

Fixed point theorems for R- weakly commuting mappings of type (A_g) in intuitionistic fuzzy metric space satisfying integral type inequality

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Abstract: The purpose of this paper, using the idea of intuitionistic fuzzy set due to Atanassov [12], we define the notion of intuitionistic fuzzy metric spaces due to Kramosil and Michalek [16] and Jungck's common fixed point theorem [7] is generalized to intuitionistic fuzzy metric spaces. Further, we prove the common fixed point theorem in Intuitionistic Fuzzy Metric Space using the property (S-B), by using the notion of R-weakly commuting mappings of type (A_g) satisfying integral type inequality.

Keywords: Compatible maps, Fuzzy metric spaces, intuitionistic fuzzy metric space, Compatible maps of type (A_g) , implicit relation.

1. INTRODUCTION:

Since the introduction of the concept of fuzzy sets by Zadeh [23] in 1965, many authors have introduced the concept of fuzzy metric in different ways ([1], [15], [16]). George and Veeramani [1], modified the concept of fuzzy metric space introduced by Kramosil and Michalek [16] and defined a Hausdorff topology on this fuzzy metric space. Many authors ([2], [7],[9],[10], [18], [19],[21]) obtained common fixed point theorems for weakly commuting maps and R-weakly commuting mappings.

Atanassov [12] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Coker [6] introduced the concepts of "Intuitionistic fuzzy topological spaces". There has been much progress in the study of intuitionistic fuzzy sets by many authors. Park [11] used the idea of intuitionistic fuzzy sets and defines the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorms as a

generalization of fuzzy metric space due to George and Veeramani [1].

Pant [18] introduced the concept of non commuting maps in metric spaces. Later Pathak et al. [9] generalized this concept and gave the concept of R-weakly commuting mappings of type (C). Recently Hosseini [23], Vasuki [19], Singh et al. [22], Malviya et al. [17] and Singh B. et al. [3] prove the Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space Satisfying Integral Type Inequality.

In this paper we prove the common fixed point theorem in Intuitionistic Fuzzy Metric Space using the property (S-B), by using the notion of R-weakly commuting mappings of type (A_g) satisfying integral type inequality.

2. PRELIMINARIES

Definition 2.1 [2] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norms if "*" is satisfying the following conditions:

- (i) * is commutative and associative
- (ii) * is continuous
- (iii) $a * 1 = a$ for all $a \in [0, 1]$
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Basic example of t – norm are the Lukasiewicz

t – norm $T_1, T_1(a, b) = \max(a+b-1, 0)$,

t -norm $T_p, T_p(a, b) = a b$, and t -norm T_M ,

$$T_M(a, b) = \min \{a, b\}.$$

Definition 2.2[16]. A binary operation $\diamond: [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t -co norms if “ \diamond ” satisfying following conditions:

- (i) \diamond is commutative and associative;
- (ii) \diamond is continuous;
- (iii) $a \diamond 0 = a$ for all $a \in [0, 1]$
- (iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Note. The concepts of triangular norms (t -norms) and triangular conorms (t -conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [13] in his study of statistical metric spaces.

Definition 2.3 [16] : A 3-tuple $(X, M, *)$ is said to be a fuzzy Metric space if X is an arbitrary set, “ $*$ ” is a continuous t -norm and M is a fuzzy set of $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$

$$(FM_1) M(x, y, t) > 0$$

$$(FM_2) M(x, y, t) = 1 \text{ if and only if } x = y$$

$$(FM_3) M(x, y, t) = M(y, x, t)$$

$$(FM_4) M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$$

$$(FM_5) M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1] \text{ is continuous.}$$

Then M is called a fuzzy metric on X . Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t .

A sequence $\{x_n\}$ in X converges to x if and only if for each $t > 0$ there exist $n_o \in N$ such that, $M(x_n, x, t) = 1$, for all $n \geq n_o$.

Lemma 2.1. Let $(X, M, *)$ be a fuzzy metric space, then M is a continuous function on $X^2 \times (0, \infty)$.

Definition-2.4[5]: A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t -norm, \diamond is a

continuous t -conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X, s, t > 0$,

$$(IFM-1) M(x, y, t) + N(x, y, t) \leq 1$$

$$(IFM-2) M(x, y, t) > 0$$

$$(IFM-3) M(x, y, t) = 1 \text{ if and only if } x = y$$

$$(IFM-4) M(x, y, t) = M(y, x, t)$$

$$(IFM-5) M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$$

$$(IFM-6) M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1] \text{ is continuous}$$

$$(IFM-7) N(x, y, t) > 0$$

$$(IFM-8) N(x, y, t) = 0 \text{ if and only if } x = y$$

$$(IFM-9) N(x, y, t) = N(y, x, t)$$

$$(IFM-10) N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$$

$$(IFM-11) N(x, y, \cdot) : (0, \infty) \rightarrow (0, 1] \text{ is continuous}$$

Then (M, N) is called an *Intuitionistic fuzzy metric* on X .

Note: $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non nearness between x and y with respect to ‘ t ’ respectively.

Definition 2.5.[5] :Let $(X, M, N, *, \diamond)$ be an Intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$, $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$ and $\lim_{n \rightarrow \infty} N(x_n, x, t) = 0$

(b) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$
 $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0$

(c) An Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Remark -2.1: Every fuzzy metric space $(X, M, *)$ is an Intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t -norm $*$ and t -conorm \diamond are associated

i.e. $x \diamond y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in [0, 1]$.

Remark -2.2: An intuitionistic fuzzy metric space $X, M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-

increasing for all $x, y \in X$.

Example- 2.1: Let (X, d) be a metric space. Denote $a * b = a.b$ and $a \diamond b = \min \{1, a + b\}$ for all $a, b \in [0, 1]$. Let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M(x, y, t) = \frac{t}{t + md(x, y)},$$

and
$$N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

in which $m > 1$. Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space.

Definition 2.6[19]: Two maps A and S are called R -weakly commuting at a point x if $d(ASx, SAx) \leq Rd(Ax, Sx)$ for some $R > 0$. A and S are called point wise R -weakly commuting on X if given x in X , there exists $R > 0$ such that $d(ASx, SAx) \leq Rd(Ax, Sx)$.

Definition 2.7 : Two mappings A and S of a fuzzy metric space $(X, M, *)$ into itself are R -weakly commuting provided there exists some real number R such that

$$M(ASx, SAx, t) \geq M(Ax, Sx, t/R)$$

and $N(ASx, SAx, t) \leq N(Ax, Sx, t/R)$ for each $x \in X$ and $t > 0$.

Definition 2.8.: Two self mappings A and S of a metric space (X, d) are called R -weakly commuting of type (A_g) if there exists some positive real number R such that

$$d(AAx, SAx) \leq Rd(Ax, Sx) \text{ for all } x \text{ in } X.$$

Definition 2.9.: Two mappings A and S of Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself are R -weakly commuting of type (A_g) provided there exists some real number R such that

$$M(AAx, SAx, t) \geq M(Ax, Sx, t/R)$$

and $N(AAx, SAx, t) \leq N(Ax, Sx, t/R)$ for each $x \in X$ and $t > 0$.

Definition 2.10.: Let S and T be two self mappings of an Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. We say that S and T satisfy the property $(S-B)$ if there exist a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Example 2.2. Let $X = [0, +\infty)$. Define $S, T: X \rightarrow X$ by $Tx = x/5$ and $Sx = 3x/5$, for all x in X . Consider the

sequence $\{x_n\} = \{1/n\}$.

Clearly $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = 0$. Then S and T satisfy the property $(S-B)$.

3. Main results:

Theorem 3.1. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and f and g be point wise R -weakly commuting self mappings of the type (A_g) of X satisfying the following conditions:

- (1) $f(X) \subset g(X)$
- (2) There exist a constant $k \in (0, 1)$ such that

$$M(fx, fy, kt) \geq \int_0^{\min\{M(gx, gy, t), M(fx, gx, t), M(fy, gy, t), M(fy, gx, t), M(fx, gy, t)\}} \varphi(t) dt$$

and

$$N(fx, fy, kt) \leq \int_0^{\max\{N(gx, gy, t), N(fx, gx, t), N(fy, gy, t), N(fy, gx, t), N(fx, gy, t)\}} \varphi(t) dt$$

Where $\varphi: R^+ \rightarrow R^+$ is Lebesgue-integrable mapping which is summable, non- negative, such that $\int_0^\varepsilon \varphi(t) dt > 0$ for each $\varepsilon > 0$.

If f and g satisfy the property $(S-B)$ and the range of either of $f(X)$ or $g(X)$ is a complete subspace of X , then f and g have a unique common fixed point.

Proof: Since f and g satisfy the property $(S-B)$, there exists a sequence $\{x_n\}$ in X such that,

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z \text{ for some } z \in X.$$

Since $f(X) \subset g(X)$, there exists some point u in

X such that $z = gu$ where $z = \lim_{n \rightarrow \infty} gx_n$. If $fu \neq gu$, the

Inequality

$$M(fx_n, fu, kt) \geq \int_0^{\min\{M(gx_n, gu, t), M(fx_n, gx_n, t), M(fu, gu, t), M(fu, gx_n, t), M(fx_n, gu, t)\}} \varphi(t) dt$$

and

$$N(fx_n, fu, kt) \leq \int_0^{\max\{N(gx_n, gu, t), N(fx_n, gx_n, t), N(fu, gu, t), N(fu, gx_n, t), N(fx_n, gu, t)\}} \varphi(t) dt$$

Letting $n \rightarrow \infty$

$$M(fx_n, fu, kt) \geq \int_0^{\min\{M(gu, gu, t), M(fgu, gu, t), M(fu, gu, t), M(fu, gu, t), M(fu, gu, t)\}} \varphi(t) dt$$

and

$$N(fx_n, fu, kt) \leq \int_0^{\max\{N(gu, gu, t), N(fgu, gu, t), N(fu, gu, t), N(fu, gu, t), N(fu, gu, t)\}} \varphi(t) dt$$

Hence $fu = gu$

Since f and g are R -weakly commuting of type (A_g) , there exists $R > 0$ such that

$$M(ffu, fu, t) \geq M(fu, gu, \frac{t}{R}) = 1 \text{ and } N(ffu, fu, t) \leq$$

$$N(fu, gu, t/R)=0$$

That is $ffu=ghu$ and $ffu = fgu = ghf = ggu$.

If $fu \neq gu$, then we have

$$\int_0^{M(fu,ffu,kt)} \varphi(t)dt \geq \int_0^{\min.\{M(gu,ghu,t),M(fu,gu,t),M(ffu,ghu,t),M(ffu,ghu,t),M(gu,ffu,t)\}} \varphi(t)dt$$

And

$$\int_0^{N(fu,ffu,kt)} \varphi(t)dt \leq \int_0^{\max.\{N(gu,ghu,t),N(fu,gu,t),N(ffu,ghu,t),N(ffu,ghu,t),N(gu,ffu,t)\}} \varphi(t)dt$$

$$\int_0^{M(fu,ffu,kt)} \varphi(t)dt \geq \int_0^{M(fu,ffu,t)} \varphi(t)dt$$

$$\text{and} \quad \int_0^{N(fu,ffu,kt)} \varphi(t)dt \leq \int_0^{N(fu,ffu,t)} \varphi(t)dt$$

a contradiction.

Hence $fu=fghu$ and $fu=fghu=fghu=fghu=ggu$.

Hence fu is a common fixed point of f and g . The case when $f(X)$ is a complete subspace of X is similar to the above case since $f(X) \subset g(X)$. Hence we have the theorem.

Theorem3.2. Let $(X, M, N, *, \diamond)$ be an Intuitionistic fuzzy metric space and f and g be non compatible point-wise R -weakly commuting self maps of type A_g of X satisfying the following conditions:

- (1) $f(X) \subset g(X)$.
- (2) There exist a constant $k \in (0,1)$ such that

$$\int_0^{M(fx,fy,kt)} \varphi(t)dt \geq \int_0^{\min.\{M(gx,gy,t),M(fx,gt,t),M(fy,gy,t),M(fy,gt,t),M(fx,gy,t)\}} \varphi(t)dt$$

and

$$\int_0^{N(fx,fy,kt)} \varphi(t)dt \leq \int_0^{\max.\{N(gx,gy,t),N(fx,gt,t),N(fy,gy,t),N(fy,gt,t),N(fx,gy,t)\}} \varphi(t)dt$$

Where $\varphi: R^+ \rightarrow R^+$ is Lebesgue-integrable mapping which is summable, non- negative, and such that

$$\int_0^\epsilon \varphi(t)dt > 0 \text{ for each } \epsilon > 0$$

If the range of f or g is a complete subspace of X , then f and g have a unique common fixed point and the fixed point is the point of discontinuity.

Proof. Since f and g are non-compatible maps, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z \text{ for some } z \in X.$$

For some z in X , $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) \neq 1$ and $\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t) \neq 0$, or the limit does not exist.

Since $z \in f(X)$ and $f(X) \subset g(X)$, there exists some point u in X such that $z=gu$ where $z = \lim_{n \rightarrow \infty} gx_n$,

If $fu \neq gu$, then inequality becomes

$$\int_0^{M(fx_n,fu,kt)} \varphi(t)dt \geq \int_0^{\min.\{M(gx_n,gu,t),M(fx_n,gu,t),M(fx_n,gu,t)\}} \varphi(t)dt$$

$$\int_0^{N(fx_n,fu,kt)} \varphi(t)dt \leq \int_0^{\max.\{N(gx_n,gu,t),N(fx_n,gu,t),N(fx_n,gu,t)\}} \varphi(t)dt$$

Letting $n \rightarrow \infty$ we have $M(fu, fu, t) \geq M(gu, fu, t)$ and $N(fu, fu, t) \leq N(gu, fu, t)$

Hence $fu = gu$

Since f and g are R -weakly commuting mappings of type (A_g) , there exists $R > 0$ such that

$$M(ffu,ghu,t) \geq M(fu,gu,t/R) = 1$$

and $N(ffu,ghu,t) \leq N(fu,gu,t/R) = 0$

That is, $fu = gfu$ and $ffu = gfu = ggu$. If $fu \neq ffu$ using (2) we have

$$\int_0^{M(fu,ffu,kt)} \varphi(t)dt \geq \int_0^{\min.\{M(gu,ghu,t),M(fu,gu,t),M(ffu,ghu,t),M(ffu,ghu,t)\}} \varphi(t)dt$$

And

$$\int_0^{N(fu,ffu,kt)} \varphi(t)dt \leq \int_0^{\max.\{N(gu,ghu,t),N(fu,gu,t),N(ffu,ghu,t),N(ffu,ghu,t)\}} \varphi(t)dt$$

Hence

$$\int_0^{M(fu,ffu,kt)} \varphi(t)dt \geq \int_0^{M(fu,ffu,t)} \varphi(t)dt$$

$$\int_0^{N(fu,ffu,kt)} \varphi(t)dt \leq \int_0^{N(fu,ffu,t)} \varphi(t)dt$$

And

a contradiction. Hence $fu = fghu$ and $fu = fghu = fghu = fghu = ggu$

Hence fu is a common fixed point of f and g . The case when $f(X)$ is a complete subspace of X , is similar to the above case since $f(X) \subset g(X)$.

Now we show that f and g are discontinuous at the common fixed point $z = fu = gu$. If possible, suppose f is continuous. Then considering the sequence $\{x_n\}$ we get $\lim_{n \rightarrow \infty} ffx_n = fz = z$. R -weakly commuting of type (A_g) implies that,

$$M(ffx_n, gfx_n, t) \geq M(fx_n, gx_n, t/R) = 1$$

And $N(ffx_n, gfx_n, t) \leq N(fx_n, gx_n, t/R) = 0$

Which on letting $n \rightarrow \infty$, yields $\lim_{n \rightarrow \infty} gfx_n = fz = z$

This, in turn, yields $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t)$ and $\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t)$ are either non zero or nonexistent for the sequence $\{x_n\}$. Hence f is discontinuous at the fixed point.

Next, suppose that g is continuous, then for the sequence $\{x_n\}$,

We get $\lim_{n \rightarrow \infty} gfx_n = gz = z$ and $\lim_{n \rightarrow \infty} gg x_n = gz = z$.

In view of these limits, the inequality

$$\int_0^{M(fx_n,fgx_n,Kt)} \varphi(t)dt \geq \int_0^{\min.\{M(gx_n,ggx_n,t),M(fx_n,gx_n,t),M(fgx_n,ggx_n,t),M(fgx_n,ggx_n,t)\}} \varphi(t)dt$$

and

$$\int_0^{N(fx_n,fgx_n,Kt)} \varphi(t)dt \leq \int_0^{\max.\{N(gx_n,ggx_n,t),N(fx_n,gx_n,t),N(fgx_n,ggx_n,t),N(fgx_n,ggx_n,t)\}} \varphi(t)dt$$

yield a contradiction unless

$$\lim_{n \rightarrow \infty} fgx_n = fz = z$$

$$\text{but } \lim_{n \rightarrow \infty} fgx_n = gz \text{ and } \lim_{n \rightarrow \infty} gfx_n = gz$$

Contradicts the fact that $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t)$ and

$\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t)$ is either nonzero or nonexistent.

Thus both f and g are discontinuous at their common fixed point. Hence we have the theorem.

We now give an example to illustrate the above theorem.

Example 3.1. Let $X = [2, 20]$. Define $f, g : X \rightarrow X$ as

$$f(x) = \begin{cases} 2, & \text{if } x = 2 \text{ or } > 5 \\ 6, & \text{if } 2 < x \leq 5 \end{cases}$$

$$g(x) = \begin{cases} 7, & 2 < x < 5 \\ \frac{4x+10}{15}, & \text{if } x > 5 \end{cases} \text{ and } g(2)=2$$

$$\text{Also we define } M(fx, gy, t) = \frac{t}{t + md(fx, gy)}$$

$$\text{and } N(fx, gy, t) = \frac{d(fx, gy)}{t + d(fx, gy)}$$

for every $x, y \in X$ and $t > 0$.

Then f and g satisfy all the conditions of the above theorem and have a common fixed point at $x = 2$.

In this example $f(x) = \{2\} \cup \{6\}$ and $g(x) = [2, 6] \cup \{7\}$.

It may be seen that $f(X) \subset g(X)$.

It can be verified also that f and g are point wise R-weakly commuting maps of type (A_g) .

To see that f and g are non compatible, let us consider a sequence

$$\{x_n = 5 + 1/n : n > 1\} \text{ then } \lim_{n \rightarrow \infty} fx_n = 2, \lim_{n \rightarrow \infty} gx_n = 2$$

$$\text{and } \lim_{n \rightarrow \infty} fgx_n = 6, \lim_{n \rightarrow \infty} gfx_n = 2$$

Hence f and g are Non compatible.

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