

# Binary semi continuous functions

S.Nithyanantha Jothi

Department of Mathematics, Aditanar College, Tiruchendur – 628216, India.

**Abstract:** The authors [4] introduced the concept of binary topology between two sets and investigate its basic properties where a binary topology from  $X$  to  $Y$  is a binary structure satisfying certain axioms that are analogous to the axioms of topology. In this paper we introduce and study various types of binary continuous functions and binary semi continuous functions in binary topological spaces.

**Keywords:** Binary topology, binary semi continuous function, totally binary continuous function, totally binary semi continuous function, strongly binary continuous function, and strongly binary semi continuous function.

## 1. Introduction

Levine[3] introduced semi open and semi continuous functions in topological spaces. The authors [4] introduced the concept of binary topology and discussed some of its basic properties. The purpose of this paper is to introduce various types of binary semi continuous functions and study their relationship. Section 2 deals with basic concepts. Binary semi continuous functions in binary topological spaces are discussed in section 3. Throughout the paper,  $\wp(X)$  denotes the power set of  $X$ .

## 2. Preliminaries

Let  $X$  and  $Y$  be any two nonempty sets. A binary topology [4] from  $X$  to  $Y$  is a binary structure  $\tilde{\mathcal{M}} \subseteq \wp(X) \times \wp(Y)$  that satisfies the axioms namely (i)  $(\emptyset, \emptyset)$  and  $(X, Y) \in \tilde{\mathcal{M}}$ , (ii)  $(A_1 \cap A_2, B_1 \cap B_2) \in \tilde{\mathcal{M}}$  whenever  $(A_1, B_1) \in \tilde{\mathcal{M}}$  and  $(A_2, B_2) \in \tilde{\mathcal{M}}$ , and (iii) If  $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$  is a family of members of  $\tilde{\mathcal{M}}$ , then  $\left( \bigcup_{\alpha \in \Delta} A_\alpha, \bigcup_{\alpha \in \Delta} B_\alpha \right) \in \tilde{\mathcal{M}}$ . If  $\tilde{\mathcal{M}}$  is a binary topology from  $X$  to  $Y$  then the triplet  $(X, Y, \tilde{\mathcal{M}})$  is called a binary topological space and the members of  $\tilde{\mathcal{M}}$  are called the binary open subsets of the binary topological space  $(X, Y, \tilde{\mathcal{M}})$ . The elements of  $X \times Y$  are called the binary points of the binary topological space  $(X, Y, \tilde{\mathcal{M}})$ . If  $Y=X$  then  $\tilde{\mathcal{M}}$  is called a binary topology on  $X$  in which case we write  $(X, \tilde{\mathcal{M}})$  as a binary topological space. The examples of binary topological spaces are given in [4].

**Definition 2.1.**[4] Let  $X$  and  $Y$  be any two nonempty sets and let  $(A, B)$  and  $(C, D) \in \wp(X) \times \wp(Y)$ . We say that  $(A, B) \subseteq (C, D)$  if  $A \subseteq C$  and  $B \subseteq D$ .

**Definition 2.2.**[4] Let  $(X, Y, \tilde{\mathcal{M}})$  be a binary topological space and  $A \subseteq X$ ,  $B \subseteq Y$ . Then  $(A, B)$  is called binary closed in  $(X, Y, \tilde{\mathcal{M}})$  if  $(X \setminus A, Y \setminus B) \in \tilde{\mathcal{M}}$ .

**Definition 2.3.** Let  $X$  and  $Y$  be any two nonempty sets and let  $(A, B)$  and  $(C, D) \in \wp(X) \times \wp(Y)$ . We say that  $(A, B) \not\subseteq (C, D)$  if one of the following holds :

(i)  $A \subseteq C$  and  $B \not\subseteq D$  (ii)  $A \not\subseteq C$  and  $B \subseteq D$  (iii)  $A \not\subseteq C$  and  $B \not\subseteq D$ .

**Definition 2.4.**[4] Let  $(X, Y, \tilde{\mathcal{M}})$  be a binary topological space and let  $(Z, \tau)$  be a topological space. Let  $f: Z \rightarrow X \times Y$  be a function. Then  $f$  is called binary continuous if  $f^{-1}(A, B)$  is open in  $Z$  for every binary open set  $(A, B)$  in  $(X, Y, \tilde{\mathcal{M}})$ .

**Definition 2.5.**[3] A subset  $A$  of a topological space  $X$  is said to be semi open if there exists an open set  $U$  such that  $U \subseteq A \subseteq ClU$  or equivalently  $A \subseteq Cl(IntA)$ .

**Definition 2.6.**[1] The complement of a semi open set is called semi closed.

**Definition 2.7.[3]** Let  $f: X \rightarrow X^*$  be a function where  $X$  and  $X^*$  are topological spaces. Then  $f$  is said to be semi continuous if  $f^{-1}(U)$  is semi open in  $X$  for every open set  $U$  in  $X^*$ .

**Definition 2.8.** A subset  $A$  of a topological space  $X$  is said to be clopen if is both open and closed.

In this paper, we define varies types of functions in binary topological spaces by using semi open and semi closed sets in topological spaces. Further we establish the relationship between these functions.

### 3 . Binary semi continuous functions

In this section, we introduce binary semi continuous function and study its basic properties. Now we start with the definition of binary semi continuous function.

**Definition 3.1.** Let  $(Z, \tau)$  be a topological space and  $(X, Y, \mathcal{M})$  be a binary topological space. Then the map  $f: Z \rightarrow X \times Y$  is called binary semi continuous if  $f^{-1}(A, B)$  is semi open in  $Z$  for every binary open set  $(A, B)$  in  $(X, Y, \mathcal{M})$ .

**Example 3.2.** Consider  $Z = \{a, b, c\}$ ,  $X = \{x_1, x_2\}$  and  $Y = \{y_1, y_2\}$ . Let  $\tau = \{\emptyset, Z, \{b, c\}\}$  and  $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\})\}$ . Clearly  $\tau$  is a topology on  $Z$  and  $\mathcal{M}$  is a binary topology from  $X$  to  $Y$ . Define  $f: Z \rightarrow X \times Y$  by  $f(a) = (x_1, y_1)$  and  $f(b) = (x_2, y_2) = f(c)$ . The closed sets in  $Z$  are  $\emptyset, Z, \{a\}$ . The semi open sets in  $Z$  are  $\emptyset, Z, \{b, c\}$ . We shall find the inverse image of every binary open sets in  $(X, Y, \mathcal{M})$ . Now,  $f^{-1}(\emptyset, \emptyset) = \{z \in Z: f(z) \in (\emptyset, \emptyset)\} = \emptyset$ ,  $f^{-1}(X, Y) = \{a, b, c\} = Z$  and  $f^{-1}(\{x_1\}, \{y_2\}) = \emptyset$ . This shows that the inverse image of every binary open sets in  $(X, Y, \mathcal{M})$  is semi open in  $Z$ . Hence  $f$  is binary semi continuous.

**Example 3.3.** Consider  $Z = \{a, b, c\}$ ,  $X = \{x_1, x_2\}$  and  $Y = \{y_1, y_2\}$ . Let  $\tau = \{\emptyset, Z, \{b\}, \{a, b\}\}$  and  $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_1\})\}$ . Clearly  $\tau$  is a topology on  $Z$  and  $\mathcal{M}$  is a binary topology from  $X$  to  $Y$ . The closed sets in  $Z$  are  $\emptyset, Z, \{c\}, \{a, c\}$ . The semi open sets in  $Z$  are  $\emptyset, Z, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}$ . Define  $f: Z \rightarrow X \times Y$  by  $f(a) = (x_1, y_1)$  and  $f(b) = (x_2, y_2) = f(c)$ . We shall find the inverse image of every binary open sets in  $(X, Y, \mathcal{M})$ . Now,  $f^{-1}(\emptyset, \emptyset) = \{z \in Z: f(z) \in (\emptyset, \emptyset)\} = \emptyset$ ,  $f^{-1}(X, Y) = \{a, b, c\} = Z$  and  $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$ . Thus  $f$  is not binary continuous, since  $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$  which is not open in  $Z$ . Also  $f$  is not binary semi continuous.

The proof of the following Proposition is straight forward.

**Proposition 3.4.** Every binary continuous function is binary semi continuous.

**Proof.** Let  $(A, B)$  be a binary open set in  $(X, Y, \mathcal{M})$ . Since  $f$  is binary continuous, we have  $f^{-1}(A, B)$  is open in  $Z$ . We know that every open set is semi open. Hence  $f^{-1}(A, B)$  is semi open in  $Z$ . Thus  $f$  is binary semi continuous.

The converse of Proposition 3.4 need not be true which is shown in the following example.

**Example 3.5.** Consider  $Z = \{a, b, c\}$ ,  $X = \{x_1, x_2\}$  and  $Y = \{y_1, y_2\}$ . Let  $\tau = \{\emptyset, Z, \{a\}, \{b\}, \{a, b\}\}$  and  $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\})\}$ . Clearly  $\tau$  is a topology on  $Z$  and  $\mathcal{M}$  is a binary topology from  $X$  to  $Y$ . The closed sets in  $Z$  are  $\emptyset, Z, \{c\}, \{a, c\}, \{b, c\}$ . The semi open sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}$ . Define  $f: Z \rightarrow X \times Y$  by  $f(a) = (x_1, y_1)$  and  $f(b) = (x_1, y_2) = f(c)$ . Clearly  $f$  is binary semi continuous but not binary continuous. For,  $f^{-1}(\emptyset, \emptyset) = \{z \in Z: f(z) \in (\emptyset, \emptyset)\} = \emptyset$ ,  $f^{-1}(X, Y) = \{a, b, c\} = Z$  and  $f^{-1}(\{x_1\}, \{y_2\}) = \{b, c\}$  which is semi open in  $Z$  but not open set  $Z$ .

**Definition 3.6.** Let  $(Z, \tau)$  be a topological space and  $(X, Y, \mathcal{M})$  be a binary topological space. Then the map  $f: Z \rightarrow X \times Y$  is called totally binary continuous if  $f^{-1}(A, B)$  is clopen in  $Z$  for every binary open set  $(A, B)$  in  $(X, Y, \mathcal{M})$ .

**Definition 3.7.** Let  $(Z, \tau)$  be a topological space and  $(X, Y, \mathcal{M})$  be a binary topological space. Then the map  $f: Z \rightarrow X \times Y$  is called totally binary semi continuous if  $f^{-1}(A, B)$  is semi clopen in  $Z$  for every binary open set  $(A, B)$  in  $(X, Y, \mathcal{M})$ .

It is evident that every totally binary continuous function is totally binary semi continuous. But the converse need not be true as can be seen from the following example.

**Example 3.8.** Consider  $Z = \{a, b, c\}$ ,  $X = \{x_1, x_2\}$  and  $Y = \{y_1, y_2\}$ . Let  $\tau = \{\emptyset, Z, \{a\}, \{b\}, \{a, b\}\}$  and  $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\})\}$ . Clearly  $\tau$  is a topology on  $Z$  and  $\mathcal{M}$  is a binary topology from  $X$  to  $Y$ . The closed sets in  $Z$  are  $\emptyset, Z, \{c\}, \{a, c\}, \{b, c\}$ . Hence the clopen sets in  $Z$  are  $\emptyset, Z$ . Now the semi open sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}$ . Hence the semi closed sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{c\}, \{a, c\}$  and  $\{b, c\}$ . Thus the semi clopen sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{a, c\}, \{b, c\}$ . Define  $f: Z \rightarrow X \times Y$  by  $f(a) = (x_1, y_1)$  and  $f(b) = (x_1, y_2) = f(c)$ . For,  $f^{-1}(\emptyset, \emptyset) = \{z \in Z: f(z) \in (\emptyset, \emptyset)\} = \emptyset$ ,  $f^{-1}(X, Y) = \{a, b, c\} = Z$  and  $f^{-1}(\{x_1\}, \{y_2\}) = \{b, c\}$  which is semi clopen in  $Z$  and hence,  $f$  is totally binary semi continuous, but  $f$  is not totally binary continuous, since  $\{b, c\}$  is not clopen in  $Z$ .

**Definition 3.9.** Let  $(Z, \tau)$  be a topological space and  $(X, Y, \mathcal{M})$  be a binary topological space. Then the map  $f: Z \rightarrow X \times Y$  is called strongly binary continuous if  $f^{-1}(A, B)$  is clopen in  $Z$  for every binary set  $(A, B)$  in  $(X, Y, \mathcal{M})$ .

**Definition 3.10.** Let  $(Z, \tau)$  be a topological space and  $(X, Y, \mathcal{M})$  be a binary topological space. Then the map  $f: Z \rightarrow X \times Y$  is called strongly binary semi continuous if  $f^{-1}(A, B)$  is semi clopen in  $Z$  for every binary set  $(A, B)$  in  $(X, Y, \mathcal{M})$ .

**Example 3.11.** Consider  $Z = \{a, b, c\}$ ,  $X = \{x_1, x_2\}$  and  $Y = \{y_1, y_2\}$ . Let  $\tau = \{\emptyset, Z, \{a\}, \{b, c\}\}$  and  $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\})\}$ . Clearly  $\tau$  is a topology on  $Z$  and  $\mathcal{M}$  is a binary topology from  $X$  to  $Y$ . Define  $f: Z \rightarrow X \times Y$  by  $f(a) = (x_1, y_1)$  and  $f(b) = (x_2, y_2) = f(c)$ . The closed sets in  $Z$  are  $\emptyset, Z, \{b, c\}, \{a\}$ . Hence the clopen sets in  $Z$  are  $\emptyset, Z, \{b, c\}, \{a\}$ . We shall find the inverse image of every binary sets in  $(X, Y, \mathcal{M})$ . Now,  $f^{-1}(\emptyset, \emptyset) = \{z \in Z: f(z) \in (\emptyset, \emptyset)\} = \emptyset$ ,  $f^{-1}(\emptyset, \{y_1\}) = \emptyset$ ,  $f^{-1}(\emptyset, \{y_2\}) = \emptyset$ ,  $f^{-1}(\emptyset, Y) = \emptyset$ ,  $f^{-1}(\{x_1\}, \emptyset) = \emptyset$ ,  $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$ ,  $f^{-1}(\{x_1\}, \{y_2\}) = \emptyset$ ,  $f^{-1}(\{x_1\}, Y) = \{a\}$ ,  $f^{-1}(\{x_2\}, \emptyset) = \emptyset$ ,  $f^{-1}(\{x_2\}, \{y_1\}) = \emptyset$ ,  $f^{-1}(\{x_2\}, \{y_2\}) = \{b, c\}$ ,  $f^{-1}(\{x_2\}, Y) = \{b, c\}$ ,  $f^{-1}(X, \emptyset) = \emptyset$ ,  $f^{-1}(X, \{y_1\}) = \{a\}$ ,  $f^{-1}(X, \{y_2\}) = \{b, c\}$ ,  $f^{-1}(X, Y) = Z$ . This gives that inverse image of every binary sets in  $(X, Y, \mathcal{M})$  is clopen in  $Z$ . Hence  $f$  is strongly binary continuous.

We observe that every strongly binary continuous function is strongly binary semi continuous. But the converse need not be true as can be seen from the following example.

**Example 3.12** Consider  $Z = \{a, b, c\}$ ,  $X = \{x_1, x_2\}$  and  $Y = \{y_1, y_2\}$ . Let  $\tau = \{\emptyset, Z, \{a\}, \{b\}, \{a, b\}\}$  and  $\mathcal{M} = \{(\emptyset, \emptyset), (X, Y), (\{x_1\}, \{y_2\})\}$ . Clearly  $\tau$  is a topology on  $Z$  and  $\mathcal{M}$  is a binary topology from  $X$  to  $Y$ . The closed sets in  $Z$  are  $\emptyset, Z, \{c\}, \{a, c\}, \{b, c\}$ . Hence the clopen sets in  $Z$  are  $\emptyset, Z$ . Now the semi open sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}$ . Hence the semi closed sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{c\}, \{a, c\}$  and  $\{b, c\}$ . Thus the semi clopen sets in  $Z$  are  $\emptyset, Z, \{a\}, \{b\}, \{a, c\}, \{b, c\}$ . Define  $f: Z \rightarrow X \times Y$  by  $f(a) = (x_1, y_1)$  and  $f(b) = (x_1, y_2) = f(c)$ . For,  $f^{-1}(\emptyset, \emptyset) = \{z \in Z: f(z) \in (\emptyset, \emptyset)\} = \emptyset$ ,  $f^{-1}(\emptyset, \{y_1\}) = \emptyset$ ,  $f^{-1}(\emptyset, \{y_2\}) = \emptyset$ ,  $f^{-1}(\emptyset, Y) = \emptyset$ ,  $f^{-1}(\{x_1\}, \emptyset) = \emptyset$ ,  $f^{-1}(\{x_1\}, \{y_1\}) = \{a\}$ ,  $f^{-1}(\{x_1\}, \{y_2\}) = \{b, c\}$ ,  $f^{-1}(\{x_1\}, Y) = Z$ ,  $f^{-1}(\{x_2\}, \emptyset) = \emptyset$ ,  $f^{-1}(\{x_2\}, \{y_1\}) = \emptyset$ ,  $f^{-1}(\{x_2\}, \{y_2\}) = \emptyset$ ,  $f^{-1}(\{x_2\}, Y) = \emptyset$ ,  $f^{-1}(X, \emptyset) = \emptyset$ ,  $f^{-1}(X, \{y_1\}) = \{a\}$ ,  $f^{-1}(X, \{y_2\}) = \{b, c\}$ ,  $f^{-1}(X, Y) = Z$ . This gives that inverse image of every binary sets in  $(X, Y, \mathcal{M})$  is semi clopen in  $Z$ . Hence  $f$  is strongly binary semi continuous, but  $f$  is not strongly binary continuous, since  $\{b, c\}$  is not clopen in  $Z$ .

$(X, \emptyset) = \emptyset$ ,  $f^{-1}(X, \{y_1\}) = \{a\}$ ,  $f^{-1}(X, \{y_2\}) = \{b, c\}$ ,  $f^{-1}(X, Y) = Z$ . This gives that inverse image of every binary sets in  $(X, Y, \mathcal{M})$  is semi clopen in  $Z$ . Hence  $f$  is strongly binary semi continuous. But  $f$  is not strongly binary continuous, since  $\{b, c\}$  is not clopen in  $Z$ .

From the above discussion we have the following,

strongly binary continuous  $\Rightarrow$  totally binary continuous  $\Rightarrow$  binary continuous.  
 $\Downarrow$   $\Downarrow$   $\Downarrow$

strongly binary semi continuous  $\Rightarrow$  totally binary semi continuous  $\Rightarrow$  binary semi continuous.

### **Conclusion**

Binary semi continuous function is introduced and its basic properties are discussed. Also totally continuous functions, totally semi continuous functions, strongly continuous functions, strongly semi continuous functions in topological spaces are extended to binary topological spaces. Further the relations between these functions are discussed.

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