

An inventory model for Gompertz deteriorating items with time-varying holding cost and price dependent demand

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Abstract

The present paper deals with a deterministic inventory model which follows the Gompertz distribution deterioration rate of items. Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. Holding cost is time dependent and demand rate is assumed as price dependent in linear form. Shortages are allowed and completely backlogged. Replenishment is instantaneous and lead time is zero. The model is solved analytically by maximizing the total profit. The results are illustrated with the numerical example and also shown graphically. The sensitivity of the solution with the changes of the values of the parameters associated with the model is discussed.

Keywords:-

Deteriorating items, Price dependent demand, shortage, Time varying holding cost.

1.Introduction

It is usually seen that the price dependent demand of the items affect the delivery of goods. Most of the customers are motivated by the attractive price of the items to buy more goods and that situation creates the greater demand of the goods. Due to this condition, retailers want to increase their order quantity and the retailers earn the more profit to increase their revenue but the conditions become more complex when items deteriorate. Therefore deterioration of items is one of the most important factors in any inventory and production system.

A large number of work has been reported for inventory with deteriorating items in recent years because most of the physical goods undergo decay or deteriorate over time. Nahimias[1] developed a perishable inventory theory by considering deterioration of items. An order level inventory system for deteriorating items was developed by Aggarwal and Goel[2]. Raffat[3] discussed an inventory model for continuously deteriorating items. Goyal and Giri[4] developed an inventory model for deteriorating

inventory. Rao et.al[5] developed a production inventory model for deteriorating items.

Inventory models create lot of interest due to their ready applicability at various places like market yards, ware houses, production processes, transportation systems, etc. Several inventory models have been developed and analysed to study various inventory systems. The most influencing factors of the inventory systems are holding, demand and replenishment. In traditional inventory systems the holding cost is considered as constant but holding cost varies with time. Naddor[6] developed an inventory system by taking time varying holding cost. Muhlemann and Valtis Spanopoulos[7] produced an inventory model by taking variable holding cost. Goh[8] and Ajanta Roy[9] developed inventory models by considering time varying holding cost.

The most important influencing factor of inventory system is demand. In classical inventory model the demand rate is usually assumed to be constant but in reality demand rate for physical goods vary with time. Selling price plays an important role in inventory system. A discount price attracts more customers to buy the product in a super market. Burwell et.al[10] developed economic lot size model for price-dependent demand under quantity and freight discounts. An inventory system of ameliorating items for price-dependent demand rate was considered by Mondal et.al[11]. You[12] developed an inventory model with price and time dependent demand. Rao et.al[13] developed an inventory model with hypo exponential lifetime having demand is function of selling price and time. Sridevi et.al[14] determined inventory model for deteriorating items with Weibull rate of replenishment and selling price dependent demand. Chaudhry and Sharma[15] developed an inventory model for deteriorating items with time dependent demand and shortages. An inventory model for deteriorating items with shortages and time varying demand were developed by Sicilia et.al[16].

In this present paper, we have developed an inventory model by taking a new type Gompertz

distribution deterioration rate of items and demand rate is a function of selling price. Holding cost is time varying. Shortages are allowed here and are completely backlogged. We solve the model to optimize the total profit. Model is illustrated with numerical examples and verified graphically. Also the sensitivity analysis is carried out with the base of numerical example.

2. Assumptions and Notations

The fundamental assumptions and notations of this model are as follows:

- (i) The deterioration of items follows the Gompertz distribution

$$\theta(t) = \theta e^{\alpha t}, \quad 0 < \theta < 1 \text{ and } \alpha > 0$$

- (ii) Demand rate is function of selling price
- (iii) Shortages are allowed and completely backlogged
- (iv) Holding cost $h(t)$ per item per unit time is time dependent and is assumed to be

$$h(t) = h + \delta t^2 \quad \text{where } h > 0, \delta > 0$$

- (v) Selling price 's' follows an increasing trend where demand rate is

$$f(s) = (a - s) > 0$$

- (vi) T is the complete length of cycle
- (vii) Replenishment is instantaneous and lead time is zero
- (viii) Q is the order quantity in one cycle
- (ix) A is the cost of placing an order
- (x) 's' selling price per unit item
- (xi) C_1 is the unit cost of an item
- (xii) C_2 is the shortage cost per unit per unit time

3. Mathematical Formulations and Solutions

During time t_1 , inventory is depleted due to deterioration and demand of item. At time t_1 the inventory becomes zero and shortages start occurring. Let $I(t)$ be the inventory level at time t ($0 \leq t \leq T$). The differential equations to describe instantaneous state over (0,T) are given by

$$\frac{dI(t)}{dt} + \theta e^{\alpha t} I(t) = -(a - s), \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -(a - s), \quad t_1 \leq t \leq T \tag{2}$$

With $I(t) = 0$ at $t = t_1$

Neglecting the higher powers of θ , the solutions of (1) and (2) are given as

$$I(t) = (a - s) \left\{ (t_1 - t) + \frac{\theta}{\alpha} e^{\alpha t} (t - t_1) + \frac{\theta}{\alpha^2} (e^{\alpha t_1} - e^{\alpha t}) \right\}, \quad 0 \leq t \leq t_1 \tag{3}$$

$$I(t) = (a - s)(t_1 - t), \quad t_1 \leq t \leq T \tag{4}$$

Now, total number of deteriorated items is given by

$$D = \int_0^{t_1} \theta e^{\alpha t} I(t) dt$$

$$D = (a - s) \left\{ \frac{\theta}{\alpha} e^{\alpha t_1} - \frac{\theta}{\alpha} (1 + t_1) \right\}, \text{ using (3)} \tag{5}$$

Ordering quantity is given by

$$Q = D + \int_0^T (a - s) dt$$

$$Q = (a - s) \left\{ \frac{\theta}{\alpha} e^{\alpha t_1} - \frac{\theta}{\alpha} (1 + t_1) + T \right\} \tag{6}$$

Holding cost is given by

$$H = \int_0^{t_1} (\beta + \delta t^2) I(t) dt$$

$$H = \beta(a - s) \left\{ \frac{t_1^2}{2} + \frac{\theta}{\alpha^2} t_1 (1 + e^{\alpha t_1}) + \frac{2\theta}{\alpha^3} (1 - e^{\alpha t_1}) \right\} + \delta(a - s) \left\{ \frac{t_1^4}{12} + \frac{\theta t_1^3}{3\alpha^2} e^{\alpha t_1} + \frac{2\theta}{\alpha^3} t_1 e^{\alpha t_1} (3 - t_1) + \frac{2\theta}{\alpha^4} t_1 + \frac{8\theta}{\alpha^5} (1 - e^{\alpha t_1}) \right\} \tag{7}$$

Total shortage cost is given by

$$S = -C_2 \int_{t_1}^T I(t) dt$$

$$S = \frac{1}{2} C_2 (a - s) (t_1 - T)^2 \tag{8}$$

Now, total profit per unit time is given by

$$P(T, t_1) = s(a-s) - \frac{1}{T}(A + C_1Q + H + S)$$

and

$$P(T, t_1) = s(a-s) - \frac{1}{T} \left[\begin{aligned} &A + C_1(a-s) \left\{ \frac{\theta}{\alpha} e^{\alpha t_1} - \frac{\theta}{\alpha} (1+t_1) + T \right\} \\ &+ (a-s) \left\{ \begin{aligned} &h \left(\frac{t_1^2}{2} + \frac{\theta}{\alpha^2} t_1 e^{\alpha t_1} + \frac{\theta}{\alpha^2} t_1 + \right. \\ &\left. \frac{2\theta}{\alpha^3} - \frac{2\theta}{\alpha^3} e^{\alpha t_1} \right) \\ &+ \delta \left(\frac{t_1^4}{12} + \frac{\theta t_1^3}{3\alpha^2} e^{\alpha t_1} - \right. \\ &\left. \frac{2\theta t_1^2}{\alpha^3} e^{\alpha t_1} + \frac{6\theta}{\alpha^3} t_1 e^{\alpha t_1} \right. \\ &\left. + \frac{2\theta}{\alpha^4} t_1 + \frac{8\theta}{\alpha^5} - \frac{8\theta}{\alpha^5} e^{\alpha t_1} \right) \\ &\left. + \frac{1}{2} C_2(a-s)(t_1 - T)^2 \right\} \end{aligned} \right] \quad (9)$$

In order to maximize the total profit function $P(T, t_1)$ the necessary conditions are

$$\frac{\partial P(T, t_1)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial P(T, t_1)}{\partial t_1} = 0 \quad \text{which gives}$$

$$\frac{\partial P(T, t_1)}{\partial T} = \frac{1}{T^2} \left[\begin{aligned} &A + C_1(a-s) \left\{ \frac{\theta}{\alpha} e^{\alpha t_1} - \frac{\theta}{\alpha} (1+t_1) + T \right\} \\ &+ (a-s) \left\{ \begin{aligned} &h \left(\frac{t_1^2}{2} + \frac{\theta}{\alpha^2} t_1 e^{\alpha t_1} + \frac{\theta}{\alpha^2} t_1 + \frac{2\theta}{\alpha^3} - \frac{2\theta}{\alpha^3} e^{\alpha t_1} \right) \\ &+ \delta \left(\frac{t_1^4}{12} + \frac{\theta t_1^3}{3\alpha^2} e^{\alpha t_1} - \frac{2\theta t_1^2}{\alpha^3} e^{\alpha t_1} \right. \\ &\left. + \frac{6\theta}{\alpha^3} t_1 e^{\alpha t_1} + \frac{2\theta}{\alpha^4} t_1 + \frac{8\theta}{\alpha^5} - \frac{8\theta}{\alpha^5} e^{\alpha t_1} \right) \\ &\left. + \frac{1}{2} C_2(a-s)(t_1 - T)^2 \right\} \end{aligned} \right] \\ -\frac{1}{T} [C_1(a-s) - C_2(a-s)(t_1 - T)] = 0 \quad (10)$$

$$\frac{\partial P(T, t_1)}{\partial t_1} = -\frac{1}{T} \left[\begin{aligned} &C_1(a-s) \left\{ e^{\alpha t_1} - \frac{\theta}{\alpha} \right\} \\ &+ (a-s) \left\{ \begin{aligned} &h \left(t_1 + \frac{\theta}{\alpha} t_1 e^{\alpha t_1} + \frac{\theta}{\alpha^2} - \frac{\theta}{\alpha^2} e^{\alpha t_1} \right) \\ &\delta \left(\frac{t_1^3}{3} + \frac{\theta t_1^3}{3\alpha} e^{\alpha t_1} - \frac{\theta t_1^2}{\alpha^2} e^{\alpha t_1} \right. \\ &\left. + \frac{6\theta}{\alpha^2} t_1 e^{\alpha t_1} + \frac{6\theta}{\alpha^3} e^{\alpha t_1} - \frac{4\theta t_1}{\alpha^3} e^{\alpha t_1} \right. \\ &\left. - \frac{8\theta}{\alpha^4} e^{\alpha t_1} + \frac{2\theta}{\alpha^4} \right) \\ &\left. + C_2(a-s)(t_1 - T) \right\} \end{aligned} \right] = 0 \quad (11)$$

The solutions of (10) and (11) will give T^* and t_1^* . The values of T^* and t_1^* , so obtained, the optimal value $P^*(T, t_1)$ of the average net profit is determined by (9) provided they satisfy the sufficient conditions for maximizing $P(T, t_1)$ are

$$\frac{\partial^2 P(T, t_1)}{\partial T^2} < 0, \quad \frac{\partial^2 P(T, t_1)}{\partial t_1^2} < 0 \quad (12)$$

and

$$\frac{\partial^2 P(T, t_1)}{\partial T^2} \frac{\partial^2 P(T, t_1)}{\partial t_1^2} - \left(\frac{\partial^2 P(T, t_1)}{\partial T \partial t_1} \right)^2 > 0 \quad \text{at } t_1 = t_1^* \text{ and } T = T^* \quad (13)$$

4. Numerical Example

Let us consider the values of parameters in appropriate units as

$$A=200, a=98, C_1=25, C_2=5, s=60, h=2, \theta = 0.1, \alpha = 1, \delta = 2$$

Based on these input data, the computer outputs are as follows:

Profit

$$P^*(T, t_1) = 1128.348, \quad T^* = 1.880, \quad t_1^* = 0.818$$

5. Sensitivity Analysis

To study the effects of changes of the parameters on the optimal profit $P^*(T, t_1)$, T^* , t_1^* derived by the proposed model, a sensitivity analysis is performed in view of the numerical example given above. Sensitivity analysis is executed by changing (decreasing or increasing) the parameters by 10%, 20% and 30% and taking one parameter at a time, keeping the remaining parameters at their original values.

The corresponding changes in $P^*(T, t_1)$, T^* and t_1^* are shown in below (Table 1).

Parameters	% Change	T^*	t_1^*	$P^*(T, t_1)$
A	-30	1.597	0.717	1162.834
	-20	1.698	0.754	1150.699
	-10	1.792	0.788	1139.239
	+10	1.963	0.847	1117.942
	+20	2.042	0.873	1107.957
	+30	2.118	0.898	1098.344
C_1	-30	1.932	0.910	1420.946
	-20	1.913	0.877	1323.223
	-10	1.896	0.847	1225.698
	+10	1.865	0.792	1031.154
	+20	1.852	0.768	934.099
	+30	1.839	0.745	837.170
C_2	-30	2.121	0.759	1148.927
	-20	2.024	0.782	1141.177
	-10	1.946	0.802	1134.377
	-10	1.824	0.833	1122.956
	+10	1.776	0.846	1118.098
	+20	1.734	0.858	1113.693
a	-30	3.673	1.3267	200.104
	-20	2.610	1.050	500.511
	-10	2.152	0.910	811.784
	+10	1.693	0.752	1448.202
	+20	1.550	0.701	1770.358
	+30	1.446	0.660	2094.239
s	-30	1.574	0.709	709.631
	-20	1.658	0.740	920.347
	-10	1.758	0.776	1059.862
	+10	2.032	0.870	1126.051
	+20	2.233	0.935	1053.339
	+30	2.513	1.022	910.822
h	-30	1.902	0.864	1132.802
	-20	1.894	0.848	1131.268
	-10	1.887	0.833	1129.784
	+10	1.873	0.804	1126.959
	+20	1.866	0.790	1125.614
	+30	1.859	0.776	1124.313
θ	-30	1.936	0.916	1136.211
	-20	1.915	0.880	1133.378
	-10	1.897	0.848	1130.767
	+10	1.864	0.791	1126.099
	+20	1.850	0.766	1124.001
	+30	1.837	0.742	1122.037

α	-30	2.163	1.776	1256.483
	-20	1.992	1.171	1174.084
	-10	1.922	0.942	1143.785
	+10	1.850	0.739	1118.954
	+20	1.826	0.681	1112.517
	+30	1.806	0.636	1107.719
δ	-30	1.889	0.830	1128.827
	-20	1.885	0.826	1128.665
	-10	1.883	0.822	1128.505
	+10	1.877	0.815	1128.193
	+20	1.874	0.811	1128.041
	+30	1.872	0.808	1127.891

Table 1.

A careful study of above (table 1) reveals the following:

- (i) The values of $P^*(T, t_1)$ increases when the values of A decreases while the values of T^* and t_1^* decreases with decrease the value of A and increases with the increase of the value of A.
- (ii) The values of $P^*(T, t_1)$ decreases when the values of C_1 increases while the values of T^* and t_1^* increases with decrease the value of C_1 . The values of $P^*(T, t_1)$, T^* and t_1^* are slightly sensitive to change in the values of parameter C_1 .
- (iii) $P^*(T, t_1)$, t_1^* are slightly sensitive to change in the values of parameter C_2 while T^* is moderately sensitive to increase or decrease in the values of parameter C_2 .
- (iv) $P^*(T, t_1)$ is highly sensitive to change in the values of parameter a while T^* and t_1^* are moderately sensitive to change in the values of parameter a.
- (v) $P^*(T, t_1)$, t_1^* are slightly sensitive to change in the values of parameter s while T^* is moderately sensitive to decrease and decrease in the values of parameter s.
- (vi) $P^*(T, t_1)$, T^* and t_1^* are slightly sensitive to change in the values of parameter h.
- (vii) The values of $P^*(T, t_1)$, T^* and t_1^* increases when the values of parameter θ decreases and their values are slightly sensitive.
- (viii) $P^*(T, t_1)$, T^* and t_1^* are moderately sensitive to increase and decrease in values of parameter α .
- (ix) $P^*(T, t_1)$, T^* and t_1^* are slightly sensitive to increase and decrease in values of parameter δ .

6. Conclusion

In this paper, we have developed an inventory model for deteriorating items which follows the Gompertz distribution deterioration rate. The demand rate is assumed to be a function of selling price. Manager of the industry always take care of selling price parameters which affect the profit quickly. Shortages are allowed and completely backlogged in the present model. The traditional parameters of holding cost is assumed here to be time varying. As the changes in the time value of money and in the price, holding cost can not remain constant over time. Here we assumed that the holding cost is increasing function of time. Numerical example is given to illustrate the model and also verified graphically(Figure1). Comprehensive sensitive analysis has been carried out for showing the effect of variation in the parameters. The model is solved analytically by maximizing the total profit. In the numerical example, we found the optimum value of profit P^* , T^* and t_1^* .

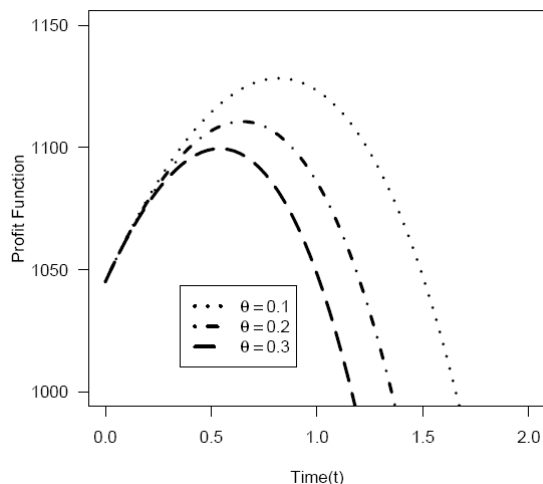


Figure-1

The present model is also extended with shortages by taking partial backlogging rate.

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