Finite integral involving the spheroidal function, a class of polynomials multivariable I-functions and Fresnel integral

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ABSTRACT

In the present paper we evaluate a generalized finite integral involving the product of the spheroidal function,the Fresnel integral, the multivariable I-functions defined by Prasad [3] and general class of polynomials of several variables with general arguments. The importance of the result established in this paper lies in the fact they involve the I-function of several variables which is sufficiently general in nature and capable to yielding a large of results merely by specializing the parameters their in.

Keywords:Multivariable I-function, general class of polynomials, spheroidal function, multivariable H-function.

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1.Introduction and preliminaries.

In the present paper we evaluate a finite integral involving the product of the spheroidal function, multivariable I-functions defined by Prasad [3] and general class of polynomials with general arguments.

The multivariable I-function is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, \ldots z_r) = I_{P_2,Q_2,P_3,Q_3;\cdots;P_r,Q_r:P',Q';\cdots;P^{(r)},Q^{(r)}}^{0,N_2;0,N_3;\cdots;0,N_r:M',N';\cdots;M^{(s)},N^{(r)}} \left(\begin{array}{c|c} z_1 & (A_{2j}; \gamma'_{2j}, \gamma''_{2j})_{1,P_2}; \cdots ; \\ . & \\ . & \\ . & \\ z_r & (B_{2j}; \delta'_{2j}, \delta''_{2j})_{1,Q_2}; \cdots ; \end{array}\right.$$

$$\left.\begin{array}{c} (A_{rj}; \gamma'_{rj}, \cdots, \gamma^{(r)}_{rj})_{1,P_r} : (A'_j, \gamma'_j)_{1,P'}; \cdots ; (A^{(r)}_j, \gamma^{(r)}_j)_{1,P^{(r)}} \\ \\ (B_{rj}; \delta'_{rj}, \cdots, \delta^{(r)}_{rj})_{1,Q_r} : (B'_j, \delta'_j)_{1,Q'}; \cdots ; (B^{(r)}_j, \delta^{(r)}_j)_{1,Q^{(r)}} \end{array}\right) \tag{1.1}$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \phi(s_1, \cdots, s_r) \prod_{i=1}^{r} \theta_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_r \tag{1.2}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_i| < \frac{1}{2}\Omega_i\pi \ , \ \text{where}$$

$$\Omega_i = \sum_{k=1}^{N^{(i)}} \gamma_k^{(i)} - \sum_{k=N^{(i)}+1}^{P^{(i)}} \gamma_k^{(i)} + \sum_{k=1}^{M^{(i)}} \delta_k^{(i)} - \sum_{k=M^{(i)}+1}^{Q^{(i)}} \delta_k^{(i)} + \left(\sum_{k=1}^{N_2} \alpha_{2k}^{(i)} - \sum_{k=N_2+1}^{P_2} \gamma_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{N_r} \gamma_{rk}^{(i)} - \sum_{k=N_r+1}^{P_r} \gamma_{rk}^{(i)} \right) - \left(\sum_{k=1}^{Q_2} \delta_{2k}^{(i)} + \sum_{k=1}^{Q_3} \delta_{3k}^{(i)} + \cdots + \sum_{k=1}^{Q_r} \delta_{rk}^{(i)} \right) \tag{1.3}$$

where $i = 1, \cdots, r$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\gamma_1'}, \cdots, |z_r|^{\gamma_r'}) , max(|z_1|, \cdots, |z_r|) \to 0$$

$$I(z_1, \cdots, z_r) = 0(|z_1|^{\delta_1'}, \cdots, |z_r|^{\beta_r'}) , min(|z_1|, \cdots, |z_r|) \to \infty$$

where $k = 1, \cdots, r : \gamma_k' = min[Re(B_j^{(k)}/\delta_j^{(k)})], j = 1, \cdots, M_k$ and

$$\delta_k' = max[Re((A_j^{(k)} - 1)/\gamma_j^{(k)})], j = 1, \cdots, N_k$$

We will use these following notations in this paper :

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

Serie representation of multivariable I-function of several variables is given by

$$I(y_1, \cdots, y_r) = \sum_{G_1, \cdots, G_r = 0}^{\infty} \sum_{g_1 = 0}^{M_1} \cdots \sum_{g_r = 0}^{M_r} \frac{(-)^{G_1 + \cdots + G_r}}{\delta_{g_1} G_1! \cdots \delta_{g_r} G_r!} \phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$$

$$\times \ \theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) y_1^{-\eta_{G_1,g_1}} \cdots y_r^{-\eta_{G_r,g_r}} \tag{1.4}$$

Where $\psi(., \cdots, .), \theta_i(.), i = 1, \cdots, r$ are defined by Prasad

$$\eta_{G_1,g_1} = \frac{B_{g_1}^{(1)} + G_1}{\delta_{g_1}^{(1)}}, \cdots, \ \ \eta_{G_r,g_r} = \frac{B_{g_r}^{(r)} + G_r}{\delta_{g_r}^{(r)}}$$

which is valid under the conditions $\delta_{g_i}^{(i)}[B_j^i + p_i] \neq \delta_j^{(i)}[B_{g_i}^i + G_i]$ \hfill (1.5)

for $j \neq M_i, M_i = 1, \cdots \eta_{G_i,g_i}; P_i, N_i = 0, 1, 2, \cdots, ; y_i \neq 0, i = 1, \cdots, r$ \hfill (1.6)

In the document , we will note :

$$G(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) = \phi(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})\theta_1(\eta_{G_1,g_1}) \cdots \theta_r(\eta_{G_r,g_r}) \tag{1.7}$$

where $\phi(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r}),\theta_1(\eta_{G_1,g_1}),\cdots,\theta_r(\eta_{G_r,g_r})$ are defined by Prasad [3]

We will use these following notations in this paper :

$$U_1 = P_2, Q_2; P_3, Q_3; \cdots; P_{r-1}, q_{r-1}; V_1 = 0, N_2; 0, N_3; \cdots; 0, N_{r-1} \tag{1.8}$$

$$W_1 = (P', Q'); \cdots; (P^{(r)}, Q^{(r)}); X_1 = (M', N'); \cdots; (M^{(r)}, N^{(r)}) \tag{1.9}$$

$$A_1 = (A_{2k}, \gamma_{2k}^{(1)}, \gamma_{2k}^{(2)}); \cdots; (A_{(r-1)k}, \gamma_{(r-1)k}^{(1)}, \gamma_{(r-1)k}^{(2)}, \cdots, \gamma_{(r-1)k}^{(r-1)}) \tag{1.10}$$

$$B_1 = (B_{2k}, \delta_{2k}^{(1)}, \delta_{2k}^{(2)}); \cdots; (B_{(r-1)k}, \delta_{(r-1)k}^{(1)}, \delta_{(r-1)k}^{(2)}, \cdots, \delta_{(r-1)k}^{(r-1)}) \tag{1.11}$$

$$\mathfrak{A}_1 = (A_{rk}; \gamma_{rk}^{(1)}, \gamma_{rk}^{(2)}, \cdots, \gamma_{rk}^{(r)}) : \mathfrak{B}_1 = (B_{rk}; \delta_{rk}^{(1)}, \delta_{rk}^{(2)}, \cdots, \delta_{rk}^{(r)}) \tag{1.12}$$

$$A'_1 = (A'_k, \gamma'_k)_{1,P'}; \cdots; (a_k^{(r)}, \gamma_k^{(r)})_{1,P^{(r)}}; B'_1 = (B'_k, \delta'_k)_{1,Q'}; \cdots; (B_k^{(r)}, \delta_k^{(r)})_{1,Q^{(r)}} \tag{1.13}$$

The multivariable I-function of r-variables write :

$$I(z_1, \cdots, z_r) = I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1} \left(\begin{array}{c|c} z_1 & \\ . & A_1; \mathfrak{A}_1; A'_1 \\ . & \\ . & \\ z_s & B_1; \mathfrak{B}_1; B'_1 \end{array} \right) \tag{1.14}$$

The multivariable I-function of s-variables is defined in term of multiple Mellin-Barnes type integral :

$$I(z_1, z_2, ...z_s) = I_{p_2,q_2,p_3,q_3;\cdots;p_s,q_s:p',q';\cdots;p^{(s)},q^{(s)}}^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(s)},n^{(s)}} \left(\begin{array}{c|c} z_1 & (a_{2j}; \alpha'_{2j}, \alpha''_{2j})_{1,p_2}; \cdots; \\ . & \\ . & \\ . & \\ z_s & (b_{2j}; \beta'_{2j}, \beta''_{2j})_{1,q_2}; \cdots; \end{array} \right.$$

$$\left. \begin{array}{c} (a_{sj}; \alpha'_{sj}, \cdots, \alpha_{sj}^{(s)})_{1,p_s} : (a'_j, \alpha'_j)_{1,p'}; \cdots; (a_j^{(s)}, \alpha_j^{(s)})_{1,p^{(s)}} \\ \\ (b_{sj}; \beta'_{sj}, \cdots, \beta_{sj}^{(s)})_{1,q_s} : (b'_j, \beta'_j)_{1,q'}; \cdots; (b_j^{(s)}, \beta_j^{(s)})_{1,q^{(s)}} \end{array} \right) \tag{1.15}$$

$$= \frac{1}{(2\pi\omega)^s} \int_{L_1} \cdots \int_{L_s} \xi(t_1, \cdots, t_s) \prod_{i=1}^{s} \phi_i(t_i) z_i^{t_i} \mathrm{d}t_1 \cdots \mathrm{d}t_s \tag{1.16}$$

The defined integral of the above function, the existence and convergence conditions, see Y,N Prasad [3]. Throughout the present document, we assume that the existence and convergence conditions of the multivariable I-function.

The condition for absolute convergence of multiple Mellin-Barnes type contour (1.9) can be obtained by extension of the corresponding conditions for multivariable H-function given by as :

$$|arg z_i| < \frac{1}{2}\Omega_i'\pi \, , \text{ where}$$

$$\Omega_i' = \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \left(\sum_{k=1}^{n_2} \alpha_{2k}^{(i)} - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} \right) + \cdots +$$

$$\left(\sum_{k=1}^{n_s} \alpha_{sk}^{(i)} - \sum_{k=n_s+1}^{p_s} \alpha_{sk}^{(i)} \right) - \left(\sum_{k=1}^{q_2} \beta_{2k}^{(i)} + \sum_{k=1}^{q_3} \beta_{3k}^{(i)} + \cdots + \sum_{k=1}^{q_s} \beta_{sk}^{(i)} \right) \tag{1.17}$$

where $i = 1, \cdots, s$

The complex numbers z_i are not zero.Throughout this document , we assume the existence and absolute convergence conditions of the multivariable I-function.

We may establish the the asymptotic expansion in the following convenient form :

$$I(z_1, \cdots, z_s) = 0(|z_1|^{\gamma_1'}, \cdots, |z_s|^{\gamma_s'}), max(|z_1|, \cdots, |z_s|) \to 0$$

$$I(z_1, \cdots, z_s) = 0(|z_1|^{\delta_1'}, \cdots, |z_s|^{\beta_s'}), min(|z_1|, \cdots, |z_s|) \to \infty$$

where $k = 1, \cdots, z : \alpha_k' = min[Re(b_j^{(k)}/\beta_j^{(k)})], j = 1, \cdots, m_k$ and

$$\beta_k' = max[Re((a_j^{(k)} - 1)/\alpha_j^{(k)})], j = 1, \cdots, n_k$$

We will use these following notations in this paper :

$$U = p_2, q_2; p_3, q_3; \cdots; p_{s-1}, q_{s-1}; V = 0, n_2; 0, n_3; \cdots; 0, n_{s-1} \tag{1.18}$$

$$W = (p', q'); \cdots; (p^{(s)}, q^{(s)}); X = (m', n'); \cdots; (m^{(s)}, n^{(s)}) \tag{1.19}$$

$$A = (a_{2k}, \alpha_{2k}^{(1)}, \alpha_{2k}^{(2)}); \cdots; (a_{(s-1)k}, \alpha_{(s-1)k}^{(1)}, \alpha_{(s-1)k}^{(2)}, \cdots, \alpha_{(s-1)k}^{(s-1)}) \tag{1.20}$$

$$B = (b_{2k}, \beta_{2k}^{(1)}, \beta_{2k}^{(2)}); \cdots; (b_{(s-1)k}, \beta_{(s-1)k}^{(1)}, \beta_{(s-1)k}^{(2)}, \cdots, \beta_{(s-1)k}^{(s-1)}) \tag{1.21}$$

$$\mathfrak{A} = (a_{sk}; \alpha_{sk}^{(1)}, \alpha_{sk}^{(2)}, \cdots, \alpha_{sk}^{(s)}) : \mathfrak{B} = (b_{sk}; \beta_{sk}^{(1)}, \beta_{sk}^{(2)}, \cdots, \beta_{sk}^{(s)}) \tag{1.22}$$

$$A' = (a'_k, \alpha'_k)_{1,p'}; \cdots; (a_k^{(s)}, \alpha_k^{(s)})_{1,p^{(s)}}; B' = (b'_k, \beta'_k)_{1,q'}; \cdots; (b_k^{(s)}, \beta_k^{(s)})_{1,q^{(s)}} \tag{1.23}$$

The multivariable I-function write :

$$I(z_1, \cdots, z_s) = I_{U:p_s,q_s;W}^{V;0,n_s;X} \left(\begin{array}{c|c} z_1 & \text{A ;} \mathfrak{A}; \text{A'} \\ \cdot & \\ \cdot & \\ \cdot & \\ z_s & \text{B; } \mathfrak{B}; \text{ B'} \end{array} \right) \tag{1.24}$$

The generalized polynomials defined by Srivastava [5], is given in the following manner :

$$S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}[y_1, \cdots, y_t] = \sum_{K_1=0}^{[N_1/M_1]} \cdots \sum_{K_t=0}^{[N_t/M_t]} \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_t)_{M_tK_t}}{K_t!}$$

$$A[N_1, K_1; \cdots; N_t, K_t] y_1^{K_1} \cdots y_t^{K_t} \tag{1.25}$$

Where $M_1, \cdots, M_s$ are arbitrary positive integers and the coefficients $A[N_1, K_1; \cdots; N_t, K_t]$ are arbitrary constants, real or complex. In the present paper, we use the following notation

$$a_1 = \frac{(-N_1)_{M_1K_1}}{K_1!} \cdots \frac{(-N_t)_{M_tK_t}}{K_t!} A[N_1, K_1; \cdots; N_t, K_t] \tag{1.26}$$

The spheroidal function $\psi_{\alpha n}(c, \eta)$ of general order $\alpha > -1$ can be expansed as ([4] an [7].

$$\psi_{\alpha n}(c, \eta) = \frac{i^n \sqrt{2\pi}}{V_{\alpha n(c)}} \sum_{k=0,or1}^{\infty *} a_k(c|\alpha n)(c\eta)^{-\alpha-\frac{1}{2}} J_{k+\alpha+\frac{1}{2}}(c\eta) \tag{1.27}$$

which represents the function uniformly on (∞, ∞), where the coefficients $a_k(c|\alpha n)$ satisfy the recursion formula [14 ,eq.67]and the asterisk over the summation sign indicates that the sum is taken over only even or odd values of k according as n is even or odd. As $c \to 0, a_k(c|\alpha n) \to 0, k \neq n$

The Fresnel integral denoted $S(z)$ is defined by : see   Abramowitz M and Stegun I.A. ([1],page 89(300))

$$S(z) = \int_0^z sin\left(\frac{\pi}{2}t^2\right) dt \tag{1.28}$$

2. Required integral

We have the following integral, see Brychkow ([2], 4.5.1, 1page 189).

Lemme

$$\int_0^a x^{s-1}(a-x)^{t-1} S\left(b\sqrt{x(a-x)}\right) dx = \frac{a^{s+t+\frac{1}{2}}}{3} \sqrt{\frac{2b^3}{\pi}} \, B\left(s+\frac{3}{4}, t+\frac{3}{4}\right)$$

$$
\times {}_3F_4\left(\begin{array}{c} \frac{3}{4},\text{s}+\frac{3}{4},\text{t}+\frac{3}{4} \\ \cdot\ \ \cdot\ \ \cdot \\ \frac{3}{2},\ \frac{7}{4},\ \frac{s+t}{2}+\frac{3}{4},\ \frac{s+t}{2}+\frac{5}{4} \end{array}; -\frac{(ab)^2}{16}\right) \tag{2.1}
$$

where $a>0, Re(s)>-\dfrac{3}{4}, Re(t)>-\dfrac{3}{4}$

3. Main integral

Let $X_{s,t}=x^s(a-x)^t$, We have the following generalized finite integral :

Theorem

$$
\int_0^a x^{s'-1}(a-x)^{t'-1} S\left(b\sqrt{x(a-x)}\right)\ \psi_{\alpha n}(c^\sigma, X_{\alpha,\beta})\ S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c} \text{y}_1 X_{\gamma_1,\mu_1} \\ \cdot\ \cdot\ \cdot \\ \text{y}_t X_{\gamma_t,\mu_t} \end{array}\right)
$$

$$
I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{array}{c} \text{z}_1 X_{\alpha_1,\beta_1} \\ \cdot\ \cdot\ \cdot \\ \text{z}_r X_{\alpha_r,\beta_r} \end{array}\right) I_{U_1:P_r,Q_r;W_1}^{V_1;0,N_r;X_1}\left(\begin{array}{c} \text{Z}_1 X_{\eta_1,\epsilon_1} \\ \cdot\ \cdot\ \cdot \\ \text{Z}_s X_{\eta_s,\epsilon_s} \end{array}\right) \mathrm{d}x = \frac{a^{s'+t'+\frac{1}{2}}}{3}\sqrt{\frac{2b^3}{\pi}}\sum_{n'=0}^{\infty}
$$

$$
\sum_{k=0,or1}^{\infty_*}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]} a_1\ \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}
$$

$$
G(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\ \frac{\left(\frac{3}{4}\right)_{n'}(-ab)^{2n'}}{16^{n'}\left(\frac{3}{2}\right)_{n'}\left(\frac{7}{4}\right)_{n'}n'!}\ x_1^{p_1}\cdots x_s^{p_s} z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}} y_1^{K_1}\cdots y_t^{K_t} c^{\sigma(2m+k)}
$$

$$
a^{(2m+k)(\alpha+\beta)+\sum_{i=1}^t K_i(\gamma_i+\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i)}\ I_{U_1:P_r+4,Q_r+3;W_1}^{V_1;0,N_r+4;X_1}\left(\begin{array}{c} \text{Z}_1 a^{\eta_1+\epsilon_1} \\ \cdot\ \cdot\ \cdot \\ ... \\ \text{Z}_s a^{\eta_s+\epsilon_s} \end{array}\right|
$$

$$
\text{A};(\tfrac{1}{4}\text{-n'-}(\text{s'}+(2m+k)\alpha + \textstyle\sum_{i=1}^t K_i\gamma_i + \sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),
$$
$$
\cdot\ \cdot\ \cdot
$$
$$
\text{B}; (\text{-}\tfrac{1}{2}\text{-}(\text{s'+t'}+(2m+k)(\alpha+\beta) + \textstyle\sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\epsilon_1+\eta_1,\cdots,\epsilon_s+\eta_s),
$$

$$
(\tfrac{1}{4}\text{-}\tfrac{1}{2}(\text{s'+t'}+(2m+k)(\alpha+\beta) + \textstyle\sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\tfrac{\epsilon_1+\eta_1}{2},\cdots,\tfrac{\epsilon_s+\eta_s}{2}),
$$
$$
\cdot\ \cdot\ \cdot
$$
$$
(\tfrac{1}{4}\text{-n'-}\tfrac{1}{2}(\text{s'+t'}+(2m+k)(\alpha+\beta) + \textstyle\sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\tfrac{\epsilon_1+\eta_1}{2},\cdots,\tfrac{\epsilon_s+\eta_s}{2}),
$$

$(-\frac{1}{4}-\frac{1}{2}(s'+t'+(2m+k)(\alpha+\beta)+\sum_{i=1}^{t}K_i(\gamma_i+\mu_i)+\sum_{i=1}^{r}\eta_{G_i,g_i}(\alpha_i+\beta_i)); \frac{\epsilon_1+\eta_1}{2}, \cdots, \frac{\epsilon_s+\eta_s}{2}),$

$\cdots$

$(-\frac{1}{4}\text{-n'}-\frac{1}{2}(s'+t'+(2m+k)(\alpha+\beta)+\sum_{i=1}^{t}K_i(\gamma_i+\mu_i)+\sum_{i=1}^{r}\eta_{G_i,g_i}(\alpha_i+\beta_i)); \frac{\epsilon_1+\eta_1}{2}, \cdots, \frac{\epsilon_s+\eta_s}{2}),$

$$\left. \begin{array}{c} (\frac{1}{4}\text{-n'}-(t'+(2m+k)\beta+\sum_{i=1}^{t}K_i\mu_i+\sum_{i=1}^{r}\eta_{G_i,g_i}\beta_i); \epsilon_1, \cdots, \epsilon_s), \mathfrak{A}:A' \\ \cdots \\ \mathfrak{B}:B' \end{array} \right) \tag{3.1}$$

Provided that

a) $min\{A, \gamma, \delta, \gamma_i, \mu_i, \alpha_j, \beta_j, \eta_k, \epsilon_k\} > 0, i = 1, \cdots, t, j = 1, \cdots, r, k = 1, \cdots, s$

b) $Re\left[s' + (2m+k)\alpha + \sum_{i=1}^{r} \alpha_i \min_{1 \leqslant j \leqslant M^{(i)}} \frac{B_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} \eta_i \min_{1 \leqslant j \leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}\right] > -\frac{3}{4}$

c) $Re\left[t' + (2m+k)\beta + \sum_{i=1}^{r} \beta_i \min_{1 \leqslant j \leqslant M^{(i)}} \frac{B_j^{(i)}}{\delta_j^{(i)}} + \sum_{i=1}^{s} \epsilon_i \min_{1 \leqslant j \leqslant m^{(i)}} \frac{b_j^{(i)}}{\beta_j^{(i)}}\right] > -\frac{3}{4}$

d)$|arg z_k| < \frac{1}{2}\Omega_i \pi$, where Ω_i is defined by (1.3) ; $i = 1, \cdots, r$

e)$|arg Z_k| < \frac{1}{2}\Omega'_i \pi$, where Ω_i is defined by (1.17) ; $i = 1, \cdots, s$

f) The series occuring on the right-hand side of (3.1) is absolutely and uniformly convergent.

Proof

First, expressing the spheroidal function $\psi_{\alpha n}(c^\sigma, X_{\alpha,\beta})$ in serie with the help of equation (1.27), the I-function of r variables in series with the help of equation (1.4), the general class of polynomial of several variables $S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}$ with the help of equation (1.25) and the I-function of s variables in Mellin-Barnes contour integral with the help of equation (1.15), changing the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process) and then evaluating the resulting integral with the help of equation (2.1) and expressing the generalized hypergeometric function $_3F_4$ in serie , use the following relations $\Gamma(a)(a)_n = \Gamma(a+n)$ and $a = \frac{\Gamma(a+1)}{\Gamma(a)}$ with $Re(a) > 0$. Finally interpreting the result thus obtained with the Mellin-barnes contour integral, we arrive at the desired result.

4. Multivariable H-function

If $U = V = A = B = U_1 = V_1 = A_1 = B_1 = 0$, the multivariable I-function defined by Prasad degenere in multivariable H-function defined by Srivastava et al [6]. Our integral contain two multivariable H-functions.

In this section, we note

$G_0(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r}) = G_{A=B=A_1=B_1=0}(\eta_{G_1,g_1}, \cdots, \eta_{G_r,g_r})$

Corollary

$$\int_0^a x^{s'-1}(a-x)^{t'-1} S\big(b\sqrt{x(a-x)}\big) \ \psi_{\alpha n}(c^\sigma, X_{\alpha,\beta}) \ S_{N_1,\cdots,N_t}^{M_1,\cdots,M_t}\left(\begin{array}{c} \mathrm{y}_1 X_{\gamma_1,\mu_1} \\ \cdots \\ \mathrm{y}_t X_{\gamma_t,\mu_t} \end{array}\right)$$

$$H_{P_r,Q_r;W_1}^{0,N_r;X_1}\left(\begin{array}{c} \mathrm{z}_1 X_{\alpha_1,\beta_1} \\ \cdots \\ \mathrm{z}_r X_{\alpha_r,\beta_r} \end{array}\right) H_{P_r,Q_r;W_1}^{0,N_r;X_1}\left(\begin{array}{c} \mathrm{Z}_1 X_{\eta_1,\epsilon_1} \\ \cdots \\ \mathrm{Z}_s X_{\eta_s,\epsilon_s} \end{array}\right)\mathrm{d}x = \frac{a^{s'+t'+\frac{1}{2}}}{3}\sqrt{\frac{2b^3}{\pi}}\sum_{n'=0}^{\infty}$$

$$\sum_{k=0,or1}^{\infty *}\sum_{m=0}^{\infty}\sum_{G_1,\cdots,G_r=0}^{\infty}\sum_{g_1=0}^{m_1}\cdots\sum_{g_r=0}^{m_r}\sum_{K_1=0}^{[N_1/M_1]}\cdots\sum_{K_t=0}^{[N_t/M_t]} a_1\ \frac{(-)^{G_1+\cdots+G_r}}{\delta_{g_1}G_1!\cdots\delta_{g_r}G_r!}\frac{(-)^m a_k(c^\sigma|\alpha n)}{m!\Gamma(m+k+\alpha+\frac{3}{2})}$$

$$G_0(\eta_{G_1,g_1},\cdots,\eta_{G_r,g_r})\ \frac{\left(\frac{3}{4}\right)_{n'}(-ab)^{2n'}}{16^{n'}\left(\frac{3}{2}\right)_{n'}\left(\frac{7}{4}\right)_{n'}n'!}\ x_1^{p_1}\cdots x_s^{p_s}\ z_1^{\eta_{G_1,g_1}}\cdots z_r^{\eta_{G_r,g_r}} y_1^{K_1}\cdots y_t^{K_t}\ c^{\sigma(2m+k)}$$

$$a^{(2m+k)(\alpha+\beta)+\sum_{i=1}^t K_i(\gamma_i+\mu_i)+\sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i)} H_{P_r+4,Q_r+3;W_1}^{0,N_r+4;X_1}\left(\begin{array}{c|c} \mathrm{Z}_1 a^{\eta_1+\epsilon_1} & \\ \cdots & \\ \ldots & \\ \mathrm{Z}_s a^{\eta_s+\epsilon_s} & \end{array}\right.$$

$$(\tfrac{1}{4}\text{-n'-(s'+(2m+k)}\alpha + \sum_{i=1}^t K_i\gamma_i + \sum_{i=1}^r \eta_{G_i,g_i}\alpha_i);\eta_1,\cdots,\eta_s),$$
$$\cdots$$
$$(-\tfrac{1}{2}\text{-(s'+t'+(2m+k)}(\alpha+\beta) + \sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\epsilon_1+\eta_1,\cdots,\epsilon_s+\eta_s),$$

$$(\tfrac{1}{4}-\tfrac{1}{2}\text{(s'+t'+(2m+k)}(\alpha+\beta) + \sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\tfrac{\epsilon_1+\eta_1}{2},\cdots,\tfrac{\epsilon_s+\eta_s}{2}),$$
$$\cdots$$
$$(\tfrac{1}{4}\text{-n'-}\tfrac{1}{2}\text{(s'+t'+(2m+k)}(\alpha+\beta) + \sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\tfrac{\epsilon_1+\eta_1}{2},\cdots,\tfrac{\epsilon_s+\eta_s}{2}),$$

$$(-\tfrac{1}{4}-\tfrac{1}{2}\text{(s'+t'+(2m+k)}(\alpha+\beta) + \sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\tfrac{\epsilon_1+\eta_1}{2},\cdots,\tfrac{\epsilon_s+\eta_s}{2}),$$
$$\cdots$$
$$(-\tfrac{1}{4}\text{-n'-}\tfrac{1}{2}\text{(s'+t'+(2m+k)}(\alpha+\beta) + \sum_{i=1}^t K_i(\gamma_i+\mu_i) + \sum_{i=1}^r \eta_{G_i,g_i}(\alpha_i+\beta_i));\tfrac{\epsilon_1+\eta_1}{2},\cdots,\tfrac{\epsilon_s+\eta_s}{2}),$$

$$(\tfrac{1}{4}\text{-n'-(t'+(2m+k)}\beta + \sum_{i=1}^t K_i\mu_i + \sum_{i=1}^r \eta_{G_i,g_i}\beta_i);\epsilon_1,\cdots,\epsilon_s), \mathfrak{A}:A'$$
$$\cdots$$
$$\left.\mathfrak{B}:B'\right)$$

$$(4.1)$$

under the same notations and validity conditions that (3.1) with $A = B = A_1 = B_1 = 0$.

5. Conclusion

In this paper we have evaluated a unified generalized infinite integral involving the multivariable I-functions, a class of polynomials of several variables and the spheroidal function and general arguments.The integral established in this paper is of very general nature as it contains multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

REFERENCES

[1]Abramowitz M and Stegun I.A. Handbook of Mathematical Functions ,1964,

[2] Brychkow Y.A. Handbook of Special Functions. Derivatives. Integrals, Series and Other Formulas. CRC. Press. Taykor and Francis Group. Boca. Raton. London. New York. 2008.

[3] Y.N. Prasad , Multivariable I-function , Vijnana Parishad Anusandhan Patrika 29 (1986) , page 231-237.

[4] Rhodes D.R. On the spheroidal functions. J. Res. Nat. Bur. Standards. Sect. B 74(1970), page187-209.

[5] Srivastava H.M. A multilinear generating function for the Konhauser set of biorthogonal polynomials suggested by Laguerre polynomial, Pacific. J. Math. 177(1985), page183-191.

[6] H.M. Srivastava And R.Panda. Some expansion theorems and generating relations for the H-function of several complex variables. Comment. Math. Univ. St. Paul. 24(1975), p.119-137.

[7] Stratton J.A. And Chu L.J. Elliptic and spheroidal wave function J. Math. And Phys. 20 (1941), page 259-309.

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