

# Mean-Variance Portfolio Optimization under Asset-Liability based on Time Series Approaches

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**Abstract.** In this paper, we analyze about Optimization of Mean-Variance portfolio under assets-liability based on time series approach. It is assumed that the asset return follows the time series pattern, where the asset return has non-constants of mean and volatility. Non-constant mean is estimated using the model of autoregressive moving average (ARMA), and non-constant volatility is estimated using the generally autoregressive conditional heteroscedastic (GARCH) model. While the mean and variance of return liabilities are estimated using the return of bonds. The surplus return is estimated using the asset liability model. The predictive value of the mean and volatility model is non-constant, then used to determine the mean and variance of surplus return following the asset liability models. Next, the value of the mean and variance of returns surplus is used for the investment portfolio optimization process. Portfolio optimization of the surplus return is done using the Mean-Variance model from Markowitz. As a numerical illustration is analyzed several assets traded on the capital market in Indonesia. Optimization of this portfolio produces a combination of optimum weight, which can be used as consideration for investors in making investment decisions on assets analyzed.

**Keywords:** Optimization, mean-variance, asset-liability, ARMA, GARCH

## I. INTRODUCTION

In general, investment is an investment strategy, either directly or indirectly, aimed at obtaining certain benefits as a result of investment [11]. There are some investors who are able to invest in financial assets, but on the other hand, the investor also has an obligation to pay from the investment [9]. Thus, the profit earned by investors is a surplus between investment returns with obligations (liabilities) to be borne [7], [14]. Every investment decision, an investor will be directed to the highest surplus rate. Investors will choose the investment that promises the highest return in order to obtain a high surplus rate as well. The problem is that the investment is risk-taking, therefore investors should take into account the risk factor [4], [9].

Investing in financial assets, there are several characteristics of the return of financial assets that need to be considered, in order to be able to analyze properly. One of its characteristics, that the return of financial assets often follows the time series pattern [8]. So the return of this financial asset has an unstable average and volatility, or its values change with time change [10]. To estimate the magnitude and volatility of these constant values Gökbulut & Pekaya [3], and Jánský, & Rippel [4], do so using the autoregressive moving average (ARMA) model and generally autoregressive conditional heteroscedastic (GARCH). Furthermore, based on the constant estimator and the unstable volatility of asset returns and returns on these liabilities, surplus returns can be determined. To determine this surplus return, Wurtz, Chalabi, & Luksan [13] and Zhao, Wei, & Wang [14], do so using an asset-liability model. Investors want a maximum surplus return with a certain level of risk, or a certain surplus return rate with a minimum risk level. According to Kirby and Ostdiek [5], a strategy often used in dealing with risky investment conditions is to establish a portfolio. The essence of forming a portfolio is to allocate funds on several investment opportunities so that investment risk will be reduced or minimized [1]. The mathematical model that can accommodate the investor's objectives, namely maximizing surplus and minimizing risk is the form of the Mean-Variance portfolio [5].

Therefore, this paper analyzes the optimization of the investment portfolio of Mean-Variance liability assets, in which both asset return and liability returns follow the time series model, which has an unstable mean and volatility. The goal is to obtain an infectious surface, ie the various points of the average pair and the risk of a viable portfolio to invest. As a numerical illustration is analyzed some of the investment assets traded on the Indonesian capital market, and several liability factors.

## II. METHODOLOGY

This section discusses the determination of asset returns, estimates of mean models, estimates of volatility models, asset liability models, and Markowitz portfolio optimization, which are further described as follows.

**A. Asset Return Determination**

Suppose  $P_t$  is a price or value of an asset-liability at a time  $t$  ( $t=1, \dots, T$  and  $T$  is the number of data observations), and  $r_t$  return of asset-liability at time  $t$ . The amount of asset-liability return can be determined by the equation [10]:

$$r_t = \ln P_t - \ln P_{t-1} \tag{1}$$

Return data  $r_t$  subsequently used in the estimation of the mean model as follows.

**B. Estimation of Mean Model**

Suppose  $r_t$  asset return on time  $t$ , in general the autoregressive moving average model, ARMA( $p, q$ ), can be expressed in the following equation [10], [2]:

$$r_t = \phi_0 + \sum_{i=1}^p \phi_1 r_{t-i} + \varepsilon_t - \sum_{j=1}^q \theta_j \varepsilon_{t-j} \tag{2}$$

Where  $\{\varepsilon_t\}$  is assumed normal white noise distribution with zero mean and variance  $\sigma_\varepsilon^2$ . Non-negative integer  $p$  and  $q$  is an ARMA order. The AR and MA models are specific model in cases ARMA( $p, q$ ). Using a back-shift operator, model (2) can be written as:

$$(1 - \phi_1 B - \dots - \phi_p B^p) r_t = \phi_0 + (1 - \theta_1 B - \dots - \theta_q B^q) \varepsilon_t \tag{3}$$

Polynomial  $(1 - \phi_1 B - \dots - \phi_p B^p)$  of AR model and polynomial  $(1 - \theta_1 B - \dots - \theta_q B^q)$  of MA model. If all solutions of characteristic equations are absolute smaller 1, then the stationary ARMA model is weak. In this case, the unconditional mean of the model is  $E(r_t) = \phi_0 / (1 - \phi_1 - \dots - \phi_p)$  [12].

**Mean Modeling Stages.** Broadly speaking, according to Tsay [10], the average modeling stage is as follows: (i) Identification of the model, determining the order value and using the ACF (autocorrelation function) and PACF (partial autocorrelation function) plot. (ii) Parameter estimation can be done by the least squares method or maximum likelihood. (iii) Diagnostic test, with white noise test and serial correlation to residual  $\varepsilon_t$ , and (iv) Prediction, if the model is suitable then it can be used for receding done recursively.

**C. Estimation of Volatility Model**

Estimation of volatility model is done by using GARCH models. The GARCH model, introduced by Bollerslev in 1986 is a general or generalized form of the ARCH model. In general, the GARCH( $m, n$ ) model can be written as follows [10], [6] :

$$\varepsilon_t = \sigma_t \mu_t,$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^m \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^n \beta_j \sigma_{t-j}^2 + u_t \tag{4}$$

Based on equation (4), the conditional expectation and the variance of  $\varepsilon_t$  is:

$$E(\varepsilon_t | F_{t-1}) = 0 \tag{5}$$

$$Var(\varepsilon_t | F_{t-1}) = E(\varepsilon_t^2 | F_{t-1}) = \sigma_t^2 \tag{6}$$

Compared to ARCH, the GARCH model is considered to provide simpler results because it uses fewer parameters [12].

**Volatility Modeling Stages.** In general, according to Tsay [8], the stages of volatility modeling are as follows: (i) Estimation of the average model with time series model (eg ARMA model). (ii) Use the residuals of the average model for the ARCH effect test. (iii) If there is an ARCH effect, estimate the volatility model, and the combined estimation form of the average model and the volatility model. (iv) Conduct diagnostic tests to test model suitability. (v) If the model matches, use for prediction is done recursively.

**D. Estimation of Asset-Liability Model**

Modeling of surplus return of asset-liability described briefly as follows. Suppose  $A_t$  assets at a time  $t$ ,  $L_t$  liability at the time  $t$ , and  $S_t$  surplus at a time  $t$ . At the beginning  $t=0$ , the initial surplus is given by:

$$S_0 = A_0 - L_0.$$

The surplus obtained after one period is:

$$S_1 = A_1 - L_1 = A_0[1 + r_A] - L_0[1 + r_L].$$

Suppose  $r_S$  surplus return expressed as [12], [11]:

$$\begin{aligned} r_S &= \frac{S_1 - S_0}{A_0} = \frac{A_0 r_A}{A_0} - \frac{L_0 r_L}{A_0} \\ &= r_A - \frac{1}{f_0} r_L \end{aligned} \tag{7}$$

with  $f_0 = \frac{L_0}{A_0}$ .

Based on equation (7) the mean of surplus return can be determined by the formula:

$$\mu_S = E[r_S] = \mu_A - \frac{1}{f_0} \mu_L \tag{8}$$

Where  $\mu_S$ ,  $\mu_A$  and  $\mu_L$  respectively is the mean of surplus returns, assets, and liabilities. Also, according to (7), the surplus variance can be determined by the formula:

$$\sigma_S^2 = \sigma_A^2 - \frac{2}{f_0} \sigma_{AL} + \frac{1}{f_0^2} \sigma_L^2 \tag{9}$$

Where  $\sigma_S^2$ ,  $\sigma_A^2$  and  $\sigma_L^2$  successive variance of surplus returns, assets, and liabilities. While  $\sigma_{AL} = Cov(r_A, r_L)$  covariance between asset return and liability return [11].

**E. Portfolio Optimization of Markowitz Model**

Suppose  $\mathbf{w}^T = (w_1; \dots; w_N)$  is the vector of portfolio weight from surplus return;  $\boldsymbol{\mu}^T = (\mu_{S1}, \dots, \mu_{SN})$  mean vector of surplus return with  $\mu_{Si} = E[r_{Si}]$ ; and  $\boldsymbol{\Sigma} = (\sigma_{ij})_{ij=1, \dots, N}$  the covariance matrix of surplus returns with  $\sigma_{ij} = Cov(r_{Si}, r_{Sj})$ . So that the mean of portfolio of surplus return can be determined by equation [11]:

$$\hat{\mu}_{Sp} = \boldsymbol{\mu}_S^T \mathbf{w}, \tag{10}$$

and the variance of portfolio of surplus return is determined by the equation:

$$\hat{\sigma}_{Sp}^2 = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}. \tag{11}$$

Furthermore, according to Panjer et al. [11], and Bjork, Murgoci, & Xun Yu Zhou [1], the optimization of surplus return is refer to the equation as follows:

$$\text{Max}_{\mathbf{w} \in R^N} \{2\boldsymbol{\mu}^T \mathbf{w} + 2\boldsymbol{\gamma}^T \mathbf{w} - \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}\}, \tag{12}$$

$$\text{Subject to: } \mathbf{e}^T \mathbf{w} = 1.$$

Where  $\boldsymbol{\gamma}^T = (\gamma_1, \dots, \gamma_N)$  vector of covariance between asset return and liability return, with  $\gamma_i = \frac{1}{f_0} Cov(r_{Ai}, r_L)$ ; and  $\mathbf{e}^T = (1, 1, \dots, 1)$  unit vector.

The Lagrangian function of (12) is given by the equation as follows:

$$L(\mathbf{w}, \lambda) = 2\boldsymbol{\mu}^T \mathbf{w} + 2\boldsymbol{\gamma}^T \mathbf{w} - \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} + \lambda(\mathbf{e}^T \mathbf{w} - 1).$$

Based on the Kuhn-Tucker theorem, the necessary and sufficient conditions for optimality conditions are achieved when:

$$\frac{\partial L}{\partial \mathbf{w}} = 2\boldsymbol{\mu} + 2\boldsymbol{\gamma} - 2\boldsymbol{\Sigma} \mathbf{w} + \lambda \mathbf{e} = 0, \tag{13}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{e}^T \mathbf{w} - 1 = 0. \tag{14}$$

Completing the system of equations (13) and (14), the following equations are obtained:

- For  $\tau = 0$ , obtained the first form of the minimum weight vector equation as:

$$\mathbf{w}^{Min} = \frac{1}{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}} \boldsymbol{\Sigma}^{-1} \mathbf{e}, \tag{15}$$

and the second form is:

$$\mathbf{z}^L = \left[ \boldsymbol{\Sigma}^{-1} \boldsymbol{\gamma} - \frac{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\gamma}}{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}} \boldsymbol{\Sigma}^{-1} \mathbf{e} \right], \tag{16}$$

$$\text{with } \sum_{i=1}^N \mathbf{z}_i^L = 0.$$

Thus, an efficient portfolio with minimum variance under liabilities is:

$$\mathbf{w}^{MinL} = \mathbf{w}^{Min} + \mathbf{z}^L. \tag{17}$$

- For  $\tau > 0$ , obtained form:

$$\mathbf{z}^* = \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \frac{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}}{\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}} \boldsymbol{\Sigma}^{-1} \mathbf{e}, \tag{18}$$

$$\text{with } \sum_{i=1}^N \mathbf{z}_i^* = 0.$$

Thus, an efficient portfolio with minimum variance under liabilities is:

$$\mathbf{w}^* = \mathbf{w}^{MinL} + \boldsymbol{\tau} \mathbf{z}^*. \tag{19}$$

Therefore, for  $\tau \geq 0$ , efficient portfolios under liabilities can be expressed as follows:

$$\mathbf{w}^* = \mathbf{w}^{MinL} + \mathbf{z}^L + \boldsymbol{\tau} \mathbf{z}^*; \tau \geq 0. \tag{20}$$

The above mathematical models are then used for the asset-liability analysis below.

**III. RESULTS AND DISCUSSION**

Asset data analyzed is accessed through the website <http://www.finance.go.id/>. The data consists of 5 (five) selected assets, for the period of January 2, 2014, up to June 4, 2017, covering shares: INDF, DEWA, AALI, and LSIP. Next, the sequence is called by  $A_1, A_2, A_3, A_4$ , and  $A_5$ . In addition to asset data, here also required liability data. The fifth asset price data, then calculated return each using the log return approach. While the liability data here assumed to be equal to the bonds associated with each asset  $A_1, A_2, A_3, A_4$ , and  $A_5$ . Call it these liabilities are  $L_1, L_2, L_3, L_4$ , and  $L_5$ .

Stages of analysis performed along with the results are described briefly as follows.

**A. Estimating the Mean and Volatility Model of Asset Return**

Referring to Johansson & Sowa [3], the estimation of mean and volatility models of asset returns were made using the ARMA-GARCH model. **First**, stationarity test is done to asset return data  $A_1, A_2, A_3, A_4$ , and  $A_5$  using unit root test statistics. The stationarity test is done with the help of Eviews-8 software, and the results show that all asset return data is stationary. **Secondly**, each stationary return data is then estimated by the mean model. Estimates are performed using ARMA models referring to equation (2). Estimation includes the following phases: the mean model identification, parameter estimation models, verification test parameters, and diagnostic testing. All the stages were performed using the help of Eviews-8 software, and the result of the mean model estimation all showed was significant.

**Third**, using the residuals from each mean model of assets return  $A_1, A_2, A_3, A_4$ , and  $A_5$ ,

conducted estimation non-constant volatility model. The non-constant volatility estimated using GARCH models refers to equation (4). The non-constant volatility model estimation stage includes: ARCH element test, model identification, model parameter estimation, parameter verification test, and diagnostic test. All stages are done with the help of Eviews-8 software, and the estimation results show that all non-constant volatility models have been significant. The estimation results of the mean and non-constant volatility models are generally outlined in Table 1, in the Model column. Estimator mean and non-constant volatility models, then used for prediction one period ahead, namely  $\hat{r}_{A_i}(1)$  and  $\hat{\sigma}_{A_i}^2(1)$ , and the results are summarized in Table 1.

**TABLE 1.**  
**TIME SERIES MODEL AND PARAMETER VALUE ESTIMATOR**

Asset $A_i$	Models	Mean $\hat{r}_{A_i}(1)$	Variance $\hat{\sigma}_{A_i}^2(1)$
$A_1$	ARMA(1, 0)-GARCH(1,1)	0.015399	0.002643
$A_2$	ARMA(2, 2)-ARCH(1)-M	0.039007	0.002797
$A_3$	ARMA(0, 1)-GARCH(3,3)	0.003315	0.001331
$A_4$	ARMA(1, 1)-GARCH(1,1)	0.008672	0.001921
$A_5$	ARMA(0, 1)-GARCH(1,1)	-0.000262	0.001873

Furthermore, it is assumed that the return liability  $L_1, L_2, L_3, L_4$ , and  $L_5$ , each has a mean and variance as given in Table 2.

**TABLE 2.**  
**MEAN AND VARIANCE OF LIABILITIES RETURN**

Liabilities $L_i$	Mean $\hat{\mu}_{L_i}$	Variance $\hat{\sigma}_{L_i}^2$
$L_1$	0.000121	0.000102
$L_2$	0.002805	0.000125
$L_3$	0.000213	0.000031
$L_4$	0.000642	0.000005
$L_5$	0.000011	0.000075

As well as having the covariance between each asset and its liabilities are given respectively as the following vectors  $\gamma^T = (0.000021, 0.000073, 0.000015, 0.000022, -0.00026)$ .

Next, the values presented in Table 1, Table 2, and the covariance vector are used to calculate estimator of the mean and variance of surplus returns.

**B. Estimation of Mean and Variance of Surplus Return**

In this section, the mean and variance values of surplus return are estimated. To estimate the mean and variance values of surplus returns it is assumed

that the ratio between initial assets and liabilities is  $f_0 = 1$ . Using the values presented in Table-1, Table-2, and the covariance vectors between the assets and liabilities respectively, for the estimation of surplus return the mean values are made by reference equation (8), whereas for the estimation of variance values of surplus return is done by equation (9). The estimation results are given in Table 3.

**TABLE 3.**  
**MEAN AND VARIANCE OF SURPLUS RETURN**

Surplus $S_i$	Mean $\hat{\mu}_{S_i}$	Variance $\hat{\sigma}_{S_i}^2$
$S_1$	0.015278	0.002703
$S_2$	0.036202	0.002776
$S_3$	0.003102	0.001332
$S_4$	0.008030	0.001882
$S_5$	-0.000273	0.002000

Estimator values of the mean and variance of surplus return in Table 3, is then used to establish the mean vector and the covariance matrix of return surplus.

**C. Establish the Mean Vector and Covariance Matrix of Surplus Return**

In part, this is done formation mean vector and the covariance matrix of surplus return. Using the mean values estimator in Table-1, the mean vector of surplus return formed as  $\mu_S^T = (0.015278, 0.036202, 0.003102, 0.008030, -0.00273)$ .

Since the covariance between surplus returns is very small, so it is assumed to be equal to zero. Furthermore, by using the estimator of the variance of surplus return in Table 3, a covariance matrix of surplus return is given as follows:

$$\Sigma_S = \begin{pmatrix} 0.002703 & 0 & 0 & 0 & 0 \\ 0 & 0.002776 & 0 & 0 & 0 \\ 0 & 0 & 0.001332 & 0 & 0 \\ 0 & 0 & 0 & 0.001882 & 0 \\ 0 & 0 & 0 & 0 & 0.002000 \end{pmatrix}$$

The inverse matrix of  $\Sigma_S$  is as follows:

$$\Sigma_S^{-1} = \begin{pmatrix} 369.9593 & 0 & 0 & 0 & 0 \\ 0 & 36.2305 & 0 & 0 & 0 \\ 0 & 0 & 75.7508 & 0 & 0 \\ 0 & 0 & 0 & 53.3496 & 0 \\ 0 & 0 & 0 & 0 & 500.0000 \end{pmatrix}$$

Inverse matrix  $\Sigma_S^{-1}$  then used for the process of optimizing the portfolio of the following surplus return.

**D. Portfolio Optimization of Surplus Return**

In this section, the portfolio optimization of the investment surplus returns. Portfolio optimization of investment surplus return is based on Mean-Variance Markowitz model. Since the five stocks are used for the formation of the portfolio, it is determined that the unit vector as  $e^T = (1, 1, 1, 1, 1)$ . Based on the values of the mean vector of surplus returns, unit vectors, and inverse matrices  $\Sigma^{-1}$ , the optimization process is

done with reference to the equation (12). In the optimization process here, the risk tolerance values are simulated and tested for some values. Starting with the risk tolerance of  $\tau = 0; 0.001; 0.002; 0.003;$  and so on, which is the value in increments of 0.001.

Next, it is used to determine the weight vector  $w^*$  calculated using equation (20).

Vector composition weights are obtained and used to determine the mean surplus return values estimator portfolio by using equation (10), and to determine the surplus of portfolio risk estimator using equation (11). A collection of points of the estimator pair of the mean values of the portfolio surplus return and the portfolio surplus value estimator is used to form an efficient surface graph, as given in Fig. 1.

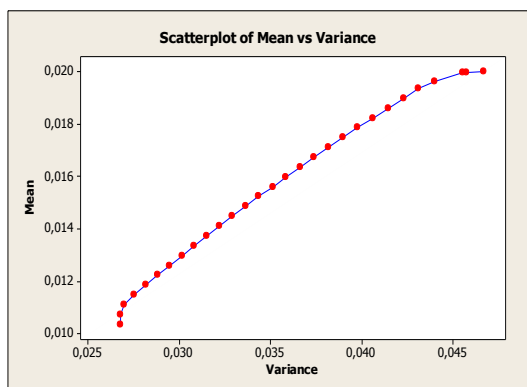


Fig. 1. Efficient surface graph

If it is assumed that short sales are not permitted, an efficient surface graph is formed along the risk tolerance interval  $0 \leq \tau < 0.029$ . Due to the risk tolerance  $\tau \geq 0.029$  produces an element in the weight vector whose value is negative, which means not feasible, or contrary to the assumption of short sales is not allowed.

The ratio between the estimator values of surplus return portfolio against the estimator values of surplus risk portfolio (variance), can be described as a graph as given in Fig. 2.

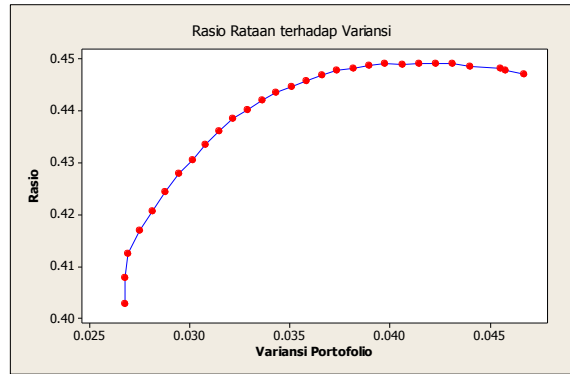


Fig. 2. Ratio vs risk graph

For risk tolerance  $\tau = 0$  obtained weight vector of minimum portfolio  $w^{*T} = (0.1485, 0.1634, 0.2969, 0.1773)$ , and when substituted into equation (10) obtained the mean portfolio of surplus return of 0.010339. When substituted into equation (11), a variance value of 0.025662 is obtained. So obtained the ratio of 0.402888 is the smallest.

Furthermore, for the value of risk tolerance  $\tau = 0.023$ , the optimum weight vector obtained as follows  $w^{*T} = (0.1975, 0.3844, 0.1861, 0.0364)$ , and the average portfolio surplus return of 0.018982; and the value of variance of 0.042264. As well as a ratio of 0.449129 is the largest, means an optimum portfolio. As for the risk tolerance  $\tau = 0.028$  weight vector obtained as  $w^{*T} = (0.2082, 0.4325, 0.1620, 0.0057)$ , and the mean portfolio of surplus return of 0.020865, and the value of variance of 0.046674. It is a portfolio that generates the largest the mean portfolio of surplus return but is not an optimum portfolio. As for risk tolerance  $\tau = 0.029$  weight vector obtained as  $w^{*T} = (0.2103, 0.4421, 0.1571, -0.0004)$ , this weight is not feasible, because there are weights that are negative value. Increased risk tolerance  $\tau = 0$  become  $\tau = 0.001; \tau = 0.002$  and so on, has brought changes in the composition of the weight vector and increase the mean portfolio of surplus return values, and increase in values of the portfolio risk (variance).

**IV. CONCLUSIONS**

This paper has discussed the mean-variance portfolio optimization under the asset-liability based on time series approaches. As a numerical illustration, analyzed return asset  $A_1, A_2, A_3, A_4$ , and  $A_5$ . Based on the analysis obtained that the fifth return of the asset in a row follows the models of ARMA(1,0)-GARCH(1,1); ARMA(2,2)-ARCH(1)-M; ARMA(0,1)-GARCH(3,3); ARMA(1,1)-GARCH(1,1); and ARMA(0,1)-GARCH(1,1). The prediction results of the period ahead of the fifth return the item, along with the mean and variance estimator returns of each asset

liabilities, is used to calculate the mean and variance of return surplus.

Based on the mean and variance of return this surplus carried portfolio optimization, in order to determine the composition of the optimum weights for some value of risk tolerance. From the optimization result obtained that the optimum portfolio occurs at the value of risk tolerance of  $\tau = 0,023$ , with vector weight composition  $\mathbf{w}^{*T} = (0.1975, 0.3844, 0.1861, 0.0364)$ . This optimum portfolio provides a mean estimate of portfolio surplus return of 0.018982 with a risk value (variance) of 0.042264; and the ratio between the average of portfolio surplus returns to its variance, of 0.449129; is the largest compared to other ratios.

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