

A STUDY ON THREE DIMENSIONAL QUASI-SASAKIAN MANIFOLD

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Abstract

In the present paper we have studied quasi conformally flat 3dimensional quasi-Sasakian manifold and 3-dimensional quasi-sasakian manifold with $\tilde{C}.S=0$ and 3-dimensional irrotational Quasi-sasakian . we also have studied that a pseudo projective ϕ recurrent 3 dimensional quasi sasakian manifold is η -Einstein if $a+b\neq 0$.

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1 Preliminaries

Let M be an almost contact metric manifold of dimension $(2n+1)$ with an almost contact metric structure (ϕ,ξ,η,g) [1] where ϕ,ξ,η are tensor fields of type $(1,1),(1,0),(0,1)$ respectively and g is a Riemannian metric on M such that

$$\begin{aligned}\phi^2 &= -I + \eta \otimes \xi, \eta(\xi) = 1, \phi\xi = 0, \eta\phi = 0, \eta(X) = g(X, \xi), \\ g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y)\end{aligned}\tag{1}$$

$\forall X,Y \in \chi(M)$, where $\chi(M)$ is the Lie algebra of the vector fields of the manifold M.

Let Φ be the second fundamental 2-form of M defined by

$$\Phi(X, Y) = g(X, \phi Y)\tag{2}$$

Then $\Phi(X, \xi) = 0$.

An almost contact metric manifold M of dimension 3 is Quasi-Sasakian if and only if[11]

$$\nabla_X \xi = -\beta \phi X\tag{3}$$

for a certain function β on M such that $\xi\beta=0$, ∇ being the operator of covariant differentiation with respect to the Levi-Civita connection of M. As a consequence of (3),we have [11]

$$(\nabla_X \phi)Y = \beta(g(X, Y)\xi - \eta(Y)X)\tag{4}$$

Because of (4) and (3), we find

$$\nabla_X(\nabla_Y\xi)=-(X\beta)\phi Y-\beta^2\{g(X,Y)\xi-\eta(Y)X\}-\beta\phi\nabla_XY \quad (5)$$

which implies that

$$R(X,Y)\xi=-(X\beta)\phi Y+(Y\beta)\phi X+\beta^2\{\eta(Y)X-\eta(X)Y\} \quad (6)$$

Thus we get from(6)

$$R(X,Y,Z,\xi)=(X\beta)g(\phi Y,Z)-(Y\beta)g(\phi X,Z)-\beta^2\{\eta(Y)g(X,Z)-\eta(X)g(Y,Z)\} \quad (7)$$

where $R(X,Y,Z,W)=g(R(X,Y)Z,W)$

Putting $X=\xi$ in (7) and using (1) we obtain

$$R(\xi,Y,Z,\xi)=\beta^2\{g(Y,Z)-\eta(Y)\eta(Z)\}+g(\phi Y,Z)\xi\beta \quad (8)$$

Interchanging Y and Z of (8) yields

$$R(\xi,Z,Y,\xi)=\beta^2\{g(Y,Z)-\eta(Y)\eta(Z)\}+g(\phi Z,Y)\xi\beta \quad (9)$$

Since $R(\xi,Y,Z,\xi)=R(Z,\xi,\xi,Y)=R(\xi,Z,Y,\xi)$, from (8) and (9) we have

$$\{g(\phi Y,Z)-g(\phi Z,Y)\}\xi\beta=0$$

Therefore , we can easily verify that $\xi\beta=0$.

In a 3-dimensional Riemannian manifold, we always have

$$R(X,Y)Z=g(Y,Z)QX-g(X,Z)QY+S(Y,Z)X-S(X,Z)Y-\frac{r}{2}[g(Y,Z)X-g(X,Z)Y] \quad (10)$$

where Q is the Ricci operator that is $g(QX,Y)=S(X,Y)$, S being the Ricci tensor and r is the scalar curvature of the manifold.

Throughout this chapter we consider β as a constant.Let M be a 3-dimensional quasi-Sasakian manifold .Since β is constant the Ricci tensor S of M is given by[11]

$$S(X,Y)=(\frac{r}{2}-\beta^2)g(X,Y)+(3\beta^2-\frac{r}{2})\eta(X)\eta(Y) \quad (11)$$

From(11),we get

$$QX=(\frac{r}{2}-\beta^2)X+(3\beta^2-\frac{r}{2})\eta(X)\xi \quad (12)$$

$$S(X,\xi)=2\beta^2\eta(X), Q\xi=2\beta^2\xi \quad (13)$$

As a consequence of (10),(13) becomes

$$R(X,Y)\xi=\beta^2(\eta(Y)X-\eta(X)Y) \quad (14)$$

Also from(11),

$$S(\phi X,\phi Y)=S(X,Y)-2\beta^2\eta(X)\eta(Y) \quad (15)$$

and

$$(\nabla_X\eta)Y=g(\nabla_X\xi,Y)=-\beta g(\phi X,Y) \quad (16)$$

$$R(\xi,X)\xi=\beta^2(\eta(X)\xi-X) \quad (17)$$

2 Quasi Conformally flat 3-dimensional Quasi Sasakian manifold

The quasi-Conformal curvature tensor in a 3-dimensional quasi-Sasakian manifold is given by[7]

$$\begin{aligned}\tilde{C}(X, Y)Z = & aR(X, Y)Z - b[S(X, Z)Y - S(Y, Z)X + g(X, Z)QY - g(Y, Z)QX] \\ & - \frac{r}{3}(\frac{a}{2} + 2b)[g(Y, Z)X - g(X, Z)Y].\end{aligned}\quad (18)$$

where $X, Y \in \chi(M)$ and a, b are constants and r is the scalar curvature of the manifold M .

Let a 3-dimensional quasi Sasakain manifold be quasi Conformally flat. Then from (18) ,[7],

$$\begin{aligned}'R(X, Y, Z, W) = & \frac{b}{a}[S(X, Z)g(Y, W) - S(Y, Z)g(X, W) + S(Y, W)g(X, Z) - S(X, W)g(Y, Z)] \\ & + \frac{r}{3a}(\frac{a}{2} + 2b)[g(Y, Z)g(X, W) - g(X, Z)g(Y, W)].\end{aligned}\quad (19)$$

where $'R(X, Y, Z, W) = g(R(X, Y)Z, W)$.

Putting $Z = \xi$

$$\begin{aligned}g(R(X, Y)\xi, W) = & \frac{b}{a}[S(X, \xi)g(Y, W) - S(Y, \xi)g(X, W) + S(Y, W)g(X, \xi) - S(X, W)g(Y, \xi)] \\ & + \frac{r}{3a}(\frac{a}{2} + 2b)[g(Y, \xi)g(X, W) - g(X, \xi)g(Y, W)].\end{aligned}\quad (20)$$

using (1) and (13) we have

$$\begin{aligned}g(R(X, Y)\xi, W) = & \frac{b}{a}[2\beta^2\eta(X)g(Y, W) - 2\beta^2\eta(Y)g(X, W) + S(Y, W)\eta(X) - S(X, W)\eta(Y)] \\ & + \frac{r}{3a}(\frac{a}{2} + 2b)[\eta(Y)g(X, W) - \eta(X)g(Y, W)].\end{aligned}\quad (21)$$

Taking $X = \xi$ in above and using (1) and (13) we get

$$\begin{aligned}g(R(\xi, Y)\xi, W) = & \frac{b}{a}[2\beta^2g(Y, W) - 2\beta^2\eta(Y)\eta(W) + S(Y, W) - 2\beta^2\eta(W)\eta(Y)] \\ & + \frac{r}{3a}(\frac{a}{2} + 2b)[\eta(Y)\eta(W) - g(Y, W)].\end{aligned}\quad (22)$$

Using (17) we find

$$S(Y, W) = \{-\frac{a}{b}\beta^2 - 2\beta^2 + \frac{r}{3a}(\frac{a}{2} + 2b)\}g(Y, W) + \{\frac{a}{b}\beta^2 + 4\beta^2 - \frac{r}{3a}(\frac{a}{2} + 2b)\}\eta(Y)\eta(W).\quad (23)$$

Thus we can state

Theorem 2.1. A quasi Conformally flat 3-dimensional quasi-Sasakian manifold is η - Einstein, provided r is constant.

3 Quasi Sasakian manifold satisfying $\tilde{C} \cdot S = 0$

Let $\tilde{C} \cdot S = 0$. Then

$$S(\tilde{C}(X, Y)Z, W) + S(Z, \tilde{C}(X, Y)W) = 0 \quad (24)$$

Putting $X=W=\xi$ in (24) we have

$$S(\tilde{C}(\xi, Y)Z, \xi) + S(Z, \tilde{C}(\xi, Y)\xi) = 0 \quad (25)$$

Putting $X=\xi$ in (18) we get

$$\begin{aligned} \tilde{C}(\xi, Y)Z &= aR(\xi, Y)Z + b\{S(Y, Z)\xi - S(\xi, Z)Y + g(Y, Z)Q\xi - g(\xi, Z)QY\} \\ &\quad - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\{g(Y, Z)\xi - g(\xi, Z)Y\}. \end{aligned} \quad (26)$$

Using (12) and (13) we get

$$\begin{aligned} \tilde{C}(\xi, Y)Z &= \left\{a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\right\}\{g(Y, Z)\xi - \eta(Z)Y\} + b\{S(Y, Z)\xi \\ &\quad - (\beta^2 + \frac{r}{2})\eta(Z)Y + 2\beta^2g(Y, Z)\xi + (\frac{r}{2} - 3\beta^2)\eta(Z)\eta(Y)\xi\}. \end{aligned} \quad (27)$$

$$\begin{aligned} S(\tilde{C}(\xi, Y)Z, \xi) &= \left\{a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\right\}\{g(Y, Z)S(\xi, \xi) - \eta(Z)S(Y, \xi)\} + b\{S(Y, Z)S(\xi, \xi) \\ &\quad - (\beta^2 + \frac{r}{2})\eta(Z)S(Y, \xi) + 2\beta^2g(Y, Z)S(\xi, \xi) + (\frac{r}{2} - 3\beta^2)\eta(Z)\eta(Y)S(\xi, \xi)\}. \end{aligned} \quad (28)$$

$$\begin{aligned} S(\tilde{C}(\xi, Y)Z, \xi) &= \left\{a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\right\}\{g(Y, Z).2\beta^2 - \eta(Z).2\beta^2\eta(Y)\} + b\{S(Y, Z).2\beta^2 \\ &\quad - (\beta^2 + \frac{r}{2})\eta(Z).2\beta^2\eta(Y) + 2\beta^2g(Y, Z).2\beta^2 + (\frac{r}{2} - 3\beta^2)\eta(Z)\eta(Y).2\beta^2\}. \end{aligned} \quad (29)$$

$$\begin{aligned} S(\tilde{C}(\xi, Y)Z, \xi) &= 2\beta^2\{2b\beta^2 + a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\}g(Y, Z) \\ &\quad + 2\beta^2[b\{\left(\frac{r}{2} - 3\beta^2\right) - (\beta^2 + \frac{r}{2})\} - \{a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\}]\eta(Y)\eta(Z) \\ &\quad + 2\beta^2bS(Y, Z). \end{aligned} \quad (30)$$

Taking $Z=\xi$ in (27) we get

$$\begin{aligned} \tilde{C}(\xi, Y)\xi &= \left\{a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\right\}\{g(Y, \xi)\xi - \eta(\xi)Y\} + b\{S(Y, \xi)\xi \\ &\quad - (\beta^2 + \frac{r}{2})\eta(\xi)Y + 2\beta^2g(Y, \xi)\xi + (\frac{r}{2} - 3\beta^2)\eta(\xi)\eta(Y)\xi\}. \end{aligned} \quad (31)$$

$$\begin{aligned}\tilde{C}(\xi, Y)\xi &= \{a\beta^2 - \frac{r}{3}(\frac{a}{2} + 2b)\}\{\eta(Y)\xi - Y\} + b\{2\beta^2\eta(Y)\xi \\ &\quad - (\beta^2 + \frac{r}{2})Y + 2\beta^2\eta(Y)\xi + (\frac{r}{2} - 3\beta^2)\eta(Y)\xi\}.\end{aligned}\tag{32}$$

$$\tilde{C}(\xi, Y)\xi = \{a\beta^2 - \frac{r}{3}(\frac{a}{2} + 2b) + b(\beta^2 + \frac{r}{2})\}\{\eta(Y)\xi - Y\}\tag{33}$$

Thus

$$S(\tilde{C}(\xi, Y)\xi, Z) = \{a\beta^2 - \frac{r}{3}(\frac{a}{2} + 2b) + b(\beta^2 + \frac{r}{2})\}\{\eta(Y)S(\xi, Z) - S(Y, Z)\}\tag{34}$$

$$S(\tilde{C}(\xi, Y)\xi, Z) = \{a\beta^2 - \frac{r}{3}(\frac{a}{2} + 2b) + b(\beta^2 + \frac{r}{2})\}\{\eta(Y).2\beta^2\eta(Z) - S(Y, Z)\}\tag{35}$$

Using (35)and(30)in (24) we get

$$\{a\beta^2 - b(\frac{r}{2} - \beta^2) - \frac{r}{3}(\frac{a}{2} + 2b)\}S(Y, Z) = 2\beta^2\{2b\beta^2 + a\beta^2 - \frac{r}{3}(\frac{a}{2} + 2b)\}g(Y, Z) + 2b\beta^2(\frac{r}{2} - 3\beta^2)\eta(Y)\eta(Z)\tag{36}$$

Thus we can state

Theorem 3.1. A 3-dimensional quasi Sasakian manifold with $\tilde{C}.S=0$ is η -Einstein manifold, provided $r \neq \frac{6\beta^2(a+b)}{(a+15b)}$

4 Irrational three dimensional Quasi Sasakian manifold

The quasi-Conformal curvature tensor in a 3-dimensional quasi-Sasakian manifold is given by[7]

$$\begin{aligned}\tilde{C}(X, Y)Z &= aR(X, Y)Z - b[S(X, Z)Y - S(Y, Z)X + g(X, Z)QY - g(Y, Z)QX] \\ &\quad - \frac{r}{3}(\frac{a}{2} + 2b)[g(Y, Z)X - g(X, Z)Y].\end{aligned}\tag{37}$$

where $X, Y \in \chi(M)$ and a, b are constants and r is the scalar curvature of the manifold M .

The rotation (curl) of quasi-Conformal curvature tensor \tilde{C} on a Riemannian manifold is given by [6]

$$Rot\tilde{C} = (\nabla_U\tilde{C})(X, Y, Z) + (\nabla_X\tilde{C})(Y, U, Z) + (\nabla_Y\tilde{C})(U, X, Z) - (\nabla_Z\tilde{C})(X, Y, U)\tag{38}$$

By virtue of second Bianchi identity, we have

$$(\nabla_U\tilde{C})(X, Y)Z + (\nabla_X\tilde{C})(Y, U)Z + (\nabla_Y\tilde{C})(U, X)Z = 0\tag{39}$$

and (38) reduces to

$$\operatorname{curl} \tilde{C} = -(\nabla_Z \tilde{C})(X, Y)U \quad (40)$$

The quasi Conformal curvature tensor is said to be irrotational if $\operatorname{curl} \tilde{C}=0$. i.e,

$$(\nabla_Z \tilde{C})(X, Y)U = 0 \quad (41)$$

From (41), we have

$$\nabla_Z \tilde{C}(X, Y)U = \tilde{C}(\nabla_Z X, Y)U + \tilde{C}(X, \nabla_Z Y)U + \tilde{C}(X, Y)\nabla_Z U \quad (42)$$

Putting $U=\xi$, we have

$$\nabla_Z \tilde{C}(X, Y)\xi = \tilde{C}(\nabla_Z X, Y)\xi + \tilde{C}(X, \nabla_Z Y)\xi + \tilde{C}(X, Y)\nabla_Z \xi \quad (43)$$

Putting $Z=\xi$ in (37), we have

$$\begin{aligned} \tilde{C}(X, Y)\xi &= aR(X, Y)\xi - b[S(X, \xi)Y - S(Y, \xi)X + g(X, \xi)QY - g(Y, \xi)QX] \\ &\quad - \frac{r}{3}\left(\frac{a}{2} + 2b\right)[g(Y, \xi)X - g(X, \xi)Y]. \end{aligned} \quad (44)$$

Using (12), (13) and (14), we have

$$\begin{aligned} \tilde{C}(X, Y)\xi &= a\beta^2\{\eta(Y)X - \eta(X)Y\} - b\{2\beta^2\eta(X)Y - 2\beta^2\eta(Y)X + \eta(X)QY - \eta(Y)QX\} \\ &\quad - \frac{r}{3}\left(\frac{a}{2} + 2b\right)\{\eta(Y)X - \eta(X)Y\}. \end{aligned} \quad (45)$$

$$\begin{aligned} \tilde{C}(X, Y)\xi &= [a\beta^2 - \frac{r}{3}\left(\frac{a}{2} + 2b\right)]\{\eta(Y)X - \eta(X)Y\} + 2b\beta^2\{\eta(Y)X - \eta(X)Y\} - b\left\{\left(\frac{r}{2} - \beta^2\right)\eta(X)Y \right. \\ &\quad \left. + (3\beta^2 - \frac{r}{2})\eta(X)\eta(Y)\xi - \left(\frac{r}{2} - \beta^2\right)\eta(Y)X - (3\beta^2 - \frac{r}{2})\eta(X)\eta(Y)\xi\right\}. \end{aligned} \quad (46)$$

$$\tilde{C}(X, Y)\xi = (a+b)(\beta^2 - \frac{r}{6})\{\eta(Y)X - \eta(X)Y\} = d\{\eta(Y)X - \eta(X)Y\}$$

where $d=(a+b)(\beta^2 - \frac{r}{6})$

Using (3)from(43) we get,

$$\begin{aligned} \nabla_Z d\{\eta(Y)X - \eta(X)Y\} &= d\{\eta(Y)\nabla_Z X - \eta(\nabla_Z X)Y\} \\ &\quad + d\{\eta(\nabla_Z Y)X - \eta(X)\nabla_Z Y\} + \tilde{C}(X, Y)(-\beta\phi Z). \end{aligned} \quad (47)$$

$$d\{\nabla_Z \eta(Y)X - \eta(\nabla_Z Y)X\} - d\{\nabla_Z \eta(X)Y - \eta(\nabla_Z X)Y\} = \tilde{C}(X, Y)(-\beta\phi Z). \quad (48)$$

$$d\{(\nabla_Z \eta)Y\}X - d\{(\nabla_Z \eta)X\}Y = \tilde{C}(X, Y)(-\beta\phi Z).$$

$$\text{or, } d\{-\beta g(\phi Z, Y)\}X + d\{\beta g(\phi Z, X)\}Y = \tilde{C}(X, Y)(-\beta\phi Z).$$

Taking $Z=\phi Z$

$$dX\{-\beta g(\phi^2 Z, Y)\} + dY\{\beta g(\phi^2 Z, X)\} = \tilde{C}(X, Y)(-\beta\phi^2 Z).$$

$$dX\{-\beta g(-Z + \eta(Z)\xi, Y)\} + dY\{\beta g(-Z + \eta(Z)\xi, X)\} = \tilde{C}(X, Y)(-\beta(-Z + \eta(Z)\xi)). \quad (49)$$

$$dg(Y, Z)X - d\eta(Y)\eta(Z)X - dg(Z, X)Y + d\eta(Z)\eta(X)Y = \tilde{C}(X, Y)Z - \tilde{C}(X, Y)\eta(Z)\xi \quad (50)$$

$$\tilde{C}(X, Y)Z = d\{g(Y, Z)X - g(X, Z)Y\} \quad (51)$$

Thus we can state

Theorem 4.1. If the quasi-Conformal curvature tensor in a 3-dimensional quasi Sasakian manifold with constant curvature be irrotational then the quasi Conformal curvature tensor satisfies (51).

Corollary 4.2. A 3-dimensional quasi-Sasakian manifold with irrotational quasi Conformal curvature tensor will be quasi conformally flat if either $a=-b$ or $r=6\beta^2$ and vice-versa.

5 Pseudo projective ϕ - recurrent curvature tensor in a 3-dimensional Quasi Sasakian manifold

The pseudo-projective curvature tensor in a 3-dimensional quasi Sasakian manifold [5] is given by

$$\bar{P}(X, Y)Z = aR(X, Y)Z + b[S(Y, Z)X - S(X, Z)Y] - \frac{r}{3}\left(\frac{a}{2} + b\right)[g(Y, Z)X - g(X, Z)Y] \quad (52)$$

where a, b are non zero constants.

A 3-dimensional quasi Sasakian manifold is said to be Pseudo projective ϕ recurrent if there exist a non zero 1-form A such that [25]

$$\phi^2((\nabla_W)\bar{P}(X, Y)Z) = A(W)\bar{P}(X, Y)Z \quad (53)$$

for $X, Y, Z, W \in \chi(M)$ and 1-form A is defined by

$$g(X, \rho) = A(X) \quad (54)$$

ρ being the vector field associated to 1-form A .

Let a 3-dimensional quasi Sasakian manifold be pseudo projective ϕ -recurrent. Then from (53), using (1), we have

$$(\nabla_W\bar{P})(X, Y)Z - \eta((\nabla_W\bar{P})(X, Y)Z)\xi = -A(W)\bar{P}(X, Y)Z \quad (55)$$

Taking inner product with respect to U , we have

$$g((\nabla_W\bar{P})(X, Y)Z, U) - \eta((\nabla_W\bar{P})(X, Y)Z)\eta(U) = -A(W)g(\bar{P}(X, Y)Z, U) \quad (56)$$

Putting $X=U=\xi$, and using(1), we get

$$-g((\nabla_W\bar{P})(\xi, Y)Z, \xi) + \eta((\nabla_W\bar{P})(\xi, Y)Z)\eta(\xi) = A(W)g(\bar{P}(\xi, Y)Z, \xi) \quad (57)$$

$$A(W)\eta(\bar{P}(\xi, Y)Z) = 0 \quad (58)$$

$$\eta(\bar{P}(\xi, Y)Z) = 0, \text{ as } A(W) \neq 0 \quad (59)$$

From (52) we have

$$\bar{P}(\xi, Y)Z = aR(\xi, Y)Z + b[S(Y, Z)\xi - S(\xi, Z)Y] - \frac{r}{3}\left(\frac{a}{2} + b\right)[g(Y, Z)\xi - g(\xi, Z)Y] \quad (60)$$

Using (12),(13),we have

$$\begin{aligned} \bar{P}(\xi, Y)Z &= a\{2\beta^2g(Y, Z)\xi - (\frac{r}{2} - \beta^2)\eta(Z)Y - (3\beta^2 - \frac{r}{2})\eta(Y)\eta(Z)\xi + S(Y, Z)\xi \\ &\quad - 2\beta^2\eta(Z)Y - \frac{r}{2}[g(Y, Z)\xi - \eta(Z)Y]\} + b[S(Y, Z)\xi - 2\beta^2\eta(Z)Y] \\ &\quad - \frac{r}{3}\left(\frac{a}{2} + 2b\right)[g(Y, Z)\xi - \eta(Z)Y]. \end{aligned} \quad (61)$$

Using (61) in (59) and we get,

$$(a+b)S(Y, Z) = \{\frac{2r}{3}(a+b) - 2a\beta^2\}g(Y, Z) + \{2\beta^2(2a+b) - \frac{2r}{3}(a+b)\}\eta(Y)\eta(Z) \quad (62)$$

$$S(Y, Z) = Cg(Y, Z) + D\eta(Y)\eta(Z) \quad (63)$$

provided $a+b \neq 0$, where $C = \frac{\frac{2r}{3}(a+b) - 2a\beta^2}{a+b}$ and $D = \frac{2(2a+b)\beta^2 - \frac{2r}{3}(a+b)}{a+b}$.

Thus we can say

Theorem 5.1. A Pseudo projective ϕ - recurrent 3 dimensional quasi Sasakian manifold with constant scalar curvature is η -Einstein if $a+b \neq 0$.

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