Mathematical Modelling on blood flow through Stenosed artery under the influence of Magnetic field

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Abstract — *Blood can be assumed as a suspension of magnetic particles in non-magnetic plasma, due to presence of hemoglobin in red blood cells. The hemodynamic and rheological property of blood could help us to diagnose and perceive the pathological condition of stenosis. Stenosis is an abnormal and unnatural growth that is due to the deposits of atherosclerotic plaques, cholesterol, lipids, fats etc. inside the lumen of artery in a cardiovascular system. The governing equation of flowing fluid is solved numerically with the help of Frobenius method. The Einstein equation, dependent on hematocrit concentration of blood also helps to develop this model. The hematocrit is the proportion, by volume, of the blood that consists of red blood cells. The essential theoretical results such as axial velocity profile, pressure gradient and wall shear stress have been calculated numerically using MATLAB. From these results we conclude that the axial velocity decreases for increasing Hartmann number while pressure gradient and wall shear stress increases with increase of it. The variation of the solutions of these theoretical results with regard to different parameters has been shown from the graphical representation and it has been observed that the flow pattern is significantly controlled by the magnetic field.*

Keywords — *Stenosis, blood, Magnetic field, axial velocity, pressure gradient, wall shear stress, magnetic number.*

I. Introduction

One of the frequently occurring cardiovascular diseases, Atherosclerosis (stenosis) is the most epidemic disease in human beings, which is the leading cause of death in many countries. Stenosis is an abnormal and unnatural growth in the arterial wall thickness at various locations of cardiovascular system. Young [6] analysed the presence of stenosis in artery and pointed that the cause of stenosis is not exactly known but its existence in artery disturbed the normal flow of blood. In eighty's researchers attracted towards the application of magnetic field in biomedical researches. Since blood is a suspension of red cells containing haemoglobin having iron oxide in composition, so it can be assumed as an electrically conducting fluid.

Kolin [2] has first time strikes the idea of use of electromagnetic field in the medical research and has established the application of external magnetic field on the biological systems. Barnothy [15] has reported that the heart rate decreases on applying the applications of external magnetic field. The possibility of regulating the flow of blood in human system by applying magnetic field has been discussed by Korchevsky and Marochnik [9]. Mathematical analysis of blood flow through an indented tube in the presence of erythrocytes is presented by Haldar and ghosh [14]. Effect of slip conditions on a couple stress fluid flows in a channel with mild stenosis in the presence of uniform magnetic field is discussed by Awgichew et al. [10]. The problem of blood flow in an artery under some pathological situation when fatty plaques of cholesterol and artery clogging blood clots are formed in the lumen of the coronary artery is studied by Dash et al. [21].

The influence of transverse magnetic field on the physiological type blood flow region through non constricted single tubes is briefly reviewed by Ramchandra Rao & Deshikachar [3]. Analysis of flow through an obstructed tube under a pulsatile pressure gradient is described by Young [7]. Keltner et al. [13] discussed the effect of the pressure change in vessels of the human vascular system under the action of strong magnetic fields. Sanyal and Maiti [5] use the series solution for solving a mathematical model on arterial blood flow in the presence of mild stenosis and magnetic field. Bali and Awasthi [20] discussed the effect of an externally applied uniform magnetic field on the multi stenosed artery with core region. They considered blood as casson fluid by properly accounting for yield stress of blood in small blood vessel. A mathematical model to see the effect of magnetic field on the blood flow through stenosis under porous medium is developed by M. Jain et al. [16] and Raja et al. [19] studied the slip velocity effect on MHD oscillatory blood flow through stenosed artery. The effect of slip velocity and stenosis shape parameter on resistance to non- Newtonian flow of blood is given by A. Bhatnagar et al. [1]. Pulsatile unsteady flow of blood through porous medium in a stenotic artery under the influence of transverse magnetic field is studied by Sharma et al. [17]. Blood flow in stenotic arteries with slip effect in the presence of transverse Magnetic Field is studied by Sut. [8]. The Magnetic field effect on steady blood flow through an inclined circular tube is proposed by Verma et al. [22]. Analysis of non-Newtonian blood flow through stenosed vessel in porous medium under the effect of magnetic field is investigated by Singh et al. [12]. Several researchers [4, 11, 18, 23] have described the mathematical modal on the blood flowing through the stenotic arteries subject to various physiological conditions.

These researches on externally applied magnetic field motivated for the present study to deal with the steady blood flow through mild stenosed artery under the influence of an externally applied uniform magnetic. The results are useful in surgical procedures of cardiovascular diseases to control the blood pressure in arteries.

II. MATHEMATICAL MODEL

Consider an axially symmetric, laminar and steady flow of blood through a rigid circular artery with a mild stenosis under the influence of an externally applied uniform magnetic field. This model is developed with the assumption that blood flowing in the tube is a suspension of red blood cells in plasma. It is also assumed that the viscosity of fluid varies radially but density is constant and an electromagnetic force produced due to the interaction of current with magnetic field, is very small.
Now from the Einstein's equation for coefficient of viscosity of blood

$$\mu(r)=\mu_0\left\{1+\beta h(r)\right\} \tag{1}$$

Where $\mu(r)$ is the coefficient of viscosity of blood, μ_0 is the coefficient of viscosity of plasma, β is a constant and $h(r)$ stands for the hematocrit concentration which varies along the radial direction (r), described by the empirical formula.

$$h(r)=H_m\left\{1-\left(\frac{r}{R_0}\right)^n\right\} \tag{2}$$

Here H_m is the maximum hematocrit concentration at the centre of the vessel, R_0 is the radius of unobstructed tube and n (≥ 2) is the parameter defined the exact shape of profile and it supposed to be spherical in shape.

The geometry of the stenosis in the arterial lumen (Fig. 1) is given by [7] as;

$$\frac{R(z)}{R_0}=\begin{bmatrix} 1-\dfrac{\delta}{2R_0}\left\{1+\cos\dfrac{2\pi}{L_0}\left(z-d-\dfrac{L_0}{2}\right)\right\} & \text{in } d\le z\le d+L_0 \\ 1 & \text{in the normal artery region}\end{bmatrix} \tag{3}$$

Where $R(z)$ is the radius of stenosed artery, d is the location of the stenosis, L_0 is the length of the stenosis and δ is the maximum height of the stenosis. We shall further assume that $\frac{\delta}{R_0} \ll 1$.

Now considering the one dimensional equation of motion in cylindrical co-ordinate system (r, θ, z), under the assumption of small electrical conductivity σ as

$$\frac{\partial P}{\partial z}-\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right)+\sigma\beta_0^2 u=0\ , \tag{4}$$

Where P is the fluid pressure, u is the axial velocity component, β_0 magnetic flux density.

The boundary conditions are

$$u=0 \qquad at\ r=R(z) \qquad \text{no slip}, \tag{5}$$

$$du/dr=0 \qquad at\ r=0 \qquad \text{symmetry about the axis}. \tag{6}$$

Let us introduce the next following transformation

$$y = r/R_0\ , \tag{7}$$

Then the governing equation (4) reduced to

$$\frac{1}{y}\frac{\partial}{\partial y}\left[y\left(a-ky^n\right)\frac{\partial u}{\partial y}\right]-\beta_0^2R_0^2\left(\frac{\sigma}{\mu_0}\right)u=\frac{R_0^2}{\mu_0}\frac{\partial p}{\partial z} \tag{8}$$

Where $k = \beta H_m$, $a = 1+k$ & Let M is the Hartmann number, defined by $M = \beta_0^2R_0^2\left(\frac{\sigma}{\mu_0}\right)^{1/2}$,

Therefore equation(8) becomes

$$\frac{1}{y}\frac{d}{dy}\left[y\left(a-ky^n\right)\frac{du}{dy}\right]-M^2u=\frac{R_0^2}{\mu_0}\frac{dp}{dz} \tag{9}$$

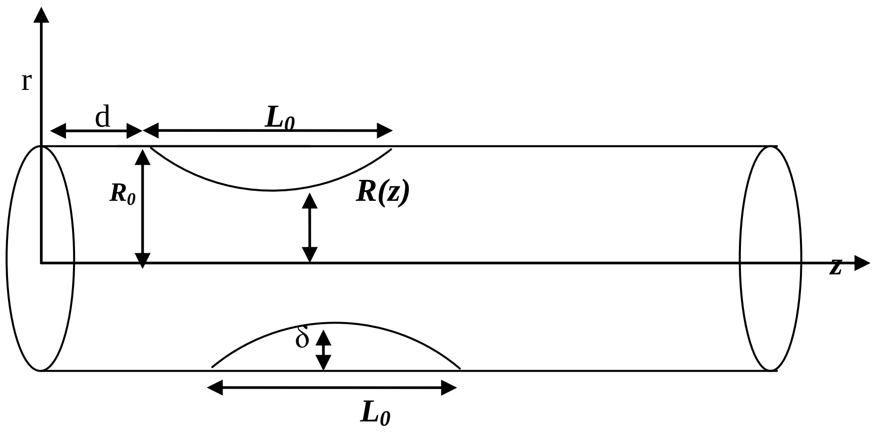

Figure 1. Geometry of construction

The corresponding boundary conditions (5) and (6) are transformed to

$$u = 0 \qquad at\ y = R(z)/R_0\,, \tag{10}$$

$$du/dy = 0 \qquad at\ y = 0. \tag{11}$$

III. Method of Solution

To solve the differential equation (9), with the boundary condition (10) and (11), we use Frobenius method. For implementing this method, it is required that u is bounded at $y = 0$ and only admissible series solution of equation (9) is,

$$u = D\sum_{m=0}^{\infty} a_m y^m + \frac{R_0^2}{4a\mu_0}\sum_{m=0}^{\infty} b_m y^{m+2} \tag{12}$$

Here D is an arbitrary constant to be determined by the boundary condition (10) and the second term of the R.H.S is the solution, corresponding to non homogeneous part of the equation (9). The values of series constant a_m and b_m involved in the solution are given by,

$$a_{m+1} = \frac{\left\{k(m+1)(m-n+1)a_{m-n+1} + M^2 a_{m-1}\right\}}{\left\{a(m+1)^2\right\}} \tag{13}$$

$$b_{m+1} = \frac{\left\{k(m+3)(m-n+3)b_{m-n+1} + M^2 b_{m-1}\right\}}{\left\{a(m+3)^2\right\}} \tag{14}$$

With $a_0 = b_0 = 1$ and $a_{-m} = b_{-m} = 0$ (15)

Applying the boundary condition (10), we have

$$D = \frac{-\dfrac{R_0^2}{4a\mu_0}\cdot\dfrac{dp}{dz}\cdot\left\{\displaystyle\sum_{m=0}^{\infty} b_m\left(R/R_0\right)^{m+2}\right\}}{\displaystyle\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m} \tag{16}$$

Then the resulting expression for u is,

$$u(y) = \frac{\dfrac{R_0^2}{4a\mu_0}\cdot\dfrac{dp}{dz}\left\{\begin{array}{l}\displaystyle\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m\sum_{m=0}^{\infty} b_m\left(y\right)^{m+2}\\ \displaystyle-\sum_{m=0}^{\infty} b_m\left(R/R_0\right)^{m+2}\sum_{m=0}^{\infty} a_m\left(y\right)^m\end{array}\right\}}{\displaystyle\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m} \tag{17}$$

In the absence of magnetic field and hematocrit, the average velocity u_0 in the normal tube is given by;

$$u_0 = -\frac{R_0^2}{8\mu_0}\left(\frac{dp}{dz}\right)_0 \tag{18}$$

Where $(dp/dz)_0$ is the pressure gradient of fluid in the unconstricted tube.
The dimensionless form of u with respect to u_0 is given by,

$$U = \frac{u(y)}{u_0}$$

$$=\left(-\frac{2}{a}\right).\frac{(dp/dz)}{(dp/dz)_0}.\left\{\frac{\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m \sum_{m=0}^{\infty} b_m\left(y\right)^{m+2} - \sum_{m=0}^{\infty} b_m\left(R/R_0\right)^{m+2} \sum_{m=0}^{\infty} a_m\left(y\right)^m}{\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m}\right\} \tag{19}$$

A.Volumetric flow rate:

The volumetric flow rate Q of the fluid in stenotic region is obtained with the help of equation (17),

$$Q = 2\pi R_0 \int_0^{R(z)/R_0} u(y).y\,dy,$$

$$Q = -\frac{\pi R_0^4}{2\mu_0 a}.\frac{dp}{dz}.\frac{\left\{\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m \sum_{m=0}^{\infty}\frac{b_m}{m+4}\left(R/R_0\right)^{m+4} - \sum_{m=0}^{\infty} b_m\left(R/R_0\right)^{m+2} \sum_{m=0}^{\infty}\frac{a_m}{m+2}\left(R/R_0\right)^{m+2}\right\}}{\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m} \tag{20}$$

Let Q_0 be the flow rate of plasma fluid in unconstructed tube in the absence of magnetic field and hematocrit, then;

$$Q_0 = \frac{\pi R_0{}^4}{8\mu_0}.\left(\frac{dp}{dz}\right)_0 \tag{21}$$

When the flow is steady and the system is closed, then the value of $(Q/Q_0) = 1$.

B. Pressure Gradient:

The expression for the relative pressure gradient can be obtained, with the help of equation (20) & (21) as

$$\frac{(dp/dz)}{(dp/dz)_0} = \left(\frac{a}{4}\right).\frac{\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m}{\left\{\sum_{m=0}^{\infty} b_m\left(R/R_0\right)^{m+2} . \sum_{m=0}^{\infty}\frac{a_m}{m+2}\left(R/R_0\right)^{m+2} - \sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m . \sum_{m=0}^{\infty}\frac{b_m}{m+4}\left(R/R_0\right)^{m+4}\right\}} \tag{22}$$

Now with the help of equation (22), the axial velocity U of flowing fluid in stenotic region from equation (19) can be easily obtained.

C. Wall shear stress:

The shear stress at the surface of stenosis is described with the help of equation (1) and equation (17)

$$\tau_r = -\left\{\mu(r)\,du/dr\right\}_{r=R(z)},$$

$$\tau_r = \frac{R_0}{4a}.\frac{dp}{dz}\frac{\left\{\sum_{m=0}^{\infty} b_m\left(R/R_0\right)^{m+2} . \sum_{m=0}^{\infty}(m+1)a_{m+1}\left(R/R_0\right)^m - \sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m . \sum_{m=0}^{\infty}(m+3)b_{m+1}\left(R/R_0\right)^{m+2}\right\}}{\sum_{m=0}^{\infty} a_m\left(R/R_0\right)^m} \tag{23}$$

If shear stress of plasma fluid at the wall of the normal tube is $\tau_w = -\frac{R_0}{2}(dp/dz)_0$, in the absence of hematocrit and magnetic field, then the non-dimensional form is obtained as

$$\tau = \tau_r/\tau_w.$$

IV. Result and Discussion

In this analysis the expression of axial velocity, pressure gradient and wall shear stress are obtained and computed for different values of Hartmann number with the help of MATLAB. The following data have been used for the numerical computation $R_0 = 1$, $L_0 = 1$, $d = 0$, $n = 2$, $\beta = 2.5$, $H_m = 0.45$. For better understanding the problem, we have shown the results graphically. The profile of axial velocity versus radial distance is shown in **Fig. 2.** This depicts that on increasing the Hartmann number, the axial velocity decreases. This means higher the value of Hartmann number leads to lower axial velocity. **Fig.3** illustrate the variation of axial velocity with axial distance and it shows that on decreasing the Hartmann number, the velocity of flowing fluid increases along the length of the tube and almost flatten at the absence of magnetic field.

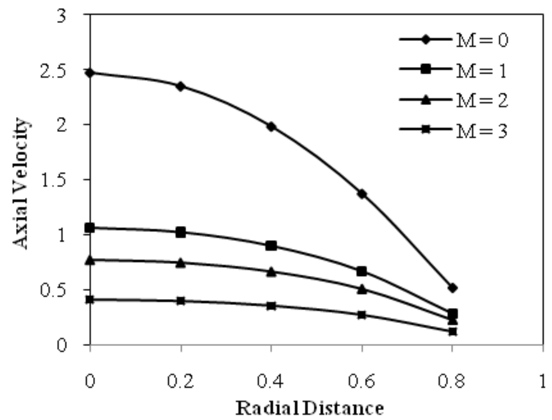

Figure 2 . Variation of axial velocity with radial distance for different values of Hartmann number (M) at $\delta = 0.1$

We observed the variation of Pressure gradient along the tube length in **Fig. 4 & Fig. 5**, for different

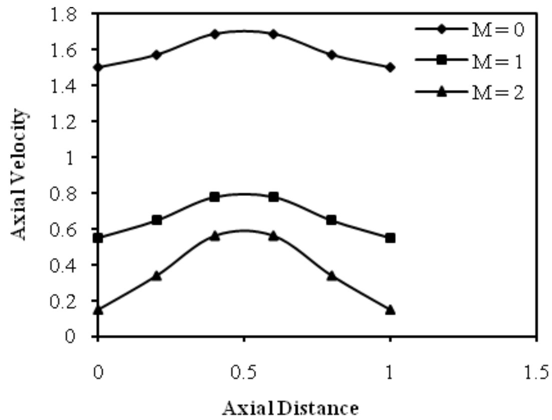

Figure 3 . Variation of axial velocity with axial distance for different values of Hartmann number (M) at $\delta = 0.1$.

numeric values of Hartmann number and stenosis height. It can be seen that the Pressure gradient increases with increase in Hartman number while curve of Pressure gradient decreases as stenosis height decreases but attains the same value at the extremities for different Hartmann number.

Fig. 6 depicts that the wall shear stress for different Hartmann number increases more rapidly. This may be the cause of breaking off stenosis and may be the

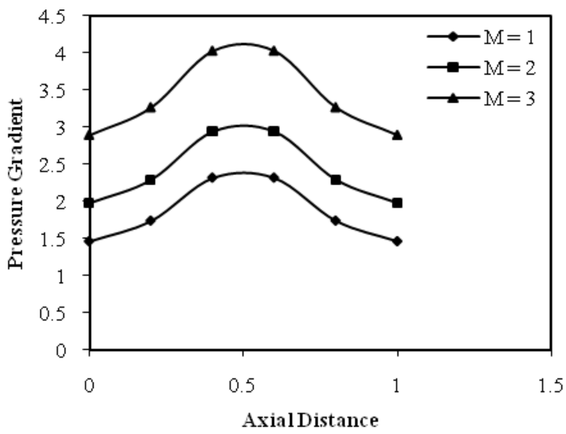

Figure 4 . Variation of Pressure gradient with axial distance for different values of Hartmann number (M) at $\delta = 0.1$

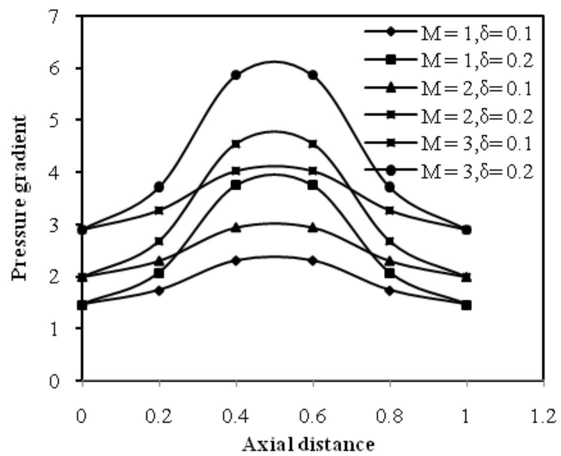

Figure 5 . Variation of Pressure gradient with axial distance for different values of Hartmann number (M) and stenosis height (δ/R_0)

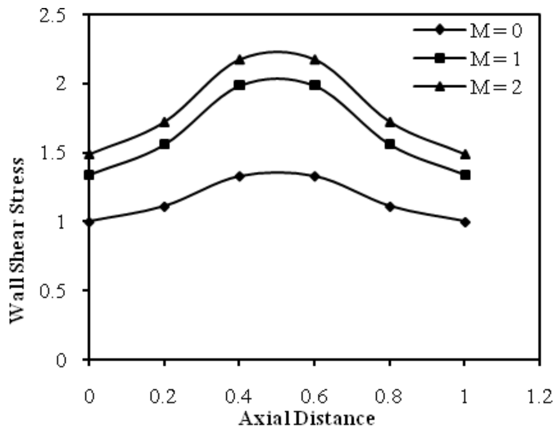

Figure 6 . Variation of wall shear stress with axial distance for different values of Hartmann number (M) at stenosis height $\delta = 0.1$

reason of different health injurious issues like stroke, paralysis or sometimes the result of sudden death. **Fig. 7** reveals that the wall shear stress for different values of stenosis height has the least value at the edge of stenosis, then it starts increasing with stenosis height along the tube length of the artery and has the maximum value at the stenosis throat, again it falls towards the least value.

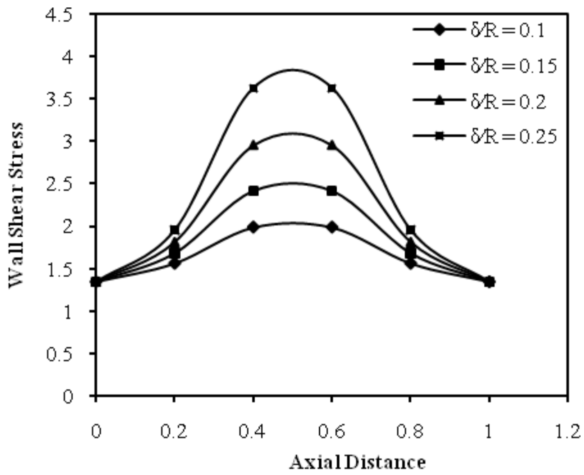

Figure 7 .Variation of wall shear stress with axial distance for different values of stenosis height .

V. CONCLUSION

In this investigation, we have developed a mathematical model on blood flow through a mild stenosed artery under the influence of magnetic field. This research supports to predict the characteristics of physiological flows and helps physicians to understand the depth of severity of stenosis. It is also useful for the surgeons to control the blood pressure as the magnetic field controls and regulates the blood flow and supply nutritions, harmones etc. smoothly to each and every body organs. It is observed from the analysis, axial velocity versus radial distance that axial velocity decreases on increasing the magnetic field intensity. While from axial velocity against axial distance we have perceive that the velocity of flowing fluid in stenosed artery increases along the tube length, on decreasing the Hartman number. The following points have been also concluded from above study:

1. The velocity profile is controlled by Magnetic field and almost flattens at the central region of stenotic artery.
2. The pressure gradient increases for magnetic field and specifies the rise in systolic pressure and fall in diastolic pressure. This shows the symptoms of frail and diseased heart.
3. Wall shear stress increases significantly with the increase of the magnetic field effect. This may be the cause of stenosis break off and ultimate result of sudden death or paralysis.
4. Magnetic field controls the flow of blood or may be deforms the velocity of fluid under observation, while treating with the patients suffering from arterial diseases or cardiovascular diseases in the treatment, to control the blood pressure.

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