SOME DEFINITIONS and INTERPRETATIONS on FUZZY SOFT GAME

Sreelekshmi Warrier

Abstract. Deli and Ca$\widetilde{(g)}$man introduce soft games and fuzzy soft games,which can apply to problems contain vagueness and uncertainty.In this paper,we present some definitions related to fuzzy soft game.Then we have extended some existing theories about Nash equilibrium under the fuzzy soft context.

1. INTRODUCTION

One of the most appropriate theory for dealing with uncertainty is the theory of fuzzy set which was developed by Zedeh in 1965. Later in 1999,Molodstov introduced a new mathematical theory, which is free from inadequacy of parameters known as soft set theory for modelling vagueness and uncertainty in decision making problems [1].It has many applications in study of Smoothness of functions, Game theory, Operations Research etc.[2].Roy and Maji presented the concept of fuzzy soft set (fs-sets) by embedding the ideas of fuzzy set theory[4].
In 1944 John Von Neuman, was introduced Game theory for modelling and designing automated decision making process[13].It is a mathematical study of strategic decision making.Qian and Abrham introduced strategic games using fuzzy set[12]. In both classical and fuzzy games,the pay off are real valued functions but in fuzzy soft games they are set valued functions[10]. The notions of soft games and fuzzy soft games given by Ca$\widetilde{(g)}$man and Deli in[7,10].Solution of the fuzzy soft game are obtained by using operations of fuzzy set and soft set that make this game very convenient and easily applicable in real life situations[10].In this paper we presented some definition and theorems about Nash equilibrium using fuzzy soft set approach.

2. Preliminaries

We present the basic definitions of soft set, fuzzy soft set theories and fuzzy soft game theory in this section.

Key words and phrases. Fuzzy Set,Soft Set,Fuzzy Soft Set,Game Theory, Nash Equilibrium,Soft Game, Fuzzy Soft Game.

Definition 2.1. [2] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and$A \subseteq E$.A soft set F_A over U is a set defined by a function f_A representing a mapping F_A from E to$P(U)$ such that $f_A = \emptyset$ if $x \notin A$. Here f_A is called approximate function of the soft set F_A. A soft set over Ucan be denoted by the set of ordered pairs

$$F_A = \{(x, f_A(x)); x \in E, f_A(x) \in P(U)\} \tag{2.1}$$

Definition 2.2. [8] A choice behaviour of a player is called an action. The the set of available actions of players can be represented by $\mathbb{X} \times \mathbb{Y}$, where X and Y be the set of strategies of two players .

Definition 2.3. [8] Consider a set strategies X and Y be of Player 1 and 2 respectively, U be a set alternatives and f_{sk} be a soft pay off function from $\mathbb{X} \times \mathbb{Y}$ to $P(U)$ for player$k(k = 1, 2)$. Then for each player k, a two person soft game (tps-game) is defined by a soft set over U as

$$S_k = \{((x, y), f_{sk}(x, y)) : (x, y) \in (\mathbb{X} \times \mathbb{Y})\} \tag{2.2}$$

That is the tps-game can be played as follows: at certain time Player 1select strategy $x_i \in X$, simultaneously Player 2 chooses a strategy $y_j \in Y$, so each player gets the soft pay off values be $f_{sk}(x_i, y_j)$.

Definition 2.4. [11] Let U be a set alternatives ,X and Y be a set strategies of Player 1 and 2 respectively.
Then fuzzy soft pay off function $\gamma^k_{X\times Y} : (X \times Y) \to F(U))$. Then a two person fuzzy soft game (tpfs-game) denoted as $\Gamma^k_{X\times Y}$ and defined by

$$\Gamma^k_{X\times Y} = \{((x, y), \gamma^k_{X\times Y}(x, y)) : (x, y) \in (X \times Y)\} \tag{2.3}$$

This can be played as follows: If Player 1 chooses strategy $x_i \in X$,and Player 2 chooses a strategy $y_j \in Y$,then each player receives the fuzzy soft pay off value is $\gamma^k_{X\times Y}(x_i, y_j)$.

Definition 2.5. [11] Let $\Gamma^k_{X\times Y}$ be a tp fs-game with its fuzzy soft payoff function $\gamma^k_{X\times Y}$ for $k = 1, 2$. If the following properties hold
$a)\gamma^1_{X\times Y}(x^*, y^*) \subseteq \gamma^1_{X\times Y}(x, y^*)$ for each $x \in X$.
b) $\gamma^2_{X\times Y}(x^*, y^*) \subseteq \gamma^2_{X\times Y}(x^*, y)$ for each $y \in Y$.
then ,$(x^*, y^*) \in (X \times Y)$ is called a fuzzy soft Nash equilibrium of a tp fs-game.

3. Fuzzy Soft Nash Equilibrium

Here we introduced some theorems and definitions on Nash equilibrium based on fuzzy soft concept.

Theorem 3.1. *Every two player, two action fuzzy soft game has at least one fuzzy soft Nash equilibrium.*

Proof. Let Player 1 and 2,having the set of strategies X and Y respectively, U be a set of alternatives and $\gamma^k_{X\times Y} : X \times Y \to F(U)$ be a fuzzy soft pay off

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function for player$k(k=1,2)$.Then consider,a two person fuzzy soft game $\Gamma^k_{X\times Y}=\{((x,y),\gamma^k_{X\times Y}(x,y)):(x,y)\in(X\times Y)\}$,for each player k.
We have to prove that $\Gamma^k_{X\times Y}$ has at least one fuzzy soft Nash equilibrium.
Here we are using reductio ad absurdum method. Assume that $\Gamma^k_{X\times Y}$ has no fuzzy soft Nash equilibrium, then $\gamma^1_{X\times Y}(x^*,y^*)\subseteq\gamma^1_{X\times Y}(x,y^*)$ for each $x\in X$ $\gamma^2_{X\times Y}(x^*,y^*)\subseteq\gamma^2_{X\times Y}(x^*,y)$ for each $y\in Y$ Consider a two person fuzzy soft game for player $k=1,2$

Two person fuzzy soft game for player k

$\mathbf{\Gamma^k_{X\times Y}}$	$\mathbf{y_1}$	$\mathbf{y_2}$
$\mathbf{x_1}$	$\gamma^{\mathbf{k}}_{\mathbf{X\times Y}}(\mathbf{x_1},\mathbf{y_1})$	$\gamma^{\mathbf{k}}_{\mathbf{X\times Y}}(\mathbf{x_1},\mathbf{y_2})$
$\mathbf{x_2}$	$\gamma^{\mathbf{k}}_{\mathbf{X\times Y}}(\mathbf{x_2},\mathbf{y_1})$	$\gamma^{\mathbf{k}}_{\mathbf{X\times Y}}(\mathbf{x_2},\mathbf{y_2})$

If $\gamma^1_{X\times Y}(x_1,y_2)\supseteq\gamma^1_{X\times Y}(x_2,y_2)$ and $\gamma^2_{X\times Y}(x_1,y_2)\supseteq\gamma^2_{X\times Y}(x_2,y_2)$ then (x_1,y_2) is fuzzy soft Nash equilibrium.
$if\gamma^1_{X\times Y}(x_1,y_1)\supseteq\gamma^1_{X\times Y}(x_2,y_1)$ and$\gamma^2_{X\times Y}(x_1,y_1)\supseteq\gamma^2_{X\times Y}(x_2,y_1)$ then (x_1,y_1) is fuzzy soft Nash equilibrium.
$if\gamma^1_{X\times Y}(x_2,y_1)\supseteq\gamma^1_{X\times Y}(x_1,y_1)$ and$\gamma^2_{X\times Y}(x_2,y_1)\supseteq\gamma^2_{X\times Y}(x_1,y_1)$ then (x_2,y_1) is fuzzy soft Nash equilibrium.
$if\gamma^1_{X\times Y}(x_2,y_2)\supseteq\gamma^1_{X\times Y}(x_2,y_1)$ and$\gamma^2_{X\times Y}(x_2,y_2)\supseteq\gamma^2_{X\times Y}(x_2,y_1)$ then (x_2,y_2) is fuzzy soft Nash equilibrium.

But our assumption ,there is no fuzzy soft Nash equilibrium. If either
$\gamma^1_{X\times Y}(x_1,y_2)\subset\gamma^1_{X\times Y}(x_2,y_2)$ and $\gamma^2_{X\times Y}(x_1,y_2)\subset\gamma^2_{X\times Y}(x_2,y_2)$
$\gamma^1_{X\times Y}(x_1,y_1)\subset\gamma^1_{X\times Y}(x_2,y_1)$ and$\gamma^2_{X\times Y}(x_1,y_1)\subset\gamma^2_{X\times Y}(x_2,y_1)$
$\gamma^1_{X\times Y}(x_2,y_1)\subset\gamma^1_{X\times Y}(x_1,y_1)$ and$\gamma^2_{X\times Y}(x_2,y_1)\subset\gamma^2_{X\times Y}(x_1,y_1)$
$\gamma^1_{X\times Y}(x_2,y_2)\subset\gamma^1_{X\times Y}(x_2,y_1)$ and$\gamma^2_{X\times Y}(x_2,y_2)\subset\gamma^2_{X\times Y}(x_2,y_1)$
which is not possible.Hence our assumption is wrong.
Therefore every two player, two action fuzzy soft game has at least one fuzzy soft Nash equilibrium. □

Example 3.1. Let $U=\{u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8,u_9,u_{10}\}$ be a set of alternatives, $P(U)$ be the power set of U, $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2,y_3\}$ be set of the strategies of player 1 and 2 respectively.

If player 1 constructs a two person fuzzy soft game as follows

Two person fuzzy soft game for player 1

$\Gamma^1_{X\times Y}$	y_1	y_2	y_3
x_1	$\{0.6/u_1,0.4/u_4,$ $0.1/u_7,0.5/u_9\}$	$\{0.3/u_5,0.7/u_6,$ $0.1/u_7\}$	$\{0.7/u_1,0.8/u_3,$ $0.9/u_8\}$
x_2	$\{0.5/u_1,0.6/u_2,0.7/u_4,0.3/u_7$ $0.8/u_8,0.2/u_9,0.1/u_{10}\}$	$\{0.5/u_8\}$	$\{0.2/u_1,0.4/u_4,$ $0.6/u_6\}$

And tpfs-game of player 2 is given as in following table.

Two person fuzzy soft game for player 2

$\Gamma^2_{X\times Y}$	y_1	y_2	y_3
x_1	$\{0.1/u_1, 0.2/u_4, 0.3/u_5, 0.5/u_6, 0.6/u_7\}$	$\{0.3/u_5, 0.5/u_6, 0.6/u_7\}$	$\{0.7/u_1, 0.8/u_3, 0.9/u_8\}$
x_2	$\{0.8/u_1, 0.6/u_2, 0.4/u_3, 0.2/u_4, 0.5/u_5, 0.7/u_7, 0.9/u_8, 0.3/u_9, 0.1/u_{10}\}$	$\{0.9/u_3, 0.1/u_5\}$	$\{0.2/u_2, 0.4/u_4, 0.7/u_7\}$

from the above tables
$\gamma^1_{X\times Y}(x_2, y_1) \supseteq \gamma^1_{X\times Y}(x, y_1)$ for each $x \in X$ and
$\gamma^2_{X\times Y}(x_2, y_1) \supseteq \gamma^2_{X\times Y}(x_2, y)$ for each $y \in Y$.
Then $(x_2, y_1) \in (X \times Y)$ is a fuzzy soft Nash equilibrium.

Definition 3.1. The preference relation $\gtrsim$ in a fuzzy soft strategic game. $\Gamma^k_{X\times Y} : \{\gamma^k_{X\times Y}, (x, y) \in (X\times Y)\}$ can be represented by a fuzzy soft pay off function. $\gamma^k_{X\times Y} : (X \times Y) \to F(U), \gamma^k_{X\times Y}(x, y) \subseteq \gamma^k_{X\times Y}(x^1, y^1)$ whenever $(x, y) \subseteq (x^1, y^1)$.In such case we denote fuzzy soft game
$\Gamma^k_{X\times Y} = \{(x, y), \gamma^k_{X\times Y}(x, y) : (x, y) \in (X \times Y)\}$

Definition 3.2. $\Gamma^k_{X\times Y} : \{\gamma^k_{X\times Y}, (x, y) \in (X \times Y)\}$ is Strictly competitive if for any(x, y) and $(x^1, y^1) \in X \times Y$, we have $(x, y) \gtrsim_1 (x^1, y^1)$ iff $(x^1, y^1) \gtrsim_2 (x, y)$

Definition 3.3. $\Gamma^k_{X\times Y} : \{\gamma^k_{X\times Y}(x, y), \gtrsim) : ((x, y) \in (X \times Y)\}$ is Strictly competitive fuzzy soft strategic two person game.
The action $x^* \in X$ is the maximizing player of 1 if,
$\cap_{y\in Y}\gamma^1_{X\times Y}(x^*, y) \subseteq \cap_{y\in Y}\gamma^1_{X\times Y}(x, y), \forall x \in X$. Similarly action $(y^* \in Y)$ is the maximizing player of player 2, $\cap_{x\in X}\gamma^2_{X\times Y}(x, y^*) \subseteq \cap_{x\in X}\gamma^2_{X\times Y}(x, y), \forall y \in Y$.

Theorem 3.2. $\Gamma^k_{X\times Y} : \{\gamma^k_{X\times Y}(x, y), \gtrsim) : ((x, y) \in (X \times Y)\}$ *be Strictly competitive fuzzy soft strategic two person game.*
1. *If* (x^*, y^*) *is a fuzzy soft Nash equilibrium of* $\Gamma^k_{X\times Y}$ *then* x^* *is a maxi-minimizer player for player* 1 *and* y^* *is a maxi-minimizer for player* 2.
2. *If* (x^*, y^*) *is a Nash equilibrium of* $\Gamma^k_{X\times Y}$
then $\cup_{x\in X}(\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y))) = \cap_{y\in Y}(\cup_{x\in X}(\gamma^1_{X\times Y}(x, y))) = \gamma^1_{X\times Y}(x^*, y^*)$

Proof. Let $\Gamma^k_{X\times Y} = \{(x, y), \gamma^k_{X\times Y}, \gtrsim) : (x, y) \in (X \times Y)\}$ be a strictly competitive fuzzy soft strategic two person game with fuzzy soft pay off function$\gamma^k_{X\times Y}(x, y)$ .Let (x^*, y^*) is fuzzy soft Nash equilibrium of $\Gamma^k_{X\times Y}$.
Then $\gamma^1_{X\times Y}(x^*, y^*) \supseteq (\cap_{y\in Y}(\gamma^1_{X\times Y}(x^*, y) \supseteq (\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y)) \supseteq \cup_{x\in X}(\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y))$
That is

$$\gamma^1_{X\times Y}(x^*, y^*) \supseteq \cup_{x\in X}(\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y))). \tag{3.1}$$

Also $\gamma^1_{X\times Y}(x^*, y^*) \subseteq \cap_{y\in Y}(\gamma^1_{X\times Y}(x^*, y) \subseteq \cup_{x\in X}(\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y)))$

$$\gamma^1_{X\times Y}(x^*, y^*) \subseteq \cup_{x\in X}(\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y))). \tag{3.2}$$

From equations $(3.1, 3.2)$ we get $\gamma^1_{X\times Y}(x^*, y^*) = \cup_{x\in X}(\cap_{y\in Y}(\gamma^1_{X\times Y}(x, y)))$

Similarly we get $\gamma^1_{X\times Y}(x^*, y^*) = \cap_{y\in Y}(\cup_{x\in X}(\gamma^1_{X\times Y}(x, y)))$ and (x^*) is maxi-minimizer for player1of Strictly competitive fuzzy soft strategic two person game.Similarly we prove that y^* is maxi-minimizer for player 2. $\square$

EXAMPLE 3.2. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be set of alternatives. $P(U)$ be the power set of U, $X = \{x_1, x_2\}$ and $Y = \{y_1, y_2, y_3\}$ be set of the strategies of player 1 and 2 respectively. If player 1 constructs a two person fuzzy soft game as follows

Two person fuzzy soft game for player 1

$\Gamma^1_{X\times Y}$	y_1	y_2	y_3
x_1	$\{0.6/u_1, 0.2/u_3, 0.1/u_7, 0.5/u_9\}$	$\{0.4/u_1, 0.3/u_3, 0.6/u_5, 0.1/u_7\}$	$\{0.5/u_1, 0.5/u_3, 0.3/u_5, 0.9/u_7, 0.4/u_9\}$
x_2	$\{0.8/u_1, 0.6/u_3, 0.7/u_4, 0.3/u_6, 0.8/u_7, 0.2/u_9, 0.1/u_{10}\}$	$\{0.3/u_1\}$	$\{0.2/u_1, 0.4/u_4, 0.6/u_6\}$

And tpfs-game of player 2 is given as in following table.

Two person fuzzy soft game for player 2

$\Gamma^2_{X\times Y}$	y_1	y_2	y_3
x_1	$\{0.1/u_1, 0.2/u_2, 0.7/u_8, 0.9/u_9\}$	$\{0.3/u_3, 0.5/u_6, 0.6/u_8\}$	$\{0.7/u_1, 0.8/u_3, 0.9/u_9\}$
x_2	$\{0.9/u_1, 0.6/u_2, 0.3/u_3, 0.2/u_4, 0.5/u_5, 0.7/u_7, 0.8/u_8, 0.3/u_9, 0.1/u_{10}\}$	$\{0.9/u_1, 0.1/u_8\}$	$\{0.1/u_1, 0.1/u_4, 0.7/u_7\}$

from the above tables
$\gamma^1_{X\times Y}(x_2, y_1) \supseteq \gamma^1_{X\times Y}(x, y_1)$ for each $x \in X$, $\gamma^2_{X\times Y}(x_2, y_1) \supseteq \gamma^2_{X\times Y}(x_2, y)$ for each $y \in Y$ Then , $(x_2, y_1) \in (X \times Y)$ is a fuzzy soft Nash equilibrium.
$\cap^3_{j=1}\gamma^1_{X\times Y}(x_1, y_j) = \{0.4/u_1, 0.2/u_3, 0.1/u_7\}$ $\cap^3_{j=1}\gamma^1_{X\times Y}(x_2, y_j) = \{0.2/u_1\}$
Therefore $\cap^3_{j=1}\gamma^2_{X\times Y}(x_1, y_j) \supseteq \cap^3_{j=1}\gamma^2_{X\times Y}(x, y_j)$ That is x_1 is the maxi-minimizer for player 1.
$\cap^2_{i=1}\gamma^2_{X\times Y}(x_i, y_1) = \{0.1/u_1, 0.2/u_2, 0.7/u_8, 0.3/u_9\}$ $\cap^2_{i=1}\gamma^2_{X\times Y}(x_i, y_2) = \{0.1/u_8\}$
$\cap^2_{i=1}\gamma^2_{X\times Y}(x_i, y_3) = \{0.1/u_1\}$ Therefore $\cap^2_{i=1}\gamma^1_{X\times Y}(x_i, y_1) \supseteq \cap^2_{i=1}\gamma^1_{X\times Y}(x_i, y)$ That is y_1 is the maxi-minimizer for player 2.

4. Conclusion

The fuzzy soft strategic games may be applied to many fields with applications to solve problems in decision making,Computer science, etc.

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6. List of References

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Department of Mathematics, Mahatma Gandhi University, Kottayam,Kerala, India
E-mail address: sreelekshmicwarrier@gmail.com