Laguerre wavelet and its programming

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Abstract
In this paper, the author constructs new Laguerre wavelet function with its program by using MATLAB program. Also the author derivative and integration with its powers in terms matrices are constructed. The efficiency of the above functions through the use of these verbs in the solution of some examples that will show us the validity of what we have said. Moreover, some of the hypothesis was proved as the theorems of orthogonality and Convergent.

Keywords- *Laguerre wavelets, MATLAB program, operational matrix of integration, operational matrix of derivative and powers in terms.*

I. Introduction

First, we show that the waveforms that cut data into different frequency components are mathematical functions [1, 2]. It is shown through many sources that each component will be studied with a resolution proportional to its range where they have traditional Fourier methods in case analysis in different Science, for example, physics [9, 10]. Over the past 10 years, there have been exchanges between scientific fields that include the development of waveforms independently in the fields of engineering, science and geology [3-5]. Interchanges between these fields during the past ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar and earthquake prediction [6, 11, 12, 13].
Wavelet analysis is a powerful mathematical tool, that has been used widely in image digital processing, quantum field theory [15-17], numerical analysis and many other fields in recent years. Today, there are many works on wavelets methods for approximating the solution of the problems [18, 19], such as Haar wavelets method [8], SAC wavelets method, Harmonic wavelets method, first and second Chebyshev wavelets [14] and Legendre wavelets method [7]. In the present paper, gave some important characteristics to Laguerre polynomials with its wave will be given including new properties. Processing image is also discussed in this paper.

II. LAGUERRE POLYNOMIALS AND ITS PROPERTIES

2.1. Laguerre's differential equation:

The differential equation of Laguerre's polynpmial given by

$$xy'' + (1-x)y' + ny = 0 \qquad (1)$$

where n=0,1,2,3,….
This equation has polynomial solutions called Laguerre polynomials is given by

$$L_n(x) = e^x \frac{d^n}{dx^n}\left(x^n e^{-x}\right) \qquad (2)$$

Which is also referred to as Rodrigue's formula for the Laguerre polynomials.
The first few Laguerre polynomials are

$$L_0(x)=1,\quad L_1(x)=1-x,$$
$$L_2(x)=x^2-4x+2,$$
$$L_3(x)=6-18x+9x^2-x^3,\ \ldots.. \qquad (3)$$

where $L_n(x)$ is a polynomial of degree n.

2.2. Programming Laguerre polynomial:

By using Matlab, function

```
L=Lag(n,t), if n==0
        L=1; else
        Sum = 0;
        for k = 0 : n
        sum = sum+(-1)^k*factorial(n)*factorial(n)*t^k/
        (factorial(k)*factorial(k)*factorial(n-k));
         end
         L=sum;
         End.
```

2.3. Some important properties of Laguerre polynomials:

In the following we list some properties of the Laguerre polynomials.

1. Generating function:

$$\frac{e^{-xt/(1-t)}}{1-t} = \sum_{n=0}^{\infty} \frac{L_n}{n!} t^n \qquad (4)$$

2. Recurrence formulae:

$$L_{n+1}(x) = (2n+1-x)L_n(x) - n^2 L_{n-1} \qquad (5)$$

$$L_n'(x) - nL_{n-1}'(x) + nL_{n-1}(x) = 0 \qquad (6)$$

$$xL_n'(x) = nL_n(x) - n^2 L_{n-1}(x) \qquad (7)$$

3. Orthogonality

$$\int_0^\infty e^{-x} L_m(x) L_n(x) dx = \begin{cases} 0 & if \;\; m \neq n \\ (n!)^2 & if \;\; m = n \end{cases}$$

(8)

4. Series expansions:

If $f(x) = \sum_{k=0}^{\infty} A_k L_k(x)$ then

$$A_n = \frac{1}{(n!)^2} \int_0^\infty e^{-x} f(x) L_n(x) dx \qquad (9)$$

2.4. Miscellaneous orthogonal polynomials and their properties:

There are many other examples of orthogonal polynomials. Some of the more important ones, together with their properties, are given in the following list.

Associated Laguerre Polynomials $L_n^m(x)$

$$L_n^m(x) = \frac{d^m}{dx^m} L_n(x) \qquad (10)$$

and satisfying the equation

$$xy'' + (m+1-x)y' + (n-m)y = 0 \qquad (11)$$

If $m > n$ then $L_n^m(x) = 0$

we have $\int_0^\infty e^{-x} L_n^m(x) L_p^m(x) dx = 0 \qquad p \neq n$

(12)

$$\int_0^\infty x^m e^{-x} \left(L_n^m(x)\right)^2 dx = \frac{(n!)^2}{(n-m)!} \qquad p = n \qquad (13)$$

II. LAGUERRE WAVELETS

In this section we constructed Laguerre wavelet from the family function

$$\rho_{s,r}(t) = |s|^{\frac{-1}{2}} \rho\left(\frac{t-r}{s}\right), \text{ for } s, r \in R,\ s \neq 0$$

(14) where $\rho(t) = [\rho_0(t), \rho_1(t), ..., \rho_{M-1}]^T$

The elements $\rho_0(t), \rho_1(t), ..., \rho_{M-1}(t)$ are the basis functions, orthogonal on the [0,1].

3.1. Constructed Laguerre Wavelets:

Laguerre wavelet is denoted by $(Lag)_{wav}$ is the type of wavelets used for solving differential equations, integral equations , variation problems, different sciences and engineering problems as well as fractional differential equations. Laguerre wavelet $\rho_{n,m}(t) = \rho_{t,n,m,k}$ have four arguments $k = 1,2,...,\quad n = 1,2,...,2^{k-1}$, m is order for Laguerre polynomials and t is normalized time. If we dilation by parameter $s = 2^{-(k+1)}$ and translation by parameter $r = 2^{-(k+1)}(2n-1)$ and use transform x in (14), $x = 2^{-(k+1)}(2^k t)$. Then we will get the following equation

$$\rho_{n,m}(t) = \begin{cases} 2^{k+1/2} \tilde{L}_m(2^k t - 2n + 1) & \frac{n-1}{2^{k-1}} \le t \le \frac{n}{2^{k-1}} \\ 0 & \text{otherwise} \end{cases}$$

(15)

where $\tilde{L}_m = \frac{1}{m!} L_m$, for k=2.

3.2 Programming of Laguerre wavelets:

By MATLAB program we can get above functions or wavelets function of order n by the following algorithms.

Case 1: functions on the interval [0, 0.5)

```
function L1 = Lag1(m,t)
if m = 0, L1 = 2*sqrt(2)
else s = 0; for k = 0:m
s = s + (-1) ^ k*factorial(m)*factorial(m)*
( ddd*1,t)
 ^k/  (factorial(k)*factorial(k)*factorial(m-k));
end
L1=(2*sqrt(2)/factorial(m))*s
End.
```

Case 2: functions on the interval [0. 5,1)

```
function L1 = Lag1(m,t)
if m==0, L1=2*sqrt(2)
else s = 0; f or k = 0:m
s = s +(-1)^k*factorial(m)*factorial(m)*(ddd(2,t)) ^k/
 (factorial(k)*factorial(k)*factorial(m-k));
 end.
 L1=(2*sqrt(2)/factorial(m))*s
 End.
```

IV. ORTHOGONALITY OF LAGUERRE WAVELETS

From section (2.3) and equation (8) we know $L_m(x)$ has orthogonality with respect to the weight function $w(t) = e^{-t}$ on the interval $[0, \infty)$. The set of laguerre wavelets are the orthogonal with respect to weight function

$$w_n(t) = e^{[-(2^k t - 2n+1)]} .$$

It is a step function taking values wavelets on $[0, 0.5)$ and $[0.5, 1)$ respectively where t is known that any continuous function approximated uniformly by Laguerre function. We will defined by using the Laguerre wavelets. Dilations and translations of the function $\rho_{n,m}(t)$ define an orthogonal basis in $L^2(R)$, the space of all square integrable functions. This means that any element in $L^2(R)$ may be represented as a linear combination (possibly in finite) of these basis functions.

Theorem 1: The orthogonal of $\rho_{n,m}(t)$ is easy to check. It is apparent that

$$\int \rho_{n,m}(t)\rho_{n'm'}(t)dt \qquad (16)$$

Whenever $n=n'$ and $m=m'$ is not satisfied simultaneously. If $m \neq m'$ (say $n' < n$), the non zero values of the wavelet $\rho_{n',m'}$ are contained in the set where the wavelet $\rho_{n,m}$ contains Laguerre function then that makes integral equal to zero.

If $m=m'$ but $n \neq n'$, then at least one factor in the product $\rho_{n',m'}, \rho_{n,m}$ is zero. Thus the function $\rho_{n,m}$ is orthogonal

$$\int_0^\infty e^{\left[-\left(2^k t-2n+1\right)\right]}\rho_{n',m'}(t)\rho_{n,m}(t)dt$$

$$=\int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} e^{\left[-\left(2^k t-2n+1\right)\right]}\rho_{n',m'}(t)\rho_{n,m}(t)dt$$

$$=\begin{cases} 0 & ,\text{if } m \neq m' \\ 8(m!)^2 & ,\text{if } m=m' \end{cases}$$

(17)

V. FUNCTION APPROXIMATION

A function approximation $f(t) \in L^2[0,1]$ may be expanded as

$$f(t)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} A_{n,m}\rho_{n,m}(t)$$

where , $A_{n,m}=\langle f(t),\rho_{n,m}(t)\rangle$.

(18)

In equation (18), $\langle .,. \rangle$ denote the inner product with weight function $w_n(t)$ on the Hilbert Space $[1,0)$. If the infinite series in above equation is truncated, then equation (17) can be written as,

$$f(t)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} A_{n,m}\rho_{n,m}(t)=A^T\rho_{n,m}(t)$$

(19)

Where, A and $\rho(t)$ are $2^{k-1}M \times 1$ matrices given by

$$A=\left[A_{1,0},A_{1,1},\ldots,A_{2,0},\ldots,A_{2,(M-1)},\ldots,A_{2^{k-1},0},\ldots,A_{2^{k-1},M-1}\right]^T$$

(20)

$$\rho(t)=\left[\rho_{1,0},\rho_{1,1}(t),\ldots,\rho_{1,M-1}(t),\rho_{2,0}(t),\ldots,\right.$$
$$\left.\rho_{2^{k-1},M-1}(t),\ldots,\rho_{2^{k-1},0}(t),\ldots,\rho_{2^{k-1},M-1}(t)\right]^T$$

(21)

VI. SHIFTED LAGUERRE WAVELETS

Shifting the Laguerre wavelets by using polynomials, the equation (15) will become

$$\rho^*_{n,m}(t)=\begin{cases} 2^{\left(\frac{k+1}{2}\right)}\tilde{L}_m\left(2^k t-2n-1\right), & \text{if } \frac{n}{2^{k-1}} \leq t \leq \frac{n+1}{2^{k-1}} \\ 0 \quad , & \text{otherwise} \end{cases}$$

(22)

Where $\tilde{L}_m = \frac{1}{m!}L_m$, for $m=0,1,\ldots,M$, $M=2$, $n=0,1,2,\ldots,2^{k-1}$, should note in dealing with Laguerre wavelets the weight function $w_n(t)$ have to dilated and translated.

$w_n(t)=\left(2^k t-2n-1\right)$, a function $f(t)$ defined over $L^2[0,1]$ can be expanded as,

$$f(t)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A_{n,m}\rho_{n,m}(t),$$

where $A_{n,m}=\langle f(t),\rho_{n,m}(t)\rangle$

(23)

If infinite series in (23) is truncated n, then it can be written as,

$$f(t)=\sum_{n=o}^{2^{k-1}-1}\sum_{m=0}^{M} A_{n,m}\rho_{n,m}(t)=A^T\rho_{n,m}(t)$$

(24)

where A and $\rho(t)$ are $2^{k-1}M \times 1$ matrices given by

$$A=\left[A_{0,0},A_{0,1},\ldots,A_{0,M},A_{2,0},\ldots,A_{2,M},\ldots,\right.$$
$$\left.A_{2^{k-1}-1,0},\ldots,A_{2^{k-1}-1,M}\right]^T$$

(25)

$$\rho(t)=\left[\rho_{0,0}(t),\rho_{0,1}(t),\ldots,\rho_{0,M}(t),\rho_{2,0}(t),\ldots,\right.$$

$$\rho_{2^{k-1}-1,M}(t),\ldots,\rho_{2^{k-1}-1,0}(t),\ldots,\rho_{2^{k-1}-1,M}\Big]^T$$

(26)

Theorem 2:

A function $f(t)\le N \in L^2([0,1])$ with limited second derivative say be limited second, say $|f''(t)\le N|$, can be widened as an unlimited aggregate of Laguerre wavelets, and the series converges uniformly to $f(t)$ that is

$$f(t)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}A_{n,m}\rho_{n,m}(t)\ ,\quad |A_{n,m}|\le \frac{N}{2^N\left(3\frac{k+1}{2}\right)}. \qquad (27)$$

Proof of the theorem:

we have $A_{n,m}=\int_0^1 f(t)\rho_{n,m}(t)w_n(t)dt$ and

$$A_{n,m}=\int_{\frac{n-1}{2^{k-1}}}^{\frac{n}{2^{k-1}}} 2^{\left(\frac{k+1}{2}\right)}\tilde{L}_m\left(2^k t-2n+1\right)f(t)w_n\left(2^k t-2n+1\right)dt$$

if m > 0, by substituting $2^k t-2n+1=x$, it yields

$$A_{n,m}=\frac{1}{2^{\left(\frac{k+1}{2}\right)}2^k}\int_0^{\infty}\tilde{L}_m(x)f\left(\frac{x+2n-1}{2^k}\right)e^{-x}dt$$

(28)

$$A_{n,m}=\frac{1}{2^{\left(\frac{3k+1}{2}\right)}}\int_0^{\infty}\tilde{L}_m(x)f\left(\frac{x+2n-1}{2^k}\right)e^{-x}dx$$

$e^{-x}\to 0$ whenever $x\to\infty$ by complete integration (n) times, $|A_{n,m}|\le \frac{N}{2^N\left(3\frac{k+1}{2}\right)}$.

This completes the proof of the theorem.

VII. EMPLOYMENT MATRICES FOR LAGUERRE WAVELET

7.1. The Employment Matrix of Derivative for Laguerre wavelet:

In this section we use shifted Laguerre wavelets in employment matrix of derivative for Laguerre wavelets. First we construct 6×6 matrix and it denoted by $D_{Lag(wav)}$ (derivative for Laguerre wavelets) for k=2 and M=2 by differentiation equation (22).

$$\left.\begin{array}{lll}\rho_1^{*\prime} & = & 0\\ \rho_2^{*\prime} & = & -4\rho_1\\ \rho_3^{*\prime} & = & -4\rho_1-4\rho_2\end{array}\right\}\ 0\le t\le\frac{1}{2},$$

And

$$\left.\begin{array}{lll}\rho_4^{*\prime} & = & 0\\ \rho_5^{*\prime} & = & -4\rho_4\\ \rho_6^{*\prime} & = & -4\rho_4-4\rho_5\end{array}\right\}\ 0\le\frac{1}{2}\le 1$$

The Employment Matrix of Derivative for Laguerre Wavelet

$$D_{Lag(wav)}=\begin{bmatrix}0&0&0&&0&0&0\\ -4&0&0&\vdots&0&0&0\\ -4&-4&0&&0&0&0\\ &\ldots&&\ldots&&\ldots&\\ 0&0&0&&0&0&0\\ 0&0&0&\vdots&-4&0&0\\ 0&0&0&&-4&-4&0\end{bmatrix},$$

$$D_{Lag(wav)}=-4\begin{bmatrix}L&\vdots&O\\ \ldots&\ldots&\ldots\\ O&\vdots&L\end{bmatrix}\quad L=\begin{bmatrix}0&0&0\\1&0&0\\1&1&0\end{bmatrix}$$

and $\quad O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

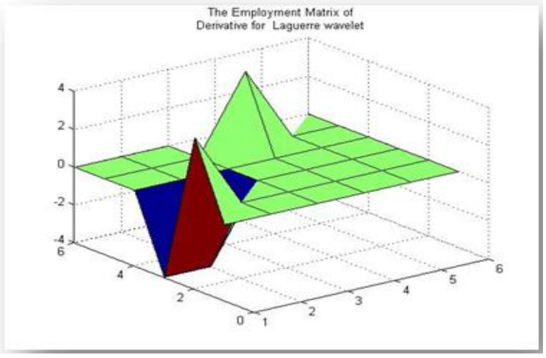


Fig.1 (The Employment Matrix of Derivative for Laguerre Wavelet)

Theorem 3:

Let $\rho_e(t)$ be the Laguerre wavelets vector defined in equation (26). Then derivative of this vector $\rho_e(t)$ can be expressed as

$$\frac{d\rho}{dt}=D_{(Lag)_{wav}}\ ,\text{where}$$

$D_{(Lag)_{wav}}$ is the $2^{k-1}(M+1)$,

$$L_{D_{(Lag)_{wav}}} = -4\begin{bmatrix} L & 0 & \dots & 0 \\ 0 & L & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & L \end{bmatrix}$$

in which is $L_{D_{(Lag)_{wav}}}$ is $(M+1)\times(M+1)$ matrix and it's (e,f), the element is defined as follows

$$L_{e,f} = \begin{cases} -2^k, & e=2,\dots,(M+1),\ f=1,\dots,e-1 \\ 0 & \text{otherwise} \end{cases}.$$

Proof: By using shifted Laguerre polynomials in $[0,1]$, the e^{th} element of vector ρ_e^* can be written as,

$$\rho_e(t) = \begin{cases} \dfrac{2^{\left(\frac{k+1}{2}\right)}}{m!} L_m\left(2^k t - n\right) & \dfrac{n}{2^{k-1}} \le t \le \dfrac{n+1}{2^{k-1}} \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

$e = 1,2,\dots,2^{k-1}(M+1)$, $m = 0,1,\dots,M$ and $n = 0,1,\dots,(2^{k-1}-1)$.

Differentiation equation (29)

$$\frac{d\rho_e^*}{dt}(t) = \begin{cases} \dfrac{2^{\left(\frac{k+1}{2}\right)}}{m!} 2^k L_m^{*\prime}\left(2^{k-1} t - n\right) & \dfrac{n}{2^{k-1}} \le t \le \dfrac{n+1}{2^{k-1}} \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

That is $\rho_e(t)$, $e = n(M+1)+n(M+1)+2,\dots,(n+1)(M+1)$.

So its Laguerre wavelets expansion has the following from

$$\frac{d\rho_e^*(t)}{dt} = \sum_{i=0}^{e-1} a_i \rho_i \tag{31}$$

This implies that, the employment matrix $D_{Lag(wav)}$ is a block matrix as defined in (7.1) moreover

$$\frac{dL_0^*(x)}{dt} = 0 \Rightarrow \frac{d\rho_e^*(t)}{dt} = 0, \text{ for}$$

$$e = 1(M+1)+1,\dots,(2^{k-1}-1)(M+1)+1,$$

Consequently the first row of matrix $L_{D_{Lag(wav)}}$ is zero.

Now by using the following equation get where

$$m = 0,1,\dots,\ L_0^{*\prime} = 0$$

$$L_m^{*\prime} = -m\left[L_{m-1} + \sum_{i=m-2}^{0} C_l L_i\right] \tag{32}$$

$$C_l = \sum_{l=0}^{m-3} (l+2)(2(l+1))$$

Substituting equation (31) and (32) in equation (30)

$$\frac{d\rho_e^*}{dt}(t) = \frac{2^{\left(\frac{k+1}{2}\right)} 2^{2k-1}}{m!}\left[-m\left[L_{m-1} + \sum_{i=m-2}^{0} C_l L_i\right]\right] \tag{33}$$

$$= -2^{\left(\frac{5k-1}{2}\right)} \sum_{f=1}^{e-1} \rho_{n(M+1)+f}(t) \tag{34}$$

Choose $L_{e,f}$ equation $\dfrac{d\rho_e'}{dt} = D_{(Lag)_{wav}}$ holds.

7.2. Employment Matrix of Integration for Laguerre Wavelets:

In this section Integration for Laguerre wavelets are discussed. For this, employment matrix of integration for Laguerre wavelets P_ρ. Now find 6×6 matrix P.

In equation (15), for M=3, the six basis functions are given by:

$$\left.\begin{aligned} \rho_{1,0}(t) &= 2\sqrt{2} \\ \rho_{1,1}(t) &= 2\sqrt{2}\,(2-4t) \\ \rho_{1,2}(t) &= \sqrt{2}\,(16t^2-24t+7) \end{aligned}\right\} t\in[0,0.5)$$

and

$$\left.\begin{aligned} \rho_{2,0}(t) &= 2\sqrt{2} \\ \rho_{2,1}(t) &= 2\sqrt{2}\,(4-4t) \\ \rho_{2,2}(t) &= \sqrt{2}\,(16t^2-40t+23) \end{aligned}\right\} t\in[0.5,1) \tag{35}$$

By integration the above six functions from 0 to t and using equation (18) we obtain

$$\left.\begin{aligned} \int_0^t \rho_{1,0}(t)dt &= \left(\tfrac{1}{2}\right)\rho_{1,0}(t) - \left(\tfrac{1}{4}\right)\rho_{1,1}(t) + \left(\tfrac{1}{2}\right)\rho_{2,0}(t) \\ \int_0^t \rho_{1,1}(t)dt &= \left(\tfrac{3}{8}\right)\rho_{1,0}(t) + \left(\tfrac{1}{4}\right)\rho_{1,1}(t) - \left(\tfrac{1}{4}\right)\rho_{1,2}(t) + \left(\tfrac{1}{2}\right)\rho_{2,0}(t) \\ \int_0^t \rho_{1,2}(t)dt &= \left(\tfrac{13}{24}\right)\rho_{1,0}(t) + \left(\tfrac{1}{4}\right)\rho_{1,2}(t) + \left(\tfrac{7}{12}\right)\rho_{2,0}(t) \end{aligned}\right\} t\in[0,0.5)$$

$$\left.\begin{aligned}
\int_0^t \rho_{2,0}(t)dt &= \left(\frac{1}{2}\right)\rho_{2,0}(t) - \left(\frac{1}{4}\right)\rho_{2,1}(t)\\
\int_0^t \rho_{2,1}(t)dt &= \left(\frac{3}{8}\right)\rho_{2,0}(t) + \left(\frac{1}{4}\right)\rho_{2,1}(t) - \left(\frac{1}{4}\right)\rho_{2,2}(t)\\
\int_0^t \rho_{21,2}(t)dt &= \left(\frac{13}{24}\right)\rho_{2,0}(t) + \left(\frac{1}{4}\right)\rho_{2,2}(t)
\end{aligned}\right\} t \in [0.5,1)$$

Thus, $\int_0^t \rho_6(t)dt = P_{6\times 6}\rho_6(t)$, where

$$\rho_6(t) = \left[\rho_{1,0}(t), \rho_{1,1}(t), \rho_{1,2}(t), \rho_{2,0}(t), \rho_{2,1}(t), \rho_{2,2}(t)\right]^T$$

By using above equations the operational matrix of integration P_ρ is Employment Matrix of Integration for Laguerre Wavelets

$$P_{(Lag)_{wav}} = \begin{bmatrix}
\frac{1}{2} & -\frac{1}{4} & 0 & \vdots & \frac{1}{2} & 0 & 0\\
\frac{3}{8} & \frac{1}{4} & -\frac{1}{4} & \vdots & \frac{1}{2} & 0 & 0\\
\frac{13}{24} & 0 & \frac{1}{4} & & \frac{7}{12} & 0 & 0\\
& \cdots & & \cdots & & \cdots & \\
& & & & \frac{1}{2} & -\frac{1}{4} & 0\\
0 & 0 & 0 & \vdots & \frac{3}{8} & \frac{1}{4} & -\frac{1}{4}\\
0 & 0 & 0 & & \frac{13}{24} & 0 & \frac{1}{4}\\
0 & 0 & 0 & & & &
\end{bmatrix},$$

$$P_{(Lag)_{wav}} = \begin{bmatrix} A_{3\times 3} & \vdots & O_{3\times 3}\\ \cdots & \cdots & \cdots\\ O_{3\times 3} & \vdots & A_{3\times 3}\end{bmatrix}$$

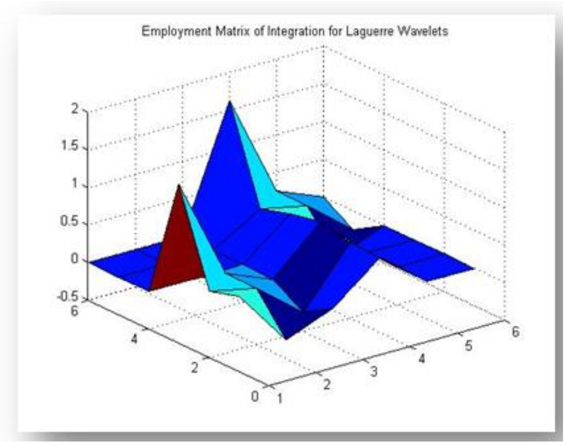


Fig.2 (The Employment Matrix of Integration for Laguerre Wavelet)

7.3. Powers in terms of Laguerre Wavelets:

In this section, we will derive powers in terms of

Laguerre Wavelets, for $k = 2$, $n = 1, 2$, $M = 1, 2, 3,$.

And t is the normalized time, will derive the powers

In terms of Laguerre Wavelets, which help to solve Problems. Let $M = 3$, $m = 0,1,2,3,\ldots$ basis

Functions are given by: In In matrix form, the powers of t can be rewritten as follows

$$Z = L_\rho W,\; L_\rho = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 0 & 0\\ \frac{1}{2} & -\frac{1}{4} & 0\\ \frac{5}{16} & -\frac{3}{8} & \frac{1}{8}\end{bmatrix}, t \in [0, 0.5)$$

and

$$L_\rho = \frac{1}{2\sqrt{2}}\begin{bmatrix} 1 & 0 & 0\\ 1 & -\frac{1}{4} & 0\\ \frac{17}{16} & -\frac{5}{8} & \frac{1}{8}\end{bmatrix}, t \in [0.5, 1)$$

Where $\left[Z = \begin{pmatrix} t^0\\ t^1\\ t^2\end{pmatrix}\right]$, $\left[W = \begin{pmatrix}\rho_{1,0}\\ \rho_{1,1}\\ \rho_{1,2}\end{pmatrix}\right]$, $t \in [0, 0.5)$,

$\left[Z = \begin{pmatrix} t^0\\ t^1\\ t^2\end{pmatrix}\right]$ and $\left[W = \begin{pmatrix}\rho_{2,0}\\ \rho_{2,1}\\ \rho_{2,2}\end{pmatrix}\right]$, $t \in [0.5, 1)$.

Powers in terms Laguerre Wavelets:

$$L_{\rho_{6\times 6}} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0\\
\frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 0\\
\frac{5}{16} & -\frac{3}{8} & \frac{1}{8} & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & -\frac{1}{8} & 0\\
0 & 0 & 0 & \frac{17}{16} & \frac{-5}{8} & \frac{1}{8}
\end{bmatrix}$$

Constructing the operation matrix of integration and differentiation that can be used in solving many problems, which are illustrated in the following examples.

VIII. APPLICATION OF MATRICES $D_{(Lag)_{wav}}$ AND $P_{(Lag)_{wav}}$ FOR SOLVING CALCULUS OF VARIATIONAL PROBLEMS

In order to solve linear or nonlinear differential equations by using the employment matrices $D_{(Lag)_{wav}}$ and $P_{(Lag)_{wav}}$, some numerical examples illustrate the

Procedure [18]. In order to solve linear or nonlinear

X	Exact solution	Approximat solution k=2,M=3	Approximat solution k=2,M=4
0	0	0.00000001	0
0.2	0.02287779	0.0228	0.02287
0.4	0.10940518	0.10945544	0.10940544
0.6	0.27115248	0.25826756	0.27826756
0.8	0.55489181	0.54330957	0.55330957
1	1	0.99999999	1

differential equation by using the above operational matrices some numerical examples illustrate the procedure which will begin with

$$y^{i}(t)=A^{T}\rho_{n,m}(t),$$

$$y^{i-1}(t)=A^{T}\int_{0}^{t}\rho_{n,m}(t)dt+y^{i-2}(0)=A^{T}P\rho_{n,m}(t)+y^{i-1}(0)$$

,. ., ………..

$$y(t)=A^{T}P^{i}\rho_{n,m}(t)+y^{i-1}(0)t^{i-2}+y^{i-2}(0)t^{i-3}+\ldots+y(0).$$

Example (1).

Consider the following variation problem [18]

$$\min \upsilon[y]=\int_{0}^{1}\left(2y^{2'}+8y\right)dt$$

With the boundary conditions $y(0)=1,\ y(1)=2$.

The corresponding Euler Lagrange equation is $y''+y'=2(t+1)$. Then the exact solution for this Problem is $y(t)=t^{2}+1$ with the above boundary conditions to solve this problem, assuming that $y(t)=A^{T}P\rho_{(Lag)_{wav}}(t)$.

Find y', y'' to go $y'(t)=A^{T}D_{(Lag)_{wav}}$, $y''(t)=A^{T}D^{2}_{(Lag)_{wav}}$ will complete this example see [18] just replace old matrices by (L)s' matrices of integration and derivative reached the exact solution.

Example (2):

Consider the following Volterra integro differential equation (VIDE) [4], [5]

$$U''(x)=e^{2x}-\int_{0}^{x}e^{2(x-t)}\,U'\,dt \quad \text{with}$$

$$U(0)=0\,,U'(0)=0.$$

The exact solution $U(x)=xe^{x}-e^{x}+1$, table.1

Shows the numerical results for this example with $k=2,\ M=3$, with $error=10^{-4}$, $k=2,\ M=4$, $error=10^{-5}$ are compared with exact solution graphically in fig.3.

Table 1 shows some numerical results for example (2)

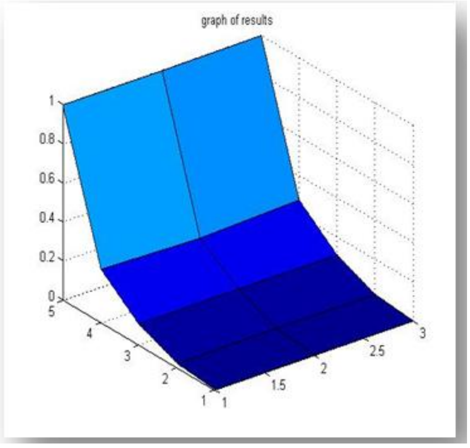
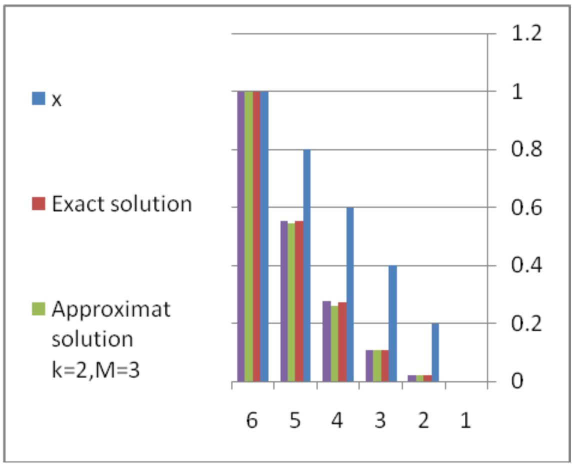


Fig.3 (3D of results of example (2))



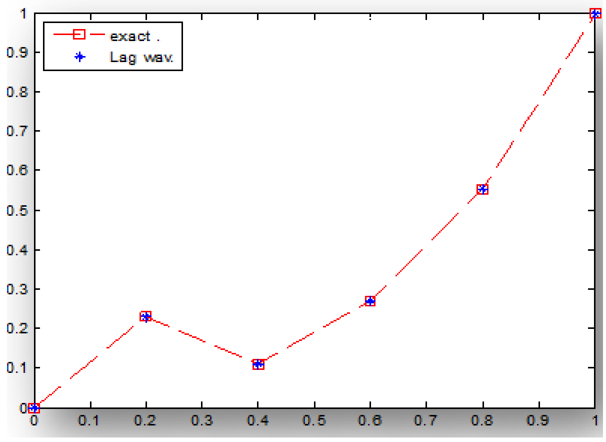


Fig.5 (compare the results with exact solution of example (2))

Example (3):
Consider the following variation problem,[18]

$$Min\,\upsilon[y]=\int_0^1\left(y'^2+y^2\right)dt$$

with the boundary conditions $y(0)=0$, $y(1)=1$,

the corresponding Euler Lagrange equation is

$y''=y$, the exact solution for this problem is

$$y(t)=\frac{e^t-e^{-t}}{e^1-e^{-1}}\ ,$$

t	exact solution	First Chebyshev wavelets	Second Chebyshev wavelets	Laguerre wavelets
0	0	0.00057584	0.00114883	0
0.1	0.0852337	0.08469184	0.08505153	0.0852337
0.2	0.17132045	0.17097404	0.17111507	0.17132045
0.3	0.25912184	0.29422446	0.25933944	0.25912184
0.4	34951660	0.35003704	0.34972464	34951660
0.5	0.44340944	0.44281784	0.44276674	0.44340944
0.6	0.54174007	0.54103704	0.54035565	0.54174007
0.7	0.64549262	0.64500906	0.64402083	0.64549262
0.8	0.75570548	0.75603258	0.75470746	0.75570548
0.9	0.87348169	0.87410757	0.87241554	0.87348169
1	1	0.99923405	0.99714506	1

with boundary conditions to solve this problem by

using $P\rho_{(Lag)_{wav}}$, will solve this problem and reached the exact solution.

IX . CONCLUSION

The Laguerre wavelets operational matrices of integrations with the aid of spectral and collocation methods are applied to solve many problems. The wavelets method authorizes the mood of very fast algorithms when compared to the algorithms app roach used (Laguerre Polynomials). Numerical results with comparisons are given to confirm the reliability of the proposed method for solving many problems.

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