On the Construction of Conference Matrices of Order 10 and 14 from Coherent Configuration

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Abstract: *— In this paper we forward a method of construction of conference matrices of order 10 and 14 by suitable combinations of adjacency matrices of suitable coherent configuration.*

Keywords:— *Coherent configuration, Weighing matrix, Conference matrix, Symmetric conference matrix.*

I. INTRODUCTION:

A Weighing matrix W of order n and weigh w is an $n \times n$ matrix with entries $(0,\pm1)$ such that $WW^T = wI_n$ Where W^T is the transpose of W and I_n is the identity matrix of order n. A Weighing matrix of order n and weight w is denoted by $W(n,w)$. A $W(n,n-1), n$ even with zeros on diagonal such that $WW^T = (n-1)I_n$ is conference matrix. A $W(n,n)$ is a Hadamard matrix. If $n \equiv 2(\bmod 4)$ such that $W = W^T$ is symmetric conference matrix.[1] and [5].

A Conference matrix of order n is an $n \times n$ matrix M with diagonal entries 0 and other entries ± 1 which satisfies $MM^T = (n-1)I_n$ where M^T is transpose of M and I_n is the identity matrix.[2].A Conference matrix M with entries $0,+1$ and -1 is called symmetric conference matrix if $MM^T = M^T M = nI_n$ where n is order of matrix M, I_n is identity matrix and M^T is the transpose of M .[2]

Let X be a finite set. A Coherent Configuration on X is a set $C = \{C_1, C_2, C_3, ..., C_n\}$ of binary relation on X (subsets of $X \times X$) satisfying the following four conditions:

(i) C is a partition of $X \times X$ that is $\bigcup_{i=1}^{n} C_i = X \times X$;

(ii) There exist a sub set C_0 of C which is a partition of the diagonal $D = ((x,x) : x \in X)$

(iii) For every relation $C_i \in C$, its converse $C_i^{'} = \{(\beta,\alpha) : (\alpha,\beta) \in C^i\}$ is in C say $C_i^{'} = C_{i^*} \in C_k$

(iv) There exist integer P_{ij}^k for $1 \le i,j,k \le m$ such that for any $(\alpha,\beta) \in C_k$ the number of points $\gamma \in X$ such that $(\alpha,\gamma) \in C_i$ and $(\gamma,\beta) \in C_j$ is equal to P_{ij}^k (and in particular , is independent of the choice of $(\alpha,\beta) \in C_k$).That is we have

$$P_{ij}^k = \left|\{C_i(\alpha) \cap C_j^{'}(\beta)\}\right| \quad \text{for} \quad (\alpha,\beta) \in C_k$$

where $C(\alpha) = \{\beta \in X : (\alpha,\beta) \in C\}$.

Coherent Configuration is also defined by adjacency matrices of classes of C .If $M_1, M_2, M_3, ..., M_n$ are adjacency matrices of $C_1, C_2, C_3, ..., C_n$ respectively then the axioms takes the following from

(i) $M_1 + M_2 + ... + M_n = J$

(ii) There exist a sub set of $\{M_1, M_2, M_3 ... M_n\}$ with sum I=Ide3ntity matrix.

(iii) Each elements of the set $\{M_1, M_2, M_3, ... M_n\}$ is closed under transposition.

(iv) $M_i M_j = \sum_{i=1}^{m} P_{ij}^k M_k$ where P_{ij}^k are non – negative integers.[6] and [10].

II. MAIN WORK:

In [6] methods of construction of weighing/conference matrices of order 6 is given.

In similar approach in this paper we forward the methods of construct a conference matrix of order 10 and 14 by suitable linear combination of adjacency matrices of suitable coherent configurations.

(A) CONSTRUCTION OF SYMMETRIC CONFERENCE MATRIX OF ORDER 10

Consider $X = \{1,2,3,4,5,6,7,8,9,10\}$ and a partition

$C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}\}$ of $X \times X$ *where*

$$C_1 = \{(i,i) : i = 1\}$$

$$C_2 = \{(1,i) : i = 2,3,4,5,6,7,8,9,10\}$$

$$C_3 = \{(i,1) : i = 2,3,4,5,6,7,8,9,10\}$$

$$C_4 = \{(i,i) : i = 2,3,4,5,6,7,8,9,10\}$$

$$C_5 = \{(2,i) : i = 3,4,5,8\} \cup \{(3,i) : i = 2,4,6,9\} \cup \{(4,i) : i = 2,3,7,10\} \cup \{(5,i) : i = 2,6,7,8\} \cup \{(6,i) : i = 3,5,7,9\} \cup \{(7,i) : i = 4,5,6,10\} \cup \{(8,i) : i = 2,5,9,10\} \cup \{(9,i) : i = 3,6,8,10\} \cup \{(10,i) : i = 4,7,8,9\}$$

$$C_6 = \{\{(2,i) : i = 6,7,9,10\} \cup \{(3,i) : i = 5,7,8,10\} \cup \{(4,i) : i = 5,6,8,9\} \cup \{(5,i) : i = 3,4,9,10\} \cup \{(6,i) : i = 2,4,8,10\} \cup \{(7,i) : i = 2,3,8,9\} \cup \{(8,i) : i = 3,4,6,7\} \cup \{(9,i) : i = 2,4,5,7\} \cup \{(10,i) : i = 2,3,5,6\}\}$$

Then the adjacency matrices M_1, M_2, M_3, M_4, M_5 and M_6 of C_1, C_2, C_3, C_4, C_5 and C_6 respectively are given below:

$$M_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 0&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0&1&0&0 \\ 0&1&0&1&0&1&0&0&1&0 \\ 0&1&1&0&0&0&1&0&0&1 \\ 0&1&0&0&0&1&1&1&0&0 \\ 0&0&1&0&1&0&1&0&1&0 \\ 0&0&0&1&1&1&0&0&0&1 \\ 0&1&0&0&1&0&0&0&1&1 \\ 0&0&1&0&0&1&0&1&0&1 \\ 0&0&0&1&0&0&1&1&1&0 \end{bmatrix}$$

$$M_6 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&0&1&1 \\ 0&0&0&0&1&0&1&1&0&1 \\ 0&0&0&0&1&1&0&1&1&0 \\ 0&0&1&1&0&0&0&0&1&1 \\ 0&1&0&1&0&0&0&1&0&1 \\ 0&1&1&0&0&0&0&1&1&0 \\ 0&0&1&1&0&1&1&0&0&0 \\ 0&1&0&1&1&0&1&0&0&0 \\ 0&1&1&0&1&1&0&0&0&0 \end{bmatrix}$$

We see that

$1. M_1 + M_2 + M_3 + M_4 + M_5 + M_6 = J_{10}$

$2. M_1 + M_2 = I_{10}$

$= 3. M_1^{'} = M_1, M_2^{'} = M_3, M_3^{'} = M_2,$

$M_4^{'} = M_4, M_5^{'} = M_5, M_6^{'} = M_6$

We see the following calculations:

$1. M_1^2 = M_1, M_1M_2 = M_2, M_1M_3 = 0,$

$M_1M_4 = 0, M_1M_5 = 0, M_1M_6 = 0$

$2.. M_2^2 = 0, M_2M_3 = 9M_1, M_2M_4 = M_2,$

$M_2M_5 = 4M_2, M_2M_6 = 4M_2$

$3. M_3^2 = 0, M_3M_4 = 0, M_3M_5 = 0, M_3M_6 = 0$

$4. M_4^2 = M_4, M_4M_5 = M_5, M_4M_6 = M_6$

$5. M_5^2 = 4M_4 + M_5 + 2M_6,$

$M_5M_6 = 2(M_5 + M_6)$

$6. M_2^6 = 4M_4 + 2M_5 + M_6$

Hence product of any two adjacency matrices is some linear combination adjacency matrices. Thus the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ is a C.C.

Consider the matrix

$$M = 0.M_1 + M_2 + M_3 + 0.M_4 + M_5 + (-1).M_6$$

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 0 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 0 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 0 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & 0 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 1 & 1 & 0 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\because MM^T = \begin{bmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$

$$= 9I_n = (10-1)I_{10}$$

$$\therefore MM^T = (10-1)I_{10}$$

$$\because M^TM = \begin{bmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix}$$

$$= 9I_n = (10-1)I_{10}$$

$$\therefore M^TM = (10-1)I_{10}$$

Thus $MM^T = M^TM = (10-1)I_{10}$.

Which show that M is asymmetric matrix of order 10.

(B) CONSTRUCTION OF SYMMETRIC CONFERENCE MATRIX OF ORDER 14

Consider $X = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$ and a partition $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ 0f $X \times X$ where

$C_1 = \{(i,i,) : i = 1\}$

$C_2 = \{(1,i) : i = 2,3,4,...,14\}$

$C_3 = \{(i,1) : i = 2,3,4,...,14\}$

$C_4 = \{(i,i) : i = 2,3,4,...,14\}$

$C_5 = \{(2,i) : i = 3,4,5,9,10,11\} \cup \{(3,i) : i = 2,6,9,10,12,13\}$
$\cup \{(4,i) : i = 2,7,10,11,13,14\} \cup \{(5,i) : i = 2,8,9,11,12,14\}$
$\cup \{(6,i) : i = 3,7,8,10,11,12\} \cup \{(7,i) : i = 4,6,8,9,11,13\}$
$\cup \{(8,i) : i = 5,6,7,9,10,14\} \cup \{(9,i) : i = 2,3,5,7,8,13\}$
$\cup \{(10,i) : i = 2,3,4,6,8,14\} \cup \{(11,i) : i = 2,4,5,6,7,12\}$
$\cup \{(12) : i = 3,5,6,11,13,14\} \cup \{(13,i) : i = 3,4,7,9,12,14\}$
$\cup \{(14,i) : i = 4,5,8,10,12,13\}$

$C_6 = \{(2,i) : i = 6,7,8,12,13,14\} \cup \{(3,i) : i = 4,5,7,8,11,14\}$
$\cup \{(4,i) i = 3,5,6,8,9,12\} \cup \{(5,i) : i = 3,4,6,7,10,13\}$
$\cup \{(6,i) : i = 2,4,5,9,13,14\} \cup \{(7,i) : i = 2,3,510,12,14\}$
$\cup \{(8,i) : i = 2,3,4,11,12,13\} \cup \{(9,i) : i = 4,6,10,11,12,14\}$
$\cup \{(10,i) : i = 5,7,9,11,12,13\} \cup \{(11,i) : i = 3,8,9,10,13,14\}$
$\cup \{(12,i) : i = 2,4,7,8,9,10\} \cup \{(13,i) : i = 2,5,6,8,10,11\}$
$\cup \{(14,i) : i = 2,3,6,7,9,11\}$

Then the adjacency matrices M_1, M_2, M_3, M_4, M_5, and M_6 of C_1, C_2, C_3, C_4, C_5 and C_6 respectively are given below:

$$M_1 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 0&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0& &0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$M_5 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0&0&1&1&1&0&0&0 \\ 0&1&0&0&0&1&0&0&1&1&0&1&1&0 \\ 0&1&0&0&0&0&1&0&0&1&1&0&1&1 \\ 0&1&0&0&0&0&0&1&1&0&1&1&0&1 \\ 0&0&1&0&0&0&1&1&0&1&1&1&0&0 \\ 0&0&0&1&0&1&0&1&1&0&1&0&1&0 \\ 0&0&0&0&1&1&1&0&1&1&0&0&0&1 \\ 0&1&1&0&1&0&1&1&0&0&0&0&1&0 \\ 0&1&1&1&0&1&0&1&0&0&0&0&0&1 \\ 0&1&0&1&1&1&1&0&0&0&0&1&0&0 \\ 0&0&1&0&1&1&0&0&0&0&1&0&1&1 \\ 0&0&1&1&0&0&1&0&1&0&0&1&0&1 \\ 0&0&0&1&1&0&0&1&0&1&0&1&1&0 \end{bmatrix}$$

$$M_6 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&1&0&0&0&1&1&1 \\ 0&0&0&1&1&0&1&1&0&0&1&0&0&1 \\ 0&0&1&0&1&1&0&1&1&0&0&1&0&0 \\ 0&0&1&1&0&1&1&0&0&1&0&0&1&0 \\ 0&1&0&1&1&0&0&0&1&0&0&0&1&1 \\ 0&1&1&0&1&0&0&0&0&1&0&1&0&1 \\ 0&1&1&1&0&0&0&0&0&0&1&1&1&0 \\ 0&0&0&1&0&1&0&0&0&1&1&1&0&1 \\ 0&0&0&0&1&0&1&0&1&0&1&1&1&0 \\ 0&0&1&0&0&0&0&1&1&1&0&0&1&1 \\ 0&1&0&1&0&0&1&1&1&1&0&0&0&0 \\ 0&1&0&0&1&1&0&1&0&1&1&0&0&0 \\ 0&1&1&0&0&1&1&0&1&0&1&0&0&0 \end{bmatrix}$$

We see that

$1. M_1 + M_2 + M_3 + M_4 + M_5 + M_6 = J_{14}$

$2. M \;\; + M \;\; = I_{14}$

$3. M_1' = M_1, M_2' = M_3, M_3' = M_2,$

$M_4' = M_4, M_5' = M_5, M_6' = M_6$

We see the following calculations:

$1. M_1^2 = M_1, M_1M_2 = M_2, M_1M_3 = 0,$

$M_1M_4 = 0, M_1M_5 = 0, M_1M_6 = 0$

$2. M_2^2 = 0, M_2M_3 = 13M_1, M_2M_4 = M_2,$

$M_2M_5 = 6M_2, M_2M_6 = 6M_2$

$3. M_3^2 = M_3M_4 = 0, M_3M_5 = 0, M_3M_6 = 0$

$4. M_4^2 = M_4, M_4M_5 = M_5, M_4M_6 = M_6$

$5. M_5^2 = 3M_3 + 6M_4 + 2M_5,$

$M_5M_6 = 3(M_5 + M_6)$

$6. M_6^2 = 6M_4 + 3M_5 + 2M_6$ Hence product of any two adjacency matrices is some linear combinations of adjacency matrices. Thus the set $C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ is a C.C.

Consider the matrix $M = 0.M_1 + M_2 + M_3 + 0.M_4 + M_5 + (-1)M_6$

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 0 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 0 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 0 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 0 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 & 0 & 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 0 & -1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 0 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 0 \end{bmatrix}$$

$$\because MM^T = \begin{bmatrix} 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 \end{bmatrix}$$

$$= 13I_{14} = (14-1)I_{14}$$

$$\therefore MM^T = (14-1)I_{14}$$

$$\because M^TM = \begin{bmatrix} 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13 \end{bmatrix}$$

$$= 13I_{14} = (14-1)I_{14}$$

$$\therefore M^TM = (14-1)I_{14}$$

Thus $MM^T = M^TM = (14-1)I_{14}$.

Which show that M is a symmetric conference matrix of order 14.

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