The Split Domination, Inverse Domination and Equitable Domination in the Middle and the Central graphs of the Path and the Cycle graphs

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Abstract

Let G=(V,E) be a simple, finite, connected and undirected graph. A non- empty subset D of V(G) in a graph G=(V,E) is a dominating set if every vertex in V-D is adjacent to atleast one vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set of G. A non-empty subset D of V(G) is called an equitable dominating set of a graph G if for every $v \in \langle V - D \rangle$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \le 1$. The minimum cardinality of such a minimal dominating set is denoted by $\gamma_e(G)$ and is called an equitable domination number of G. A dominating set D of graph G is called a split dominating set, if the induced subgraph $\langle V - D \rangle$ is disconnected. Let x denote the greatest integer not greater than xand $\begin{bmatrix} x \end{bmatrix}$ denote the least integer not less than x. In this paper, we investigated the split, inverse and equitable domination number of the middle and the central graphs of the path P_n and the cycle C_n

AMS Subject Classification: 05C, 05C72, 05C69.

Keywords: Dominating set, Domination number, Split domination, Inverse domination, equitable domination, middle graph, Central graph, Path Graph and Cycle Graph.

I. INTRODUCTION

All graphs in this paper will be simple finite and undirected. Let p=|V| and q=|E| denote the order and size of the graph G respectively.

In 1962, ore used the name "dominating set" and "domination number" for the same concept. In 1977, Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. The survey paper of Cockayne and Hedetniemi has generated a lot of interest in the study of domination in graphs.

The split domination in graphs was introduced by Kulli and Janakiram [9]. They defined

the split dominating set, the split domination number and obtained several interesting results, regarding the split domination number of some standard graphs. They have also obtained relations of split domination number with the other theoretic parameters written by Kulli and Janakiram [9 & 10].

One of the fastest growing areas in graph theory is the study of domination and related subset problems such as independence, covering matching and inverse domination. The concept of inverse domination was introduced by V. R. Kulli [11].

Swaminathan et al [13] introduced the concept of equitable domination in graph by considering the following real world problems. In a network, nodes with nearly equal capacity may interact with each other in a better way. This society persons with nearly equal status, tend to be friendly. In an industry, employees, with nearly equal powers form an association and move closely. Equitability among citizens in terms of wealth, health, status etc is the goal of a democratic nation.

For a given graph G of order n, the central graph C(G) is obtained, by subdividing each edge in E exactly once and joining all the non adjacent vertices of G. The middle graph M(G) of a graph G, is the graph whose vertex set is $V(G) \cup E(G)$ where two vertices are adjacent if and only if they are either adjacent edges of G, one is a vertex and the other is an edge incident with it. The related ideas regarding these graphs can be seen in [3].

In this paper, we introduced split domination, inverse domination and equitable domination in the middle and the central graph of C_n and P_n .

II. PRELIMINARIES

Definition 2.1 [8]

A dominating set D of a graph G is called a split dominating set, if the induced sub graph <V-D>

is disconnected. The split dominating number $\gamma_s(G)$ of G is the minimum cardinality of the minimal split dominating set of G. The minimum cardinality taken

over all the minimal split dominating sets in a graph G is called the split domination number $\gamma_s(G)$ of G. **Definition 2.2 [12]**

Let G = (V, E) be a graph with no isolated vertices. A classical observation in domination theory is that if D is the minimum dominating set of G, then V - D is also a dominating set of G. A set D' is an inverse dominating set of G if D' is a dominating set of G and $D' \subseteq V - D$. The inverse domination number of G is the minimum cardinality among all the minimal inverse dominating sets of G, denoted by $\gamma'(G)$.

Definition 2.3 [8]

A non- empty subset D of V(G) is called an equitable dominating set of a graph G if for every $v \in V - D$, there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u)-\deg(v)| \leq 1$. The minimum cardinality of an equitable dominating set of G is called equitable domination number of G and is denoted by $\gamma_e(G)$.

Definition 2.4 [3]

The middle graph M(G) is a graph which is obtained by subdividing each edge of G exactly once and join all the newly added middle vertices to the adjacent edges of G.

Definition 2.5 [3]

Let G be a simple, finite and undirected graph. Let the central graph of G be denoted by C(G) and is obtained by subdividing each edge G exactly once and joining all the non adjacent vertices of G in C(G).

III. MAIN RESULTS

The Split, Inverse and Equitable domination

number of the Middle graphs of P_n and C_n .

In this section, we obtained the split domination, inverse domination and equitable domination numbers of the middle graphs of P_n and C_n .

Example 3.1





Middle graph of P5 M(P5)

Theorem 3.2

For any Path
$$P_n$$
, $\gamma_s(M(P_n)) = \left\lceil \frac{n-1}{2} \right\rceil$.

Proof:

Let $u_1, u_2, u_3 \dots u_n$ be the vertices of path P_n and let $v_1, v_2, v_3 \dots v_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2, e_3 \dots e_{n-1}$ of P_n to obtain $M(P_n)$. Then $|V(M(P_n))| = 2n - 1$ and $|E(M(P_n))| = 3n - 1$.

By the definition of middle graph, $M(P_n)$ has vertex set $V(P_n)UE(P_n) = \{u_i \mid 1 \le i \le n+1\} U \quad \{e_i \mid 1 \le i \le n\}$

$$D = \begin{cases} \{v_i : i \equiv 2 \pmod{3} \ \bigcup \{v_{n-1}\}, if n = 3r-1, \\ r \text{ is an integer} \\ \{v_i: i \equiv 2 \pmod{3}, otherwise \end{cases}$$

where $1 \le i \le n$ with $|D| = \left\lceil \frac{n-1}{2} \right\rceil$

Since each vertex in V(M(P_n)) is adjacent to a vertex u_i . Let D be the split dominating set of M(P_n) with $D = \{ v_{2i-1} \}, 1 \le i \le n$. The vertices of D is adjacent to at least one vertex u_i of M(P_n) and the induced sub graph <V-D> is disconnected. This shows M(P_n) satisfies split domination condition. Hence the Split domination number of

$$M(P_n) = \left\lceil \frac{n-1}{2} \right\rceil.$$

Theorem: 3.3

For any Path
$$P_n$$
, $\gamma'(M(P_n)) = \left\lceil \frac{n-1}{2} \right\rceil$.

Proof

Let $u_1, u_2, u_3 \dots u_n$ be the vertices of P_n and let $v_1, v_2, v_3 \dots v_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2, e_3 \dots e_{n-1}$ of P_n to obtain M(P_n). Then $|V(M(P_n))| = 2n - 1$ and $|E(M(P_n))| = 3n - 1$. Construct the induced sub graph < V - D > of M(P_n) which is disconnected.

$$(i.e) V - D = \{ (v_{2i}, u_i), 1 \le i \le n \},$$
$$|D| = \left\lceil \frac{n-1}{2} \right\rceil.$$

Let D' be the non-empty set of V - D such that the induced subgraph $\langle V - D \rangle$ the vertex set v_{2i} is adjacent to atleast one vertex of $M(P_n)$. Let D'is the dominating set in V - D. Then $D' = \{v_{2i}/1 \le i \le n\}$ is an inverse dominating set of $M(P_n)$, we get $\gamma'(M(P_n)) = |D'|$

$$=\left\lceil \frac{n-1}{2} \right\rceil_{.}$$

Proposition 3.4

Equitable domination does not exist for the middle graphs of P_n and C_n .

Proof:

Let $D \subseteq V$ and let D be the dominating set of the middle graphs of P_n and C_n . If for every vertex $v \in \langle V - D \rangle$ there exists a vertex $u \in D$, such that $uv \in E(M(P_n))$ and $uv \in E(M(C_n))$.

Here, $\deg(u) = n-2$ (or) n-1 and $\deg(v) = 1$. (i.e) $| \deg(u) - \deg(v) | \neq 1$ or 0. This shows that $M(P_n)$ and $M(C_n)$ do not have equitable domination sets.

Theorem 3.5

For any Cycle
$$C_n$$
, $\gamma_s(M(C_n)) = \left| \frac{n}{3} \right|$

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Proof:

Let $u_1, u_2, u_3 \dots u_n$ be the vertices of C_n and let $v_1, v_2, v_3 \dots v_n$ be the added vertices corresponding to the edges $e_1, e_2, e_3 \dots e_n$ of C_n to obtain $M(C_n)$. Then $|V(M(C_n))|=2n$ and $|E(M(C_n))|=3n$. Construct the set $S = \{v_i : i \equiv 1 \mod (3)\}$ where $1 \le i \le n$ with $\left\lceil \frac{n}{3} \right\rceil$ will be a split dominating set of $M(C_n)$. The vertices of D is adjacent to at least one vertex in $M(C_n)$ and the induced subgraph < V - D > is disconnected graph. This show that the $M(C_n)$ is satisfying split domination condition.

Hence the split domination number of $M(C_n)$ is

$$\left[\frac{n}{3}\right]$$

Theorem 3.6

For any Cycle
$$C_n, \gamma'$$
 (M(C_n))= $\left\lceil \frac{n}{2} \right\rceil$

Proof:

By the definition of middle graph, $M(C_n)$ has vertex set $V(C_n) \cup E(C_n)$ in which each e_i is adjacent with e_{i+1} (i = 1, 2 ... n - 1) and e_n is adjacent with v_1 . In $M(C_n)$, $v_1, e_1, v_2, e_2 ... e_{n-1}, v_1$ induces a cycle of length 2n and $|D| = \left\lceil \frac{n}{3} \right\rceil$. Let D' be the non-

empty set of $\langle V - D \rangle$ such that the induced subgraph $\langle V - D \rangle$ of M(C_n) every vertex is adjacent to at least one vertex u_i . Since $\langle V - D \rangle$ contains a dominating set D'. Then $D' = \{v_i/v_1 \in D, i \ge 1\}$ is an inverse dominating set of C(C_n).

Thus,
$$\gamma'(\mathbf{M}(\mathbf{P}_n)) = \left| \begin{array}{c} \mathcal{D}' \\ \gamma'(\mathbf{M}(\mathbf{P}_n)) = \left[\begin{array}{c} n \\ 2 \end{array} \right].$$

IV. Split, Inverse and Equitable domination number of the Central graphs of P_n and C_n .

In this section we obtained the split, inverse and equitable domination numbers of the central graphs of P_n and C_n .

Example 4.1 Central graph of C_5



Theorem: 4.2

For any Path P_n , $\gamma_s(C(P_n)) = n - 1$ where $n \le 4$ Proof

Let P_n be any path of length n - 1 with vertices $v_1, v_2 \dots v_n$ on the process of

centralization of P_n , let u_i be the vertex of

subdivision of the edges $v_i v_{i-1} (1 \le i \le n)$.

Also let $v_i u_i = e_i$ and

$$u_i v_{i+1} = e_i' (1 \le i \le n-1)$$

In C(P_n) the vertex v_i is adjacent with all the vertices except the vertices

$$v_{i+1}$$
 and v_{i-1} for $1 \le i \le n-1$.

Construct the set

$$D = \left\{ \{v_i \} \cup \{u_i/2 \le i \le n-1\} \right\}$$

Therefore $\gamma(C(P_n)) = n-1$.

Let D will be the split dominating set of $C(P_n)$ if the induced subgraph $\langle V-D \rangle$ is disconnected graph. This shows that the $C(P_n)$ is satisfying split domination condition.

Hence, $\gamma_s(C(P_n)) = n - 1$ where $n \le 4$.

Theorem 4.3

For any Path P_n , P_n , γ_e $(C(P_n)) = n - 1$ where $n \le 4$.

Proof

It is known that $\gamma(C(P_n)) = n - 1$. Let $G = (C(P_n))$ be a graph with at least one isolated vertex. Consider $D = \{v_1, u_{i+1}\}$ and $V - D = \{v_{i+1}, u_{i+2}\}$ $1 \le i \le n$. Let D be the minimum dominating set of $C(P_n)$. Let $D \subseteq V$ if for every vertex $v \in \langle V - D \rangle$ there exist a vertex $u \in D$ such that $uv \in E(C(P_n))$.Since the degree of any vertex is n - 1 or n. Therefore $\left| deg(u) - deg(v) \right| \le 1$, and the dominating set D is the minimum equitable dominating set of G. (ie) $\gamma_{e}(C(P_n)) = |D|$. obviously the vertex must belong to any equitable dominating set. Then $\gamma_{e}(C(P_n)) = n - 1$, where $n \le 4$.

Theorem 4.4

For any Path P_n , $\gamma'(C(P_n)) = 1$ where $n \le 4$.

Proof

It is known that $\gamma(C(P_n)) = n - 1$. Let D be a minimum dominating set $D = \{v_1, u_{i+1} | i \leq 3\}$ with each $v_1, u_i \in V(C(P_n))$. Let D' be the non-empty set of V - D such that the induced subgraph $\langle V - D \rangle$ contains a dominating set D' then $D' = \{v_i | v_1 \in D, i \leq 2\}$.

Thus
$$\gamma'(C(P_n)) = |D'|$$

= 1

Therefore, $\gamma'(\mathcal{C}(P_n)) = 1$ where $n \leq 4$.

Theorem 4.5

For any Cycle $C_n, \gamma_s(C(C_n)) = n - 1$ where $n \le 4$.

Proof

We know that $\gamma(C(P_n)) = n - 1$. Let $D = \{v_i\} \cup \{u_i / 2 \le i \le n - 1\}$ will be a split dominating set if the induced subgraph $\langle V \cdot D \rangle$ is disconnected. Hence the C(C_n) graph satisfies the split domination condition giving, $\gamma_s(C(C_n)) = n - 1$ where $n \le 4$.

Theorem 4.6

For any Cycle C_n , $\gamma_e(C(C_n)) = n - 1$ where $n \le 4$.

Proof

We know that $\gamma(C(C_n)) = n - 1$. Let G=C(C_n) be a graph with at least one isolated vertex. Let $D = \{ v_1, u_2, u_3 \dots u_{i+1} \}, 1 \le i \le n$ and $V - D = \{ v_2, v_3, \dots v_{i+1}, u_{i+2} \}$.

Let D be the dominating set of $C(C_n)$. Let $D \subseteq V$ if

for every vertex $v \in \langle V - D \rangle$ there is a vertex

 $u \in D$ such that $uv \in E(C(C_n))$.

Since the degree of any vertex is n - 1. That is

 $deg(u) - deg(v) \le 1$.

Therefore D is an equitable dominating set, then

$$\gamma_{\varepsilon}(\mathcal{C}(\mathcal{C}_n)) = n - 1 \text{ where } n \leq 4.$$

Theorem 4.7

For any Cycle
$$C_n, \gamma'(C(C_n)) = \begin{cases} 2 & \text{if } n=3\\ 1 & \text{if } n=1 \end{cases}$$

Proof

It is known that $\gamma(C(C_n)) = n - 1$.

Case (i) if n = 3

Let D be the minimum dominating set of C(C_n). Let D' be the non-empty set of V - D such that the induced subgraph $\langle V - D \rangle$ contains a dominating set D'. Then $D' = \{v_i/v_1 \in D, i \ge 1\}$ is an inverse dominating set of C(C_n), Then $\gamma'(C(C_n)) = |D'|$

=2

Therefore,
$$\gamma'(C(C_n)) = 2$$
 if $n = 3$.

Case(ii) if n = 1, there exist a minimum dominating set $D = \{v_1, u_{i+1}\}, i \leq 3$ with each $v_1, u_i \epsilon V(C(P_n))$. Let D' be the non-empty set of V - D such that the induced sub graph $\langle V - D \rangle$ contains a dominating set D'. Then $D' = \{v_i\} / v_1 \epsilon D, i \leq 2$. Thus, $\gamma'(C(C_n)) = 1$ if n = 4. 5. RESULTS

| S.N | Graph | Split | Equitable | Inverse |
|-----|--------------------|--|------------------|--|
| | _ | Domination | Domination | Domination |
| | | number | number | number |
| 1. | M(P _n) | $\left\lceil \frac{n-1}{2} \right\rceil$ | Not satisfied | $\left\lceil \frac{n-1}{2} \right\rceil$ |
| 2. | M(C _n) | $\left\lceil \frac{n}{3} \right\rceil$ | Not satisfied | $\left\lceil \frac{n}{2} \right\rceil$ |
| 3. | C(P _n) | n-1 | n-1 | 1where n≤4 |
| 4. | C(C _n) | n-1 | n-1 | 2 if n=3 1 if n=4 |

VI. CONCLUSION

In this paper we have investigated split domination, inverse domination and equitable domination of the middle graphs and central graphs of cycles and paths. In this paper, we found that the equitable domination exists for $C(P_n)$ and $C(C_n)$ graphs. But the middle graphs of P_n and C_n do not satisfy the equitable domination condition.

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