

# Influence of Chemical reaction, Soret, Dufour and variable thermal conductivity effects on unsteady free convective flow between two concentric circular cylinders

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*Abstract :* In this paper, the unsteady free convective flow of viscous incompressible fluid mixture between two concentric stationary circular cylinders under the influence of Soret, Dufour, Chemical reaction and variable thermal conductivity effects have been investigated. The flow equations corresponding to momentum, energy equations and species conservation equations are converted into dimensionless form, which are then, solved numerically based on finite difference method. The effects of material parameters on transient temperature and concentration profiles are discussed graphically.

**Keywords :** Chemical reaction effect, Soret effect, Dufour effect, variable thermal conductivity effect, concentric circular cylinders, finite difference method.

**Introduction:** Free convection flow occurs normally in nature as temperature difference and also concentration difference. Free convection flow continues to attract the attention of many researchers because of its presence both in nature and engineering applications. Free convection flow has many applications including the design of electronic equipment cooling system, nuclear reactor waste transport and storage, solar collectors and thermal storage system and thermal management of aviation. Both Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in fluid binary systems, often encountered in chemical process engineering and also in high speed aerodynamics. Chemical reaction between a foreign mass and the fluid occurs in many industrial applications such as polymer production, the manufacturing of ceramics or glassware, formation and dispersion of fog, damage of crops due to freezing, distribution of temperature and moisture over groves of fruit trees and so on. Thermal conductivity of a working fluid is sensitive to the variation of temperature so it may change with temperature. Sharma and Singh [1] discussed the thermal diffusion in a binary fluid mixture confined between two concentric circular cylinders in presence of a radial magnetic field. Sreedevi et. al. [2]

analyzed the heat and mass transfer characteristics of non – Darcy mixed convective flow of a viscous electrically conducting fluid through a porous medium in a circular cylindrical annulus in the presence of temperature dependent heat source with Soret and Dufour effects. Kaladhar and Srinivasacharya [3] studied the mixed convection flow of chemically reacting couple stress fluid in an annulus with Soret and Dufour effects. Sharma and Borgohain [4] examined the Soret effect on chemically reacting natural convection between two concentric circular cylinders in a porous medium. Reddy et. al. [5] studied the influence of variable thermal conductivity on MHD boundary layer slip flow of ethylene – glycol based cu nanofluids over a stretching sheet with convective boundary condition. Jiang et. al. [6] discussed the influence of variable thermal conductivity and permeability of adsorbent on simulation : A case study of a two – stage freezing system. Manjunatha and Gireesha [7] studied the variable viscosity and thermal conductivity effect on MHD flow and heat transfer of a dusty flow . Rashad et. al [8] investigated the effect of chemical reaction on heat and mass transfer by mixed convection flow about a solid sphere in a saturated porous media. Venkateswarlu et. al. [9] studied the chemical reaction and radiation absorption effects on mixed convective flow in a circular annulus at constant heat and mass flux. Sharma and Konwar [10] discussed the chemical reaction effect on mass distribution of a binary fluid mixture in unsteady MHD couette flow.

This paper is an extension of the paper [4] in which Dufour and variable thermal conductivity are investigated.

**Mathematical formulation :**

An unsteady flow of incompressible fluids mixture between the walls of two infinite concentric circular cylinder in presence of uniform radial magnetic field of strength  $\frac{B_0 a'}{r}$  is considered where  $r'$  is the radial distance from the axis of the annulus,  $B_0$  is applied magnetic field and  $a'$  is the radius of cylinder which is insoluble in the fluid . Let  $w'$  be the fluid velocity in

axial direction and  $b'$  be the radius of cylinder which diffuses to the fluid to establish the equilibrium near the surface where  $b' > a'$ . The flow is developed due to increase in the temperature of the surface of the inner cylinder.

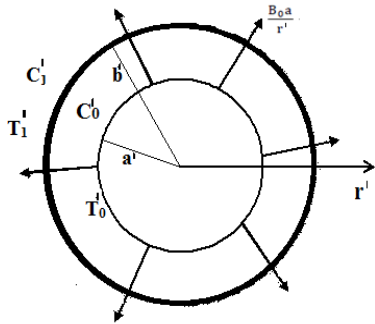


Fig. 1 Physical model

Since the cylinders are infinite in length so the flow depends on  $r'$  and  $t'$  only. As usual we take thermal conductivity as inverse linear function of temperature, i.e.,

$$\frac{1}{k} = \frac{1}{k_\infty} [1 + \beta(T' - T_1')] \text{ or } k = \frac{k_\infty}{[1 + \beta(T' - T_1')]} = \frac{k_\infty}{[1 + \beta\theta(T_0' - T_1')]} = \frac{k_\infty}{(1 + \omega\theta)} \quad (1)$$

where  $\omega = \beta(T_0' - T_1')$ ,  $k$  is the thermal conductivity,  $k_\infty$  is the reference thermal conductivity,  $\beta$  is constant.

We assume that the temperature  $T_0'$  at radius  $a'$  to be constant initially and  $T_1'$  be the constant temperature at radius  $b'$ , and the concentration  $C_0'$  at radius  $a'$  to be constant initially and  $C_1'$  be the constant concentration at radius  $b'$ . We also assume the temperature  $T'$  and concentration  $C'$  is varying as a function of time  $T_0'(t')$  and  $C_0'(t')$  respectively.

$T_0'(t')$  and  $C_0'(t')$  are defined as

$$T_0'(t') = T_0' + (T_1' - T_0') At' \quad \text{and}$$

$$C_0'(t') = C_0' + (C_1' - C_0') At'$$

where  $A = \frac{1}{t_R}$ ,  $t_R$  is the reference time

The governing equations are

$$\frac{\partial w'}{\partial z'} = 0 \quad (2)$$

$$\frac{\partial w'}{\partial t'} = \frac{-p}{\rho} + \frac{\mu}{\rho} \left( \frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'} \frac{\partial w'}{\partial r'} \right) - \frac{\alpha^2 \sigma B_0^2 w'}{r'^2 \rho} \quad (3)$$

$$\frac{\partial T'}{\partial t'} = \frac{1}{\rho C_P r'} \frac{\partial}{\partial r'} \left( k(r' \frac{\partial T'}{\partial r'}) \right) + \frac{D_m k_t}{C_S C_P} \left( \frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'} \frac{\partial C'}{\partial r'} \right) \quad (4)$$

$$\frac{\partial C'}{\partial t'} = D_m \left( \frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'} \frac{\partial C'}{\partial r'} \right) + \frac{D_m k_t}{T_m} \left( \frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'} \frac{\partial T'}{\partial r'} \right) - k_l (C' - C_1') \quad (5)$$

The initial and boundary conditions for the problem are given by

$$\left. \begin{aligned} w' = 0, \quad T' = T_0', \quad C' = C_0' \quad \text{at } t' = 0 \\ \text{At } t' > 0 \\ w' = 0, \quad T' = T_0'(t'), \quad C' = C_0'(t') \quad \text{at } r' = a' \\ w' = 0, \quad T' = T_1', \quad C' = C_1' \quad \text{at } r' = b' \end{aligned} \right\} \quad (6)$$

where  $\mu$  is coefficient of viscosity,  $T_m$  is the mean fluid temperature,  $C_P$  is the specific heat at constant pressure,  $C_S$  is the concentration susceptibility,  $D_m$  is the molecular diffusivity,  $k_t$  is thermal diffusion ratio,  $k_l$  is dimensional chemical reaction parameter.

On introducing the following non dimensional quantities,

$$\left. \begin{aligned} w' = \frac{w}{w_R}, \quad T' = T_1' + (T_0' - T_1')\theta, \quad t' = \frac{t}{t_R}, \\ C' = C_1' + (C_0' - C_1')\phi, \quad r' = \frac{r}{r_R}, \quad Pr = \frac{\mu C_P}{k_\infty}, \\ Sr = \frac{D_m k_t (T_0' - T_1')}{T_m \nu (C_0' - C_1')}, \quad Du = \frac{D_m k_t (C_0' - C_1')}{C_S C_P \nu (T_0' - T_1')}, \\ M = \frac{\alpha^2 B_0^2 \sigma}{\mu}, \quad a' = a(\beta(T_0' - T_1'))^{1/3}, \\ b' = b(\beta(T_0' - T_1'))^{1/3}, \quad \delta = \frac{k_l a'^2}{\nu}, \\ r_R = \frac{1}{a(\beta(T_0' - T_1'))^{1/3}}, \quad t_R = \frac{\nu}{a^2(\beta(T_0' - T_1'))^{2/3}}, \\ w_R = \frac{\mu}{\rho a^2} (\beta(T_0' - T_1'))^{-2/3} \end{aligned} \right\} \quad (7)$$

where  $Sr$  is Soret number,  $Du$  is Dufour number,  $M$  is Hartmann number,  $Pr$  is Prandtl number and  $\delta$  is dimensionless chemical reaction parameter.

Using eqs. (1) and (7) in eqs. (3)- (6), we have

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{M}{r^2} w - 1 \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr(1 + \omega\theta)} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - \frac{\omega}{Pr(1 + \omega\theta)^2}$$

$$\left( \frac{\partial \phi}{\partial r} \right)^2 + Du \left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) \quad (9)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) + Sr \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - \delta \phi \tag{10}$$

The initial and boundary conditions are

$$\left. \begin{aligned} w = 0, \theta = 1, \phi = 1 & \quad \text{at } t = 0 \\ \text{At } t > 0 \\ w = 0, \theta = 1-t, \phi = 1-t & \quad \text{at } r = a \\ w = 0, \theta = 0, \phi = 0 & \quad \text{at } r = \frac{b}{a} \end{aligned} \right\} \tag{11}$$

Method of Solution :

The dimensionless partial differential equations (8)-(10) subject to the initial and boundary conditions (11) are reduced to a system of difference equations using the following finite difference scheme  $\frac{\partial w}{\partial r} = \frac{w^{i+1}_j - w^i_j}{\Delta r}$ ,  $\frac{\partial^2 w}{\partial r^2} = \frac{w^{i+1}_j + w^{i-1}_j - 2w^i_j}{(\Delta r)^2}$  and then the system of difference equations are solved numerically by an iterative method.

Result and Discussion :

Fig. 2 exhibit the transient temperature profile for various values of  $Sr = (0.1, 0.4, 0.6)$  and  $t = (0.1, 0.3)$  by taking  $Du = 0.3, Pr = 0.71, Sc = 0.5, M = 1, \delta = 0.5, \omega = 0.2$ . It is observed from Fig. 2 that for constant Soret number, with increasing time temperature increases but with increasing Soret number temperature decreases. Soret number represents the contribution of the temperature gradient to concentration gradient. It can be seen that increase in Soret number causes fall in temperature. Fig. 3 exhibit the transient temperature profile for various values of  $Du = (0.1, 0.3, 0.5)$  and  $t = (0.1, 0.3)$  by taking  $Sr = 0.4, Pr = 0.71, Sc = 0.5, M = 1, \delta = 0.5, \omega = 0.2$ . It is observed from Fig. 3 that with increasing time for constant Dufour number temperature is increasing and also with increasing Dufour number temperature increases. Thermal diffusivity increases with increase in Dufour number, the process results in increasing manner of energy transfer increase thermal boundary layer.

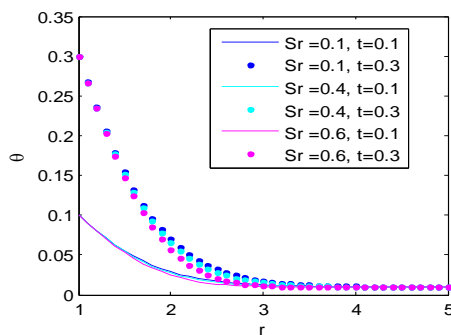


Fig. 2

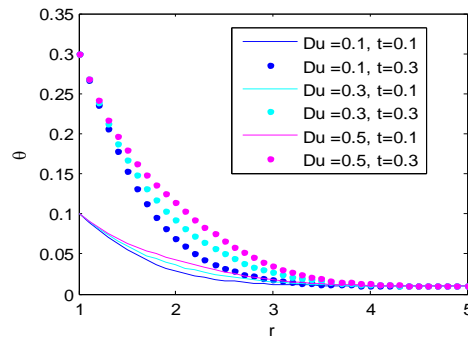


Fig. 3

Fig. 4 exhibit the transient concentration profile for various values of  $Sr = (0.1, 0.4, 0.6)$  and  $t = (0.1, 0.3)$  by taking  $Du = 0.3, Pr = 0.71, Sc = 0.5, M = 1, \delta = 0.5, \omega = 0.2$ . It is observed from Fig. 4 that with increasing time for constant Soret number concentration is increasing and also with increasing Soret number concentration increases. Increase in Soret number causes rise in concentration as a result of larger mass diffusivity. Fig. 5 exhibit the transient concentration profile for various values of  $Du = (0.1, 0.3, 0.5)$  and  $t = (0.1, 0.3)$  by taking  $Sr = 0.4, Pr = 0.71, Sc = 0.5, M = 1, \delta = 0.5, \omega = 0.2$ . It is observed from Fig. 5 that with increasing time concentration increases but with increasing Dufour number concentration decreases. Dufour number represents the contribution of concentration gradient to temperature gradient. It can be seen that increase in Dufour number causes fall in concentration.

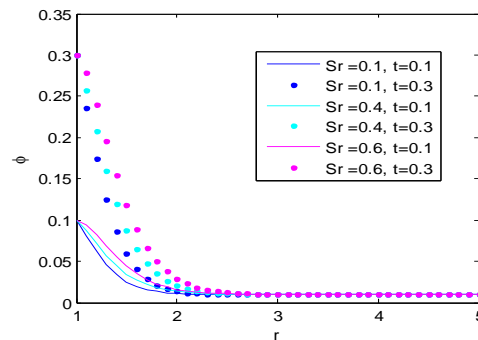


Fig. 4

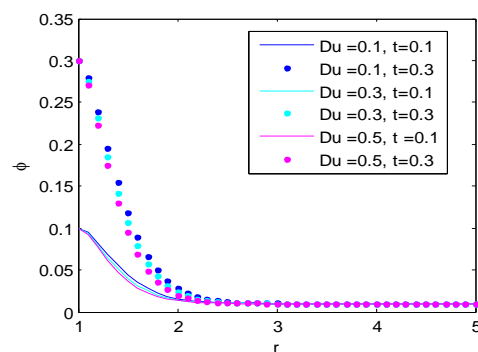


Fig. 5

Fig. 6 exhibit the transient temperature profile for various values of  $\omega = (0.1, 0.2, 0.4)$  and  $t = (0.1, 0.3)$  by taking  $Sr = 0.4, Du = 0.3, Pr = 0.71, Sc = 0.5, M = 1, \delta = 0.5$ . It is seen from Fig. 6 that with increasing time for constant thermal conductivity parameter temperature is increasing and also with increasing thermal conductivity parameter temperature increases. When thermal conductivity increases the heat transfer increases hence temperature increases. Fig. 7 exhibit the transient concentration profile for various value  $\delta = (0.1, 0.5, 1)$  and  $t = (0.1, 0.3)$  by taking  $Sr = 0.4, Du = 0.3, Pr = 0.71, Sc = 0.5, M = 1, \omega = 0.2$ . It is seen from Fig. 7 that with increasing time for constant dimensionless chemical reaction parameter concentration is increasing and also with increasing dimensionless chemical reaction parameter concentration decreases.

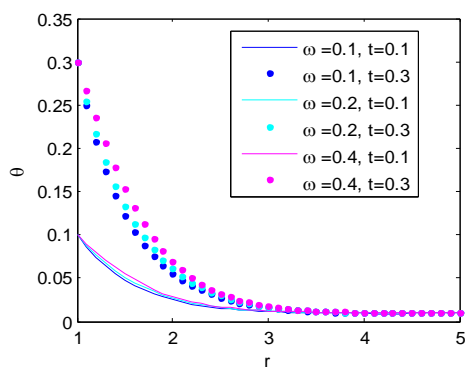


Fig. 6

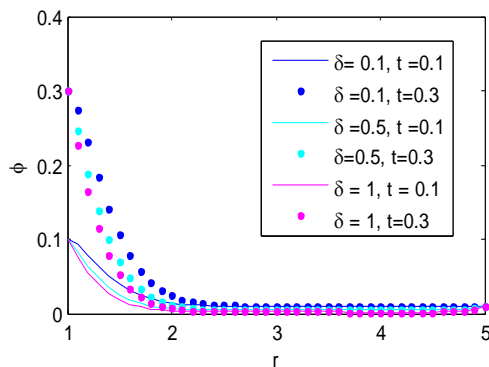


Fig. 7

Conclusion :

From the above analysis it can be concluded that the temperature of the fluid mixture increases with the

increase of Dufour number, time and thermal conductivity parameter. Also temperature decreases with the increase of Soret number. Concentration of the rarer and lighter component of the fluid mixture increases with the increase of Soret number and time. Also concentration decreases with the increase of Dufour number and dimensionless chemical reaction parameter.

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