Some Algebraic Concepts on Discrete Fuzzy Numbers

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ABSTRACT

In this paper a new definition of fuzzy ring and field with binary operations on the set of discrete fuzzy numbers are defined. Some properties related to these concepts are investigated.

KEYWORDS

Euclidean distance, Discrete fuzzy number, Fuzzy approximation ambiguity rank.

1 INTRODUCTION

Chang and Zadeh[7] introduced the concept of fuzzy number with the consideration of the properties of probability functions.Since then a lot of mathematicians have been studying on fuzzy number, and have obtained many results [6,8,10,12,21,23] .

In 2001 Voxman [20] introduced the concept of discrete fuzzy numbers, the discrete fuzzy number can be used to represent the pixel value in the centre point of a window [8] Also he gave out the canonical representation of discrete fuzzy numbers.

In 1971,Rosenfeld[19] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structure.Therefore the notion of different fuzzy algebraic strutures such as fuzzy ideals in rings and semi-rings etc. have been seriously studied by many mathematicians.

In 1982 W.J.Liu[15] introduced the concept of fuzzy ring and fuzzy ideal. In 1985 Ren [18] studied the notions of fuzzy ideal, in the case of Liu and Ren were actually a rational extension of Rosenfeld's fuzzy group by starting with an ordinary ring and then define a fuzzy sub-ring based on the ordinary operations of the given ring. Based on the notion of fuzzy space which play the role of universal set in ordinary algebra and using fuzzy binary operation K.A Dib[9] obtained a new formulation for fuzzy rings and fuzzy ideals. Since then many mathematicians such as Aktaÿs and Cÿaćgman[5],Malik and Mordeson[16,17], Yuan and Lee[24] have studied about them and more recently in S.Abdullah et al[1-4,13]

In this paper , the basic definitions and concepts related to the topic is discussed. Further a new definition of fuzzy ring and field with binary operations on the set of discrete fuzzy numbers are defined. Some of its characteristic properties are investigated.

2 PRELIMINARIES

Definition 2.1 Let X be a universe of discourse, a fuzzy set is defined as $A = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$ characterized by a membership function $\mu_A(x): X \to [0,1]$, where $\mu_A(x)$ denotes the degree of membership of the element x to the set A.

Definition 2.2 The α-cut, ${}^{\alpha}A$ of a set A is the crisp subset of A with membership grades of at least α. That is, ${}^{\alpha}A = \{x \mid \mu_A(x) \geq \alpha\}$.

Definition 2.3 A fuzzy set on the real line R is a fuzzy number if it at least possess the following properties.

(i) A must be a normal fuzzy set,

(ii) The alpha levels ${}^{\alpha}A$ must be closed for every $\alpha \in [0,1]$.

(iii) The support of A, ${}^{0+}A$ must be bounded.

Definition 2.4 A fuzzy subset u of R with membership mapping $u: R \to [0,1]$ is called discrete fuzzy number (*DFN*) if its support is finite (ie) there are $x_1, x_2, \dots \dots \dots .., x_n \in R$ with $x_1 < x_2 <.. \dots < x_n$

such that $supp(u) = \{x_1, x_2, \dots \dots \dots \dots x_n\}$, and there are natural numbers s, t with$1 \leq s \leq t \leq n$

such that

$(i) u(x_i) = 1$ for any natural number i with $s \leq i \leq t$
$(ii) u(x_i) \leq u(x_j)$ for each natural numbers i, j with $1 \leq i \leq j \leq s$
$(iii) u(x_i) \geq u(x_j)$ for each natural number i, j with $t \leq i \leq j \leq n$.

Definition2.5[16](Equal fuzzy sets) Two fuzzy sets $\tilde{u}$ and $\tilde{v}$ are said to be equal (denoted $\tilde{u} = \tilde{v}$)if and only if $\forall x \in X,\ \mu_{\tilde{u}}(x) = \mu_{\tilde{v}}(x)$

Definition2.6(Euclidean distance) Let $X = (x_1, x_2, x_3, \dots\dots\dots x_n)$ and $Y = (y_1, y_2, y_3, \dots\dots\dots\dots y_n)$ are two points in Euclidean n-space, then the distance from *X* to *Y* or from *Y* to *X* is given by

$$d(\mathrm{X}, \mathrm{Y}) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{\frac{1}{2}}$$

3.AMBIGUITY RANKS ON FUZZY GROUP OF DISCRETE FUZZY NUMBERS

Definition 3.1

For $A \subset R$ denote $max\ A = max\ \{ x : x \in A\}$, and$min\ A = min\ \{x : x \in A\}$,Let *u* be a discrete fuzzy number a fuzzy set on *R*withits support $[u]^0$ finite

For any $r \in [0,1]$ and $x_0 \in [u]^0$ denote$\bar{u}(r) = max [u]^r$, $\underline{u}(r) = min\ [u]^r$ for$r \leq 1$

where$[u]^r = \{x / u(x) \geq r\}$

Let us define some sets as follows

$$\text{(i)}\ [\overline{u}]_{r \leq 1} = \left\{ x \in [u]^0 / x \leq \underline{u}(r) \right\}$$

$$\text{(ii)}\ [\underline{u}]_{r \leq 1} = \left\{ x \in [u]^0 / x \geq \bar{u}(r) \right\}$$

$$\text{(iii)}\ [\underline{u}(r), \bar{u}(r)]_{r \leq 1} = \left\{ x \in [u]^0 / \ \underline{u}(r) \leq x \leq \bar{u}(r) \right\}$$

$$\text{(iv) av}[u]_r = \frac{\bar{u}(r) + \underline{u}(r)}{2}$$

$$\text{(v)}\ [\tilde{u}](x) = \begin{cases} [\ddot{u}]_{r\leq 1}(x) & if \quad x \in [\overline{u}]_{r\leq 1} \\ (u[r])(x) & if\ x \in [\underline{u}(r), \bar{u}(r)] \\ [\underline{u}]_{r\leq 1}(x) & if \quad x \in [\underline{u}]_{r\leq 1} \end{cases}$$

Definition3.2 Let $\tilde{u}, \tilde{v}$ be two discrete fuzzynumber.then, $\tilde{u} \sim \tilde{v}$ if and only if

(i) For every $x \in [\overline{u}]_{r \le 1}, y \in [\overline{v}]_{r \le 1}$, if $d(x,\underline{u}(r)) = d(y,\underline{v}(r))$ then $[\ddot{u}]_{r\le 1}(x) = [\ddot{v}]_{r\le 1}(y)$

(ii) $d(\underline{u}(r),\overline{u}(r)) = d(\underline{v}(r),\overline{v}(r))$ then $(u[r])(x) = (v[r])(y)$

for $x \in [\underline{u}(r),\overline{u}(r)], y \in [\underline{v}(r),\overline{v}(r)]$

(iii) For every $x \in [\underline{u}]_{r \le 1}, y \in [\underline{v}]_{r \le 1}$, if $d(x,\overline{u}(r)) = d(y,\overline{v}(r))$ then $[\underline{u}]_{r\le 1}(x) = [\underline{v}]_{r\le 1}(y)$

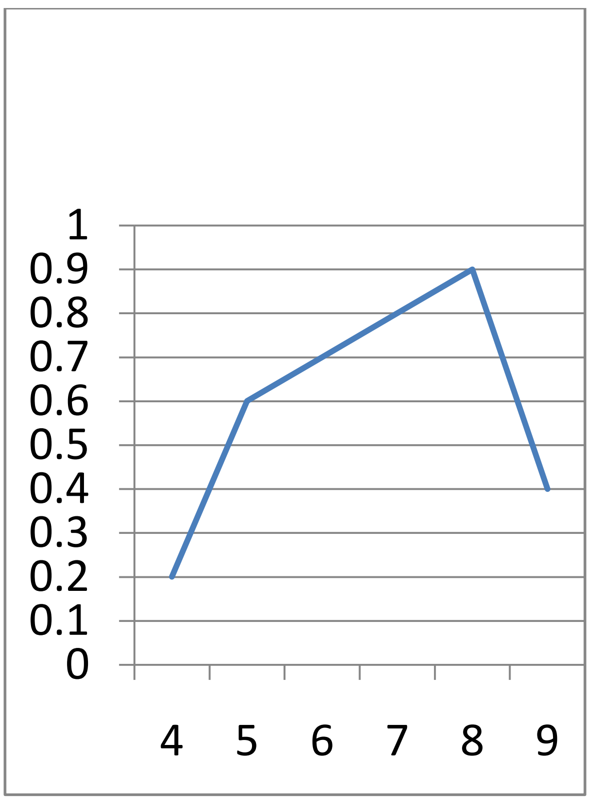
Definition 3.3 Two discrete fuzzy number $\tilde{u}$ and $\tilde{v}$ are said to be equal (denoted $\tilde{u} = \tilde{v}$) if and only if

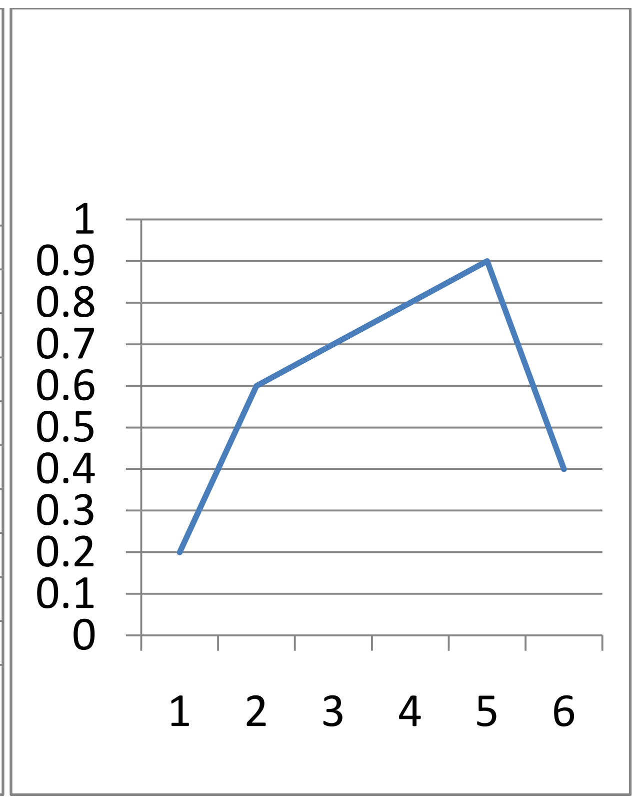
for every $x \in X, [u]_{r\le1}(x) = [v]_{r\le1}(x)$

Example 3.4 Consider the discrete fuzzy number

$$u = \left(\frac{0.2}{4},\frac{0.6}{5},\frac{0.7}{6},\frac{0.8}{7},\frac{0.9}{8},\frac{0.4}{9}\right), v = \left(\frac{0.2}{1},\frac{0.6}{2},\frac{0.7}{3},\frac{0.8}{4},\frac{0.9}{5},\frac{0.4}{6}\right)$$

By using definition 3.2, u □ v





Definition 3.5 (Fuzzy approximation) Let $\tilde{u}, \tilde{v}$ be two discrete fuzzy number then $\tilde{u} \cong \tilde{v}$ if and only if av$[u]_r$ = av $[v]_r$

Definition 3.6 (Fuzzy group) A fuzzy group is an ordered pair $(\tilde{G},*)$ where $\tilde{G}$ is a set of discrete fuzzynumberand $*$ is a binary operation on $\tilde{G}$ satisfying the following properties

(i) For every $\tilde{u},\tilde{v},\tilde{w}\in\tilde{G}$, $(\tilde{u}*\tilde{v})*\tilde{w}\cong\tilde{u}*(\tilde{v}*\tilde{w})$

(ii) For every $\tilde{u},\tilde{v}\in\tilde{G}$ if $\tilde{u}\,\square\,\tilde{v}$ then $\tilde{u}*\tilde{v}\,\square\,\tilde{u}$ (or) $\tilde{u}*\tilde{v}\,\square\,\hat{v}$

(iii) For every $\tilde{u}\in\tilde{G}$ there exists $e_{\tilde{u}}\in\tilde{G}$; $e_{\tilde{u}}\,\square\,\tilde{u}$ and $\tilde{u}*e_{\tilde{u}}=e_{\tilde{u}}*\tilde{u}=\tilde{u}$

(iv) For every $\tilde{u}\in\tilde{G}$ there exists $\widetilde{u^{-1}}\in\tilde{G}$; $\widetilde{u^{-1}}\,\square\,\tilde{u}$ and $\tilde{u}*\widetilde{u^{-1}}=\widetilde{u^{-1}}*\tilde{u}=e_{\tilde{u}}$

Definition 3.7 A fuzzy group $(\tilde{G},*)$ is said to be abelian if $\tilde{u}*\tilde{v}=\tilde{v}*\tilde{u}$ for all $\tilde{u}$ and $\tilde{v}$ in $\tilde{G}$ .A fuzzy group is said to be non-abelian if it is not abelian.

Definition 3.8(Fuzzy ring)A fuzzy ring is a triplet $(\tilde{G},*,\circ)$ where $\tilde{G}$ is a set of the discrete fuzzy numbers $*$ and $\circ$ are binary operations on $\tilde{G}$ satisfying the following properties.

(i) For every $\tilde{u},\tilde{v},\tilde{w}\in\tilde{G}$, $(\tilde{u}*\tilde{v})*\tilde{w}\cong\tilde{u}*(\tilde{v}*\tilde{w})$

(ii) For every $\tilde{u},\tilde{v}\in\tilde{G}$ if $\tilde{u}\,\square\,\tilde{v}$ then $\tilde{u}*\tilde{v}\,\square\,\tilde{u}$ (or) $\tilde{u}*\tilde{v}\,\square\,\tilde{v}$

(iii) For every $\tilde{u}\in\tilde{G}$ there exists $e_{\tilde{u}}\in\tilde{G}$; $e_{\tilde{u}}\,\square\,\tilde{u}$ and $\tilde{u}*e_{\tilde{u}}=e_{\tilde{u}}*\tilde{u}=\tilde{u}$

(iv) For every $\tilde{u}\in\tilde{G}$ there exists $\widetilde{u^{-1}}\in\tilde{G}$; $\widetilde{u^{-1}}\,\square\,\tilde{u}$ and $\tilde{u}*\widetilde{u^{-1}}=\widetilde{u^{-1}}*\tilde{u}=e_{\tilde{u}}$

(v) For every $\tilde{u},\tilde{v}\in\tilde{G}$, $\tilde{u}*\tilde{v}=\tilde{v}*\tilde{u}$

(vi) For every $\tilde{u},\tilde{v},\tilde{w}\in\tilde{G}$, $(\tilde{u}\circ\tilde{v})\circ\tilde{w}\cong\tilde{u}\circ(\tilde{v}\circ\tilde{w})$

(vii) For every $\tilde{u},\tilde{v}\in\tilde{G}$ if $\tilde{u}\,\square\,\tilde{v}$ then $\tilde{u}\circ\tilde{v}\,\square\,\tilde{u}$ (or) $\tilde{u}\circ\tilde{v}\,\square\,\tilde{v}$

(viii) For every $\tilde{u},\tilde{v},\tilde{w}\in\tilde{G}$, $\tilde{u}\circ(\tilde{v}*\tilde{w})\cong(\tilde{u}\circ\tilde{v})*(\tilde{u}\circ\tilde{w})$ and $(\tilde{v}*\tilde{w})\circ\tilde{u}\cong(\tilde{v}\circ\tilde{u})*(\tilde{w}\circ\tilde{u})$

Definition 3.9 (Fuzzy commutative ring)A fuzzy ring $(\tilde{G},*,\circ)$ is said to be commutative if $\tilde{u}\circ\tilde{v}=\tilde{v}\circ\tilde{u}$ for all $\tilde{u},\tilde{v}$ in $\tilde{G}$

Definition3.10 (Fuzzy division ring)A fuzzy ring $(\tilde{G},*,\circ)$ is said tobe division if its non zero(non the neutral elements related $*$ binary operation)elements form a fuzzy group under $\circ$ binary operation.

Definition3.11 (Fuzzy field)A fuzzy field is a fuzzy commutative division ring.

Definition3.12(Fuzzy approximation ambiguity rank) Let $\tilde{G}$ be a fuzzy group then, ambiguity rank for $\tilde{u},\tilde{v}$ defined with $ar_{1uv}=(l_{11\tilde{u},\tilde{v}},r_{11\tilde{u},\tilde{v}},lr_{*\tilde{u},\tilde{v}},l_{12\tilde{u},\tilde{v}},r_{12\tilde{u},\tilde{v}})$ as follows

$$l_{11\tilde{u},\tilde{v}}=d[\sup\{d([\underset{....}{u}]^{-1}_{r\le 1}(\alpha),\text{av}[u]_r)/\alpha\in(0,1]\},\sup\{d([\underset{....}{v}]^{-1}_{r\le 1}(\alpha),\text{av}[v]_r)/\alpha\in(0,1]\}]$$

$$r_{11\tilde{u},\tilde{v}}=d[\sup\{d([\ddot{u}]^{-1}_{r\le 1}(\alpha),\text{av}[u]_r)/\alpha\in(0,1]\},\sup\{d([\ddot{v}]^{-1}_{r\le 1}(\alpha),\text{av}[v]_r)/\alpha\in(0,1]\}]$$

$lr_{*\tilde{u},\tilde{v}} = \sup\{d(u[r](x), v[r](y)) /\ x \in [\underline{u}(r), \overline{u}(r)], y \in [\underline{v}(r), \overline{v}(r)]\}$ suchthat

$$d(\underline{u}(r), \overline{u}(r)) = d(\underline{v}(r), \overline{v}(r))$$

$$l_{12\tilde{u},\tilde{v}} = \sup\{[\underset{\dots}{u}]_{r\le 1}(x), [\underset{\dots}{v}]_{r\le 1}(y) / \forall\ x \in [\overline{u}]_{r\le 1}, y \in [\overline{v}]_{r\le 1}; d(x, av[u]_r) = d(y, av[v]_r)\}$$

$$r_{12\tilde{u},\tilde{v}} = \sup\{[\ddot{u}]_{r\le 1}(x), [\ddot{v}]_{r\le 1}(y) / \forall\ x \in [\underline{u}]_{r\le 1}, y \in [\underline{v}]_{r\le 1}; d(x, av[u]_r) = d(y, av[v]_r)\}$$

Definition 3.13(Ambiguity rank of fuzzy group) Let $\tilde{G}$ be a fuzzy group then, ambiguity rank for $\tilde{G}$ is defined with $\tilde{G}_{ar1} = (l_{11\tilde{G}}, r_{11\tilde{G}}, lr_{*\tilde{G}}, l_{12\tilde{G}}, r_{12\tilde{G}})$ as follows

$$l_{11\tilde{G}} = \sup\{l_{11\tilde{u},\tilde{v}} / \tilde{u}, \tilde{v} \in \tilde{G}\}$$
$$r_{11\tilde{G}} = \sup\{r_{11\tilde{u},\tilde{v}} / \tilde{u}, \tilde{v} \in \tilde{G}\}$$
$$lr_{*\tilde{G}} = \sup\{lr_{*\tilde{u},\tilde{v}} / \tilde{u}, \tilde{v} \in \tilde{G}\}$$
$$l_{12\tilde{G}} = \sup\{l_{12\tilde{u},\tilde{v}} / \tilde{u}, \tilde{v} \in \tilde{G}\}$$
$$r_{12\tilde{G}} = \sup\{r_{12\tilde{u},\tilde{v}} / \tilde{u}, \tilde{v} \in \tilde{G}\}$$

Lemma3.14

Let $\tilde{G}$ be a fuzzy group with the ambiguity rank $\tilde{G}_{ar1}$.Then, $l_{11\tilde{G}}, r_{11\tilde{G}}, lr_{*\tilde{G}}, l_{12\tilde{G}}, r_{12\tilde{G}} \ge 0$ **Proof**

By definition $l_{11\tilde{G}} = \sup\{l_{11\tilde{u},\tilde{v}} / \tilde{u}, \tilde{v} \in \tilde{G}\}$

=M(say)>0

$\Rightarrow l_{11\tilde{G}} \ge 0$

Similarly $r_{11\tilde{G}}, lr_{*\tilde{G}}, l_{12\tilde{G}}, r_{12\tilde{G}} \ge 0$

Lemma3.15

Let $\tilde{G}$ be a fuzzy group with the ambiguity rank $\tilde{G}_{ar1}$.

Then, $(l_{12\tilde{G}}, r_{12\tilde{G},} lr_{*\tilde{G}}) = (0,0,0) \Rightarrow (l_{11\tilde{G}}, r_{11\tilde{G}}, lr_{*\tilde{G}}) = (0,0,0)$

Proof

Suppose for the sake of contradiction that $(l_{11\tilde{G}}, r_{11\tilde{G},}) \ne (0,0)$,The following three modes can be investigated

(i) $l_{11\tilde{G}} \ne 0$ and $r_{11\tilde{G}} = 0$

(ii) $l_{11\tilde{G}} = 0$ and $r_{11\tilde{G}} \neq 0$

(iii) $l_{11\tilde{G}} \neq 0$ and $r_{11\tilde{G}} \neq 0$

Case(i)

If $l_{11\tilde{G}} \neq 0$ then there exists $\tilde{u}, \tilde{v}$ in $\tilde{G}$

$$\sup\{d([\underset{....}{u}]^{-1}_{r\le1}(\alpha), \text{av}[u]_r) / \alpha \in (0,1]\} \neq \sup\{d([\underset{....}{v}]^{-1}_{r\le1}(\alpha), \text{av}[v]_r) / \alpha \in (0,1]\}$$

Without loss of generality ,we may assume that

$$\sup\{d([\underset{....}{u}]^{-1}_{r\le1}(\alpha), \text{av}[u]_r) / \alpha \in (0,1]\} > \sup\{d([\underset{....}{v}]^{-1}_{r\le1}(\alpha), \text{av}[v]_r) / \alpha \in (0,1]\}$$

Define, $x = \sup\{d([\underset{....}{u}]^{-1}_{r\le1}(\alpha), \text{av}[u]_r) / \alpha \in (0,1]\}$

$y = \sup\{d([\underset{....}{v}]^{-1}_{r\le1}(\alpha), \text{av}[v]_r) / \alpha \in (0,1]\}$

From definition3.13, there exists $s \subseteq \underline{\text{support}}(\tilde{u}); |s| = x - y$ so, $\forall t \in s$ there exists t_y

such that $d(t, av[u]_r) = d(t_y, av[v]_r)$, $[\underset{....}{u}]^{-1}_{r\le1}(t) > 0, [\underset{....}{v}]^{-1}_{r\le1}(t_y) = 0$

Therefore,$0 < [\underset{....}{u}]_{r\le1}(t) \le l_{12\tilde{G}}$ namely $l_{12\tilde{G}} \neq 0$ This is a contradiction.Similar to the above, the cases (ii),(iii) leads to $l_{12\tilde{G}} \neq 0$ and $l_{12\tilde{G}}, r_{12\tilde{G},} \neq (0,0)$ respectively, which are inconsistent..

Lemma3.16

Let $(\tilde{G}, *)$ be a fuzzy group with ambiguity rank $G_{ar1} = (0,0,0,0,0)$. Then

(i) $\forall \tilde{u}, \tilde{v} \in \tilde{G}$; $\tilde{u} \square \tilde{v}$

(ii) $\forall \tilde{u}, \tilde{v} \in \tilde{G}$; $\tilde{u} * \tilde{v} \square \tilde{u}$ (or) $\tilde{u} * \tilde{v} \square \hat{v}$

Proof

Suppose $\tilde{u}, \tilde{v} \in \tilde{G}$ with $(l_{11\tilde{G}}, r_{11\tilde{G}}, lr_{*\tilde{G}}) = (0,0,0)$

Then

$$\sup\{d([\underset{....}{u}]^{-1}_{r\le1}(\alpha), \text{av}[u]_r) / \alpha \in (0,1]\} = \sup\{d([\underset{....}{v}]^{-1}_{r\le1}(\alpha), \text{av}[v]_r) / \alpha \in (0,1]\}$$

$$\Rightarrow \sup\{d([\underset{....}{u}]^{-1}_{r\le1}(\alpha), \text{av}[u]_r) / \alpha \in (0,1]\} = \sup\{d([\underset{....}{v}]^{-1}_{r\le1}(\alpha), \text{av}[v]_r) / \alpha \in (0,1]\}$$

So$|\{x \in \underline{u}(r) / [\underline{u}]_{r\leq 1}(x) > 0\}| = |\{x \in \underline{v}(r) / [\underline{v}]_{r\leq 1}(x) > 0\}|$,

$|\{x \in \overline{u}(r) / [\ddot{u}]_{r\leq 1}(x) > 0\}| = |\{x \in \overline{v}(r) / [\ddot{v}]_{r\leq 1}(x) > 0\}|$

Therefore, the prerequisite for $\tilde{u} \square \tilde{v}$

Also from $(l_{12\tilde{G}}, r_{12\tilde{G},} lr_{*\tilde{G}}) = (0,0,0)$

For every $x \in \underline{u}(r), y \in \underline{v}(r); d(x, av[u]_r) = d(y, av[v]_r)$

$\Rightarrow d[[\underline{u}]_{r\leq 1}(x), [\underline{v}]_{r\leq 1}(y)] = 0$

$\Rightarrow [\underline{u}]_{r\leq 1}(x) = ([\underline{v}]_{r\leq 1}(y)$ ....(1)

And For every $x \in \overline{u}(r), y \in \overline{v}(r); d(x, av[u]_r) = d(y, av[v]_r)$

$\Rightarrow d[[\ddot{u}]_{r\leq 1}(x), [\ddot{v}]_{r\leq 1}(y)] = 0$

$\Rightarrow [\ddot{u}]_{r\leq 1}(x) = [\ddot{v}]_{r\leq 1}(y)$ ....(2)

And $x \in [\underline{u}(r), \overline{u}(r)], y \in [\underline{v}(r), \overline{v}(r)]$;

$\Rightarrow d(\underline{u}(r), \overline{u}(r)) = d(\underline{v}(r), \overline{v}(r))$

$\Rightarrow (u[r])(x) = (v[r])(y)$ ....(3)

Finallyfrom $(1), (2), (3)$ and by definition 3.1 it follows that $\tilde{u} \square \tilde{v}$

(ii) According to the second condition of definition3.6 and case(i) it is obvious.

Lemma3.17 Let $\tilde{u}, \tilde{v}$ be two discrete fuzzy number. Then $\tilde{u} \cong \tilde{v}$ and $\tilde{u} \square \tilde{v}$ if and only if $\tilde{u} = \tilde{v}$

Proof

If $\tilde{u} \cong \tilde{v}$ and $\tilde{u} \square \tilde{v}$ then by the definition (3.1) it follows that

(i) $\forall x \in [\overline{u}]_{r\leq 1}, y \in [\overline{v}]_{r\leq 1}$,if $d(x, \underline{u}(r)) = d(y, \underline{v}(r))$ then $[\ddot{u}]_{r\leq 1}(x) = [\ddot{v}]_{r\leq 1}(y)$

(ii) $d(\underline{u}(r), \overline{u}(r)) = d(\underline{v}(r), \overline{v}(r))$ then $(u[r])(x) = (v[r])(y)$

for $x \in [\underline{u}(r), \overline{u}(r)], y \in [\underline{v}(r), \overline{v}(r)]$

(iii) $\forall\, x \in \underline{[u]}_{r\le 1}, y \in \underline{[v]}_{r\le 1}$, if $d(x,\bar{u}(r)) = d(y,\bar{v}(r))$ then $[\underset{....}{u}]_{r\le 1}(x) = [\underset{....}{v}]_{r\le 1}(y)$

This implies $[u]_{r\le 1}(x) = [v]_{r\le 1}(y)$, $\forall x \in X, \forall y \in Y.$

Conversely $\tilde{u} = \tilde{v}$ by the definition (3.3) it is obvious.

Lemma3.18 Let $(\tilde{G},*)$ be a fuzzy group with ambiguity rank $G_{ar1} = (0,0,0,0,0).$

Then $\forall \tilde{u},\tilde{v} \in \tilde{G}\,; e_{\tilde{u}} = e_{\tilde{v}}$

Proof

According to the lemma 3.16 and by definition3.5, we have $e_{\tilde{u}}$ □ $e_{\tilde{v}}$

And $\begin{cases} \tilde{u} * e_{\tilde{u}} = e_{\tilde{u}} * \tilde{u} = \tilde{u} \\ \overline{v} * e_{\tilde{v}} = e_{\tilde{v}} * \tilde{v} = \tilde{v} \end{cases}$

Then $\begin{cases} av[u]_r * av[e_{\tilde{u}}] = av[e_{\tilde{u}}] * av[u]_r = av[u]_r \\ av[\mathrm{v}]_r * av[e_{\tilde{v}}] = av[e_{\tilde{v}}] * av[\mathrm{v}]_r = av[\mathrm{v}]_r \end{cases}$ ....(4)

The system (4) is a system at the crisp space, thus $av[e_{\tilde{u}}] = av[e_{\tilde{v}}]$ ....(5)

Therefore from the case (5) and the definition (3.4) we have $e_{\tilde{u}} \cong e_{\tilde{v}}$

Finally, by lemma3.17 , $e_{\tilde{u}} \cong e_{\tilde{v}}$

Theorem3.19 Algebraic group is a special case of the fuzzy group with ambiguity rank $G_{ar1} = (0,0,0,0,0).$

Proof

Let $(\tilde{G},*)$ be a fuzzy group with ambiguity rank $G_{ar1} = (0,0,0,0,0).$

Then to show that

(i) $\forall \tilde{u},\tilde{v},\tilde{w} \in \tilde{G}$, $(\tilde{u} * \tilde{v}) * \tilde{w} \cong \tilde{u} * (\tilde{v} * \tilde{w})$ ....(6)

and

(ii) there exists $e \in \tilde{G}\,; \tilde{u} \in \tilde{G}$ and $\tilde{u} * e = e * \tilde{u} = \tilde{u}$ ....(7)

According to the (4) and (5) conditions on fuzzy group,using lemma3.16 and lemma3.17we get the case (6). Also the case (7)holds bylemma 3.18,with definition $e = e_{\tilde{u}}$ for each the arbitrary $\tilde{u} \in \tilde{G}$.

It should mentioned, the inverse condition $\tilde{G}$ is obvious by (7).

4. REFERENCES

[1] S.Abdullah.M.Aslam,T.A. Khan and M.Naeem, A new type of fuzzy normal subgroups and fuzzy cosets,*Journal of Intelligent and Fuzzy Systems* 25(2013),37-47.

[2] S.Abdullah.M.Aslam and K.Hila, A new generalization of fuzzy bi-ideals in semigroups its Applications in fuzzy finite state machines, *Multiple-Valued Logic and Soft Computing* **24** (2015),599-623.

[3] S.Abdullah,N-dimensional $(\alpha;\beta)$ -Fuzzy Ideals of hemirings, *International journal of Machine Learning and Cybernetics***5** (2014),635-645.

[4]S.Abdullah.M.Aslam and H.Hedayathi,Interval valued $(\alpha;\beta)$ - intuitionstic fuzzy ideals in hemirings, *Journal of Intelligent and fuzzy Systems* **26** (2014),2873-2888 .

[5] H.Aktaÿs and Cÿaćgman, Atype of fuzzy ring ,*Arch Math Logic* **46** (2007), 165-177.

[6] J.J. Buckley ," Fuzzy complex numbers" *Fuzzy Sets and Systems*,33(1989), 333-345,1989.

[7]S.S.L.Chang , and L.A.Zadeh," On fuzzy mapping and control ", *IEEE Trans Man Cybernet.,* 2(1972),30-34.

[8] Diamond.D and Kloeden, "Metric space of fuzzy sets", *Fuzzy Sets and Systems*,100(1999), 63-71.

[9] K.A.Dib,N.Gaalhum and A.AM.Hassan,Fuzzy ring and fuzzy ideals,*Fuzzy Math* **4**(2)(1996),245-261.

[10] Dubois and Prade .H "Operations on fuzzy numbers",*Internet J Systems*,9(1978),613-626.

[11] D.Dubois and H.Prade,Fuzzy Sets Systems: Theory and Applications,1998.

[12] Goetschel R. andW.Voxman, "Elementry calculus",*Fuzzy Sets and Systems*,18(1986),31-43.

[13] K.Hila and S.Abdullah,A study on intuitionistic fuzzy sets in Γ - semihypergroups, *Journal of Intelligent and Fuzzy Systems* **26**(2014),1695-1710.

[14]G.J.Klir and B.Yuan,*Fuzzy sets and Fuzzy Logic:theory and applications*,Prentice-Hall PTR, Upper Saddlie River,1995.

[15]W.Liu,Fuzzy invariant subgroups and fuzzy ideals,*Fuzzy Sets and Systems* (1982),133-139.

[16] D.S.Malik and J.N. Mordeson,*Fuzzy commutative Algebra*, World Scientific Publishing, Singapore,1998.

[17] D.S.Malik and J.N.Mordeson, Fuzzy relations on rings and groups. *Fuzzy Setsand Systems* **43**(1991),117-123.

[18] Y.C.Ren,Fuzzy Ideals and quotient rings,*Fuzzy math* **4**(1985).19-26

[19] A.Rosenfield,Fuzzygroups,*J Math Anal Appl***35** (1971), 512-517

[20] VoxmanW ,"canonical representations of dicrete fuzzy numbers", *Fuzzy Sets and Systems* 11(2001), 457-466.

[21] Wang Guixiang, Li Youming and Wen Chenglin, "On fuzzy n- cell numbers and n- dimension fuzzy vectors", *Fuzzy Sets and Systems*, 158(2007), 71-84.

[22] WangGuixiang,Wu Cong and Chunhui Zhao, "Representation and Operations of Discrete Fuzzy Numbers", *Southeast Asian Bulletin of Mathematics*, 28(2005),1003-1010.

[23] Wang Guixiang, and Wu Congxin, " Fuzzy n-cell numbers and the differential of fuzzy n-cell number valuemappings",*Fuzzy Sets and Systems*,130(2002), 367-381.

[24] X.Yuan and E.S.Lee, Fuzzy group based on fuzzy binary operational, *Comput Math App* **47** (2004), 631-641.