3)+11+n+2(i-1)+2.

= 19n+16i+41.

Thus , we obtained that the sum of the values – assigned to all the edges incident to a given vertex v∈V(G) are all distinct integers.

Thus the vertex – weight induced f^+ from V to {0,1,2…4n-1} admits an edge – even graceful labeling .

Conclusion:

In this paper , we obtained an Edge – Even graceful labeling on circulant graphs with generating sets (1,2),(1,2,3) and (1,2,3,4) . In future , we proposed to extend the study for circulant graphs with higher order generating sets.

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