Mathematical Model for Unemployment Control-A Numerical Study

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Abstract: *The paper presented a nonlinear mathematical model for unemployment using system of dynamic differential equations. This paper analyzed the situation of job competition between native unemployed and new migrant workers. We observed the effect of efforts for creating new vacancies made by government and private sector with delay and without delay as well as efforts of native unemployed and new migrant workers to become self-employed. We studied the stability of equilibrium points and carried out numerical simulation to compare with analytical result.*

Key words: Employed persons, unemployed persons, self-employment, migration, dynamic variables, delay.

I. INTRODUCTION

Unemployment is one of the annoying problem of the world which is spread day by day. Many people leave their home town, their country and go to other country for getting job. Because of this, that government realize more burden of unemployment of native unemployed and new migrant workers.

Nikolopoulos and Tzanetis ([6]) presented a model for a housing allocation of homeless families due to natural disaster. Using some concept of this paper, Misra and Singh ([1, 2]) developed a nonlinear mathematical model for unemployment. In ([2]) the model considered three dynamic variables number of unemployed persons, employed persons and newly created vacancies by government intervention. Inspired by this paper G.N.Pathan and P.H.Bhathawala ([7]) developed a mathematical model for unemployment with effect of self-employment. In ([4]) M. Neamtu presented a model for unemployment basedon some concept of ([2]) with adding two new variables said number of present jobs in the market and number of immigrants.

Based on concept of above models G.N.Pathan and P.H.Bhathawala ([9]) developed a new model of unemployment with four dynamic variables (i) Number of unemployed persons *U(t),*(ii) Number of new migrant workers *M(t),* (iii) Number of Employed persons *E(t)* and (iv) Number of newly created vacancies by government and private sector*V(t).*Using this concept we developed a new model with this four variables and analyzed the result with delay and without delay. We assumed that native unemployed and new migrant workers can apply for available vacancies and get chances equally. Therefore new migrant attracts to the territory and government realized more burden of native unemployed workers and migrant workers. So, government tried to take a step of creating new vacancies with the help of private sector. We consider the situation that native unemployed and migrant both try for their independent work and taking a step of self-employment to survive.

The paper is organized as follows: Section 2 describes Model for unemployment, Section 3 describes an equilibrium analysis, Section 4 describes the stability of equilibrium point, Numerical simulation describes in section 5 and Conclusion is given in section 6.

II . MATHEMATICAL MODEL

In the process of making a model we assume that all entrants of the category unemployment are fully qualified to do any job at any time t. Number of unemployed personsU, increases with constant rate a_1.The rate of movement from unemployed class to employed class is jointly proportional to U and (P+V - E).Where present jobs in the market provided by government and private sector is constant denoted by P. Government and private sector try to create new vacancies denoted by V and number of employed persons denoted by E. So, total available vacancies in the market are P+V-E.

We assumed that job search is open for native unemployed as well as new migrant. So, new migrant also become part of the labor workforce of the territory denoted by M.Number of migrant increases with constant rate m_1. The rate of movement of migrant workers in employment is jointly proportional to M and (P+V-E). Native unemployed and migrant both try for self-employment to survive which is proportional to number of unemployed and migrant with the rate a_5 and a_7 respectively. Employed persons joint unemployed class with rate a_4 because offired from the job or leave the job. The death and retirement rate of employed person is a_8 .The death rate of

unemployed and migrant are a_3 and m_3 respectively. The rate of migrantwho registered as unemployed or fired from their job is a_6.

$$\frac{dU}{dt} = a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U + a_6M \quad \text{_____(1)}$$

$$\frac{dM}{dt} = m_1 - m_2M(P+V-E) - a_6M - m_3M - a_7M \quad \text{_____(2)}$$

$$\frac{dE}{dt} = a_2U(P+V-E) + m_2M(P+V-E) - a_4E + a_5U + a_7M - a_8E \quad \text{_____(3)}$$

$$\frac{dV}{dt} = \alpha U(t-\tau) + \beta M(t-\tau) - \delta V \quad \text{_____(4)}$$

Here, α and β is the rate of newly created vacancies by government and Private sector respectively and δ is the diminution rate of newly created vacancies.

Lemma:

Theset $\Omega = \{(U,M,E,V):$
$0 \le U+M+E \le \frac{a_1+m_1}{\gamma}, 0 \le V \le \frac{(\alpha+\beta)(a_1+m_1)}{\gamma\delta}\}$,
where $\gamma = \min(a_3, m_3, a_8)$ is a region of attraction for the system (1) – (4) and it attracts all solutions initiating in the interior of the positive octant.

Proof:

From equation (1) – (3) we get,

$$\frac{d}{dt}(U(t)+M(t)+E(t)) = a_1 + m_1 - a_3U(t) - m_3M(t) - a_8E(t)$$

Which gives

$$\frac{d}{dt}(U(t)+M(t)+E(t)) \le a_1 + m_1 - \gamma(U(t)+M(t)+E(t))$$

Where $\gamma = \min(a_3, m_3, a_8)$.

By taking limit supremum

$$\limsup_{t\to\infty}(U(t)+M(t)+E(t)) \le \frac{a_1+m_1}{\gamma}$$

from (4) we have

$$\frac{dV}{dt} = \alpha U(t) + \beta M(t) - \delta V(t)$$

$$\therefore \frac{dV}{dt} \le (\alpha+\beta)U(t) - \delta V(t)$$

By taking limit supremum which leads to,

$$\limsup_{t\to\infty} V(t) \le \frac{(\alpha+\beta)(a_1+m_1)}{\delta\gamma}$$

This proves the lemma.

III. EQUILIBRIUM ANALYSIS

The model system (1) - (4) has only one non- negative equilibrium point $E_0(U^*, M^*, E^*, V^*)$ which obtained by solving the following set of algebraic equations.

$$a_1 - a_2U(P+V-E) - a_3U + a_4E - a_5U + a_6M = 0 \quad \text{_____(5)}$$

$$m_1 - m_2M(P+V-E) - a_6M - m_3M - a_7M = 0 \quad \text{_____(6)}$$

$$a_2U(P+V-E) + m_2M(P+V-E) - a_4E + a_5U + a_7M - a_8E = 0 \quad \text{_____(7)}$$

$$\alpha U + \beta M - \delta V = 0 \quad \text{_____(8)}$$

Taking addition of equation (5), (6) and (7)

$$a_1 + m_1 - a_3U - m_3M - a_8E = 0$$

$$\therefore E = \frac{a_1 + m_1 - a_3U - m_3M}{a_8} \quad \text{_____(9)}$$

From equation (8)

$$V = \frac{\alpha U + \beta M}{\delta} \quad \text{_____(10)}$$

$$\therefore P + V - E = \frac{aa_8U + ba_8M - (a_1 + m_1 - Pa_8)}{a_8} \quad \text{_____(11)}$$

Where $a = \frac{\alpha}{\delta} + \frac{a_3}{a_8}$, $b = \frac{\beta}{\delta} + \frac{m_3}{a_8}$

Put values of equations (9) and (11) in (5) and (6) we get,

$$A_0U^2 + A_1UM - A_2U - A_3M - A_4 = 0 \quad \text{_____(12)}$$

$$B_0M^2 + B_1UM - B_2M - B_3 = 0 \quad \text{_____(13)}$$

Where,

$A_0 = aa_2a_8,\ A_1 = a_2a_8b,$

$A_2 = [a_2(a_1 + m_1 - Pa_8) - a_3(a_4 + a_8) - a_5a_8]$

$A_3 = [a_6a_8 - m_3a_4],$

$A_4 = [a_1a_8 + a_4(a_1 + m_1)],$

$B_0 = ba_8m_2,\ B_1 = aa_8m_2,$

$B_2 = [m_2(a_1 + m_1 - Pa_8) - a_8(a_6 + m_3 + a_7)],$

$B_3 = m_1a_8.$

equation (12) and (13) represent the equation of hyperbolas.

from equation (12) ,

$$M = \frac{A_4 + A_2U - A_0U^2}{A_1U - A_3} \quad \text{_____(14)}$$

put value of equation (14) in (13) we get,

$$H_0U^3 + H_1U^2 - H_2U - H_3 = 0 \quad \text{_____(15)}$$

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Where

$H_0 = A_2A_1B_1 - A_0(A_3B_1 + A_1B_2),$

$H_1 = A_0B_0A_4 - B_0A_2^2 + A_3(A_2B_1 + A_0B_2) + A_1A_2B_2 + B_3A_1^2,$

$H_2 = 2A_2A_4B_0 - A_4(A_3B_1 + A_1B_2) + A_2A_3B_2 + 2A_1A_3B_3$

$H_3 = B_0A_4^2 + A_3A_4B_2 - A_3^2B_3.$

Since H_i , $i=1,2,3,4$ all are positive andnumber of changes in signs of equation (15) is only one. By Descart's rule equation (15) has only one positive solution say U^*. So, we get the non-negative equilibrium point of model with coordinates:

$$M^* = \frac{A_4 + A_2U^* - A_0(U^*)^2}{A_1U^* - A_3}$$

$$E^* = \frac{a_1 + m_1 - a_3U^* - m_3M^*}{a_8}$$

$$V^* = \frac{\alpha U^* + \beta M^*}{\delta}$$

So, $E_0(U^*, M^*, E^*, V^*)$ is required non negative solution of the Model.

IV. STABILITY ANALYSIS

Stability of equilibrium point without any delay:

To check the local stability for $\tau = 0$ atequilibrium point $E_0(U^*, M^*, E^*, V^*)$ we calculate the variational matrix T of the model system (1) – (4) correspondingto $E_0(U^*, M^*, E^*, V^*)$.

$$T = \begin{bmatrix} C_{11} & a_6 & C_{13} & -p_3 \\ 0 & C_{23} & p_4 & -p_4 \\ C_{31} & C_{32} & C_{33} & C_{34} \\ \alpha & \beta & 0 & -\delta \end{bmatrix}$$

Where

$p_1 = a_2(P + V - E),\ p_2 = m_2(P + V - E),$

$p_3 = a_2U,\ p_4 = m_2M,$

$C_{11} = -p_1 - a_3 - a_5,\ C_{13} = p_3 + a_4,$

$C_{23} = -p_2 - a_6 - m_3 - a_7,$

$C_{31} = p_1 + a_5,\ C_{32} = p_2 + a_7,$

$C_{33} = -p_3 - p_4 - a_4 - a_8,$

$C_{34} = p_3 + p_4.$

The characteristic equation of above matrix is

$$\lambda^4 + d_1\lambda^3 + d_2\gamma^2 + d_3\lambda + d_4 = 0 \quad \text{_____(16)}$$

Where

$d_1 = \delta - C_{33} - C_{23} - C_{11},$

$d_2 = C_{11}(C_{23} + C_{33}) - \delta(C_{11} + C_{33} + C_{23}) + C_{23}C_{33} + p_4(\beta - C_{32}) - C_{13}C_{31} + \alpha p_3,$

$d_3 = C_{33}\delta(C_{11} + C_{23}) + C_{23}C_{11}(\delta - C_{33}) - \beta p_4(C_{11} + C_{34} + C_{33}) + C_{32}p_4(C_{11} - \delta) + C_{13}C_{31}(C_{23} - \delta) + p_4a_6(\alpha - C_{31}) - \alpha(C_{13}C_{34} + p_3C_{33} + p_3C_{23})$

$d_4 = (C_{34} + C_{33})(\beta p_4C_{11} - \alpha a_6p_4) + (p_3 - C_{13})(\beta p_4C_{31} - \alpha p_4C_{32}) + C_{11}\delta(C_{32}p_4 - C_{23}C_{33}) + C_{31}\delta(C_{13}C_{23} - p_4a_6) + \alpha C_{23}(C_{13}C_{34} + p_3C_{33})$

Since, d_1, d_2, d_3, d_4 are positive then all coefficients of equation(16) are positive and some algebraic manipulation convey that $d_1d_2 > d_3$ and $d_1d_2d_3 > d_3^2 + d_1^2d_4$. So,by Routh Hurwitz criteria all roots of equation (16) are negative or having a negative real part. Therefore equilibrium point

$E_0 = (U^*, M^*, E^*, V^*)$ is locally asymptotically stable.

Stability of equilibrium point with delay:

To check the local stability for $\tau \neq 0$ at equilibrium point $E_0 = (U^*, M^*, E^*, V^*)$ we calculate the variational matrix T_1 and T_2 of the model system (1) – (4) corresponding to $E_0 = (U^*, M^*, E^*, V^*)$.

$$\frac{dx}{dt} = T_1 x(t) + T_2 x(t-\tau) \quad \text{_____(17)}$$

Where $x(t) = [u(t) \quad m(t) \quad e(t) \quad v(t)]^T$

$u(t)$, $m(t)$, $e(t)$ and $v(t)$ are small perturbations around the equilibrium point E_0.

$$T_1 = \begin{bmatrix} C_{11} & a_6 & C_{13} & -p_3 \\ 0 & C_{23} & p_4 & -p_4 \\ C_{31} & C_{32} & C_{33} & C_{34} \\ 0 & 0 & 0 & -\delta \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha & \beta & 0 & 0 \end{bmatrix}$$

Where $p_1 = a_2(P+V-E)$,

$p_2 = m_2(P+V-E)$,

$p_3 = a_2 U$, $p_4 = m_2 M$,

$C_{11} = -p_1 - a_3 - a_5$, $C_{13} = p_3 + a_4$,

$C_{23} = -p_2 - a_6 - m_3 - a_7$,

$C_{31} = p_1 + a_5$, $C_{32} = p_2 + a_7$,

$C_{33} = -p_3 - p_4 - a_4 - a_8$,

$C_{34} = p_3 + p_4$.

The characteristic equation of system (17) is

$$\psi^4 + j_1\psi^3 + j_2\psi^2 + j_3\psi + j_4 + (k_1\psi^2 + k_2\varphi + k_3)e^{-\psi\tau} = 0$$

_____(18)

Where

$j_1 = \delta - C_{33} - C_{23} - C_{11}$,

$j_2 = C_{11}(C_{33} + C_{23}) - \delta(C_{11} + C_{33} + C_{23})$
$+ C_{23}C_{33} - p_4 C_{32} - C_{13}C_{31}$,

$j_3 = C_{33}\delta(C_{11} + C_{23}) + C_{23}C_{11}(\delta - C_{33})$
$+ C_{32}p_4(C_{11} - \delta) + C_{13}C_{31}(C_{23} - \delta)$
$- p_4 a_6 C_{31}$,

$j_4 = C_{11}\delta(C_{32}p_4 - C_{23}C_{33}) + C_{31}\delta(C_{13}C_{23} - p_4 a_6)$

$k_1 = p_4\beta + p_3\alpha$,

$k_2 = \alpha(p_4 a_6 - (C_{13}C_{34} + p_3 C_{33} + p_3 C_{23}))$
$- \beta p_4(C_{11} + C_{34} + C_{33})$

$k_3 = (C_{34} + C_{33})(p_4\beta C_{11} - \alpha a_6 p_4)$
$+ (p_3 - C_{13})(\beta p_4 C_{31} - \alpha p_4 C_{32})$
$+ \alpha C_{23}(C_{13}C_{34} + p_3 C_{33})$

Now to check the stability of Eq. (18) we should not directly use Routh-Hurwitz criterion.We check that Hopf -bifurcation occurs for that we have to show that Eq. (18) has a pair of purely imaginary roots. For this we substitute $\psi = i\omega$ in Eq. (18) and we get

$$\omega^4 - ij_1\omega^3 - j_2\omega^2 + ij_3\omega + j_4 + (-k_1\omega^2 + ik_2\omega + k_3)e^{-i\omega\tau} = 0$$

_____(19)

$$\therefore \omega^4 - j_1\omega^3 i - j_2\omega^2 + ij_3\omega + j_4$$
$$+ (-k_1\omega^2 + ik_2\omega + k_3)(\cos\omega\tau - i\sin\omega\tau) = 0$$

$$\therefore \omega^4 - j_2\omega^2 + j_4 - (k_1\omega^2 - k_3)\cos\omega\tau + k_2\omega\sin\omega\tau$$
$$+ i(-j_1\omega^3 + j_3 w - (-k_1\omega^2 + k_3)\sin\omega\tau + k_2\omega\cos\omega\tau) = 0$$

_____(20)

Separating real and imaginary part of Eq. (20) we get

$$\omega^4 - j_2\omega^2 + j_4 = (k_1\omega^2 - k_3)\cos\omega\tau - k_2\omega\sin\omega\tau$$

_____(21)

$$j_1\omega^3 - j_3\omega = (k_1\omega^2 - k_3)\sin\omega\tau + k_2\omega\cos\omega\tau$$

_____(22)

By squaring and adding Eq. (21) and Eq. (22)

$$(\omega^4 - j_2\omega^2 + j_4)^2 + (j_1\omega^3 - j_3\omega)^2 = (k_1\omega^2 - k_3)^2 + k_2^2\omega^2$$

_____(23)

By taking expansion of this

$$\omega^8 + r_1\omega^6 + r_2\omega^4 + r_3\omega^2 + r_4 = 0$$

_____(24)

$r_1 = (j_1^{\,2} - 2j_2)$,
$r_2 = (j_2^{\,2} + 2j_4 - 2j_1 j_3 - k_1^{\,2})$,
$r_3 = (j_3^{\,2} - 2j_2 j_4 + 2k_1 k_3 - k_2^{\,2})$,
$r_4 = (j_4^{\,2} - k_3^{\,2})$

Substituting $\omega^2 = \sigma$ in above Eq. then we have

$$f(\sigma) = \sigma^4 + e_1\sigma^3 + e_2\sigma^2 + e_3\sigma + e_4 = 0 \quad \text{_____(25)}$$

Where
$e_1 = (j_1^{\,2} - 2j_2)$,
$e_2 = (j_2^{\,2} + 2j_4 - 2j_1 j_3 - k_1^{\,2})$,
$e_3 = (j_3^{\,2} - 2j_2 j_4 + 2k_1 k_3 - k_2^{\,2})$,
$e_4 = (j_4^{\,2} - k_3^{\,2})$

If all $e_i > 0$ (i=1,2,3,4) and satisfies Routh-Hurwitz criterion then there is no positive root of Eq. (25) i.e. all roots of Eq. (24) are negative or having a negative real part. So, by Routh-Hurwitz criterion equilibrium E_0 is asymptotically stable for all delay $\tau > 0$.

Contrary if all e_i does not satisfy the Routh-Hurwitz criterion then there is at least one positive root ω_0 exist of Eq. (24) for $e_4 < 0$. From this we get that $(j_4^{\,2} - k_3^{\,2}) < 0$ since $j_4 + k_3 > 0$ so , $j_4 - k_3 < 0$. Which gives the condition for the existence of pair of purely imaginary roots ($\pm i\omega$) of Eq. (19).

$j_4 - k_3 < 0$
i.e.

$$\begin{aligned}&\delta((p_2 + a_6 + m_3 + a_7)(a_8(p_1 + a_5) + a_3(p_3 + a_4 + a_8)))\\ &+ \delta p_4(m_3(p_1 + a_5) + a_3(a_6 + m_3))\\ &- \beta p_4(a_3 a_4 + a_8(p_1 + a_5 + a_3))\\ &- \alpha(p_4(a_6 a_8 - a_4 m_3) + p_3 a_8(p_2 + a_6 + m_3 + a_7)) < 0\end{aligned} \quad \text{_____(26)}$$

From Eq. (21) and Eq. (22) we get

$$\tan\omega\tau = \frac{(k_1\omega^2 - k_3)(j_1\omega^3 - j_3\omega) - k_2\omega(\omega^4 - j_2\omega^2 + j_4)}{k_2\omega(j_1\omega^3 - j_3\omega) + (k_1\omega^2 - k_3)(\omega^4 - j_2\omega^2 + j_4)}$$

For positive ω_0 we have corresponding τ_0 is given by

$$\tau_n = \frac{n\pi}{\omega_0} + \frac{1}{\omega_0}\tan^{-1}\left(\frac{(k_1\omega_0^{\,2} - k_3)(j_1\omega_0^{\,3} - j_3\omega_0) - k_2\omega_0(\omega_0^{\,4} - j_2\omega_0^{\,2} + j_4)}{k_2\omega_0(j_1\omega_0^{\,3} - j_3\omega_0) + (k_1\omega_0^{\,2} - k_3)(\omega_0^{\,4} - j_2\omega_0^{\,2} + j_4)}\right) \quad \text{_____(27)}$$

n=0, 1, 2, 3…..

By Butler's lemma we can say that equilibrium E_0 remains stable for $\tau < \tau_0$.

Now to check that Hopf- bifurcation occurs at τ_0 we have to check that τ_0 satisfies the transversality condition.

Lemma 2: Transversality condition is

$$\operatorname{sgn}\left[\frac{d(\operatorname{Re}(\psi))}{d\tau}\right]_{\tau=\tau_0} > 0$$

Proof: By differentiating Eq. (18) with respect to τ, we have

$$\left(\frac{d\psi}{d\tau}\right)^{-1} = \frac{4\psi^3 + 3j_1\psi^2 + 2j_2\psi + j_3 + (2k_1\psi + k_2)e^{-\psi\tau}}{\psi(k_1\psi^2 + k_2\psi + k_3)e^{-\psi\tau}} - \frac{\tau}{\psi}$$

Now,

$$\operatorname{sgn}\left[\frac{d(\operatorname{Re}(\psi))}{d\tau}\right]_{\tau=\tau_0} = \operatorname{sgn}\left[\frac{d(\operatorname{Re}(\psi))}{d\tau}\right]^{-1}_{\tau=\tau_0}$$

$$= \operatorname{sgn}\left[\operatorname{Re}\left(\frac{d\psi}{d\tau}\right)^{-1}\right]_{\psi=i\omega}$$

$$= \operatorname{sgn}\left[\frac{4\omega_0^{\,8} + 3m_1\omega_0^{\,6} + 2m_2\omega_0^{\,4} + m_3\omega_0^{\,2}}{(k_1\omega_0^{\,2} - k_3)^2 + k_2^{\,2}\omega_0^{\,2}}\right]$$

Here $m_1 = (j_1^{\,2} - 2j_2)$,
$m_2 = (2j_4 + j_2^{\,2} + k_1^{\,2} - 2j_1 j_3)$,
$m_3 = (j_3^{\,2} + k_2^{\,2} - 2j_2 j_4 - 2k_1 k_3)$

Since condition (26) is satisfied then we have positive ω_0 and for that Transversality condition is satisfies.

This shows that if condition (26) satisfies then equilibrium E_0 is asymptotically stable for $\tau < \tau_0$ (i.e. $\tau \in [0, \tau_0)$) and unstable for $\tau > \tau_0$. The condition of Hopf-bifurcation is satisfied so, periodic solution occurs when τ passes to the τ_0 for equilibrium E_0.

V. NUMERICAL SIMULATION

For the Numerical simulation using MATLAB 7.6.0 we consider the following data,

$a_1 = 5000, a_2 = 0.00004, a_3 = 0.04, a_4 = 0.004$, $a_5 = 0.03$, $a_6 = 0.1$,
$a_7 = 0.01$, $a_8 = 0.07$, $m_1 = 3000$,
$m_2 = 0.00002$, $m_3 = 0.05$, $\alpha = 0.2$,
$\beta = 0.001$, $\delta = 0.08$, $P = 10000$

The equilibrium values of the model are:

$U^* = 31978, M^* = 13101,$ $E^* = 86654,$
$V^* = 80108$

The eigenvalues of the variationalmatrix corresponding to the equilibrium point $E_0 = (U^*, M^*, E^*, V^*)$ of model system (1) - (4) are:

$-1.6004, -0.2318 + 0.0302i,$
$-0.2318 - 0.0302i, -0.0683.$

All eigenvalues are negative or having negative real part. So, equilibrium $E_0 = (U^*, M^*, E^*, V^*)$ is locally asymptotically stable.

Using above parameter and equation (24) we get $\omega = 0.1357$. From equation (27) we get $\tau = 13.66$

Figure-1:

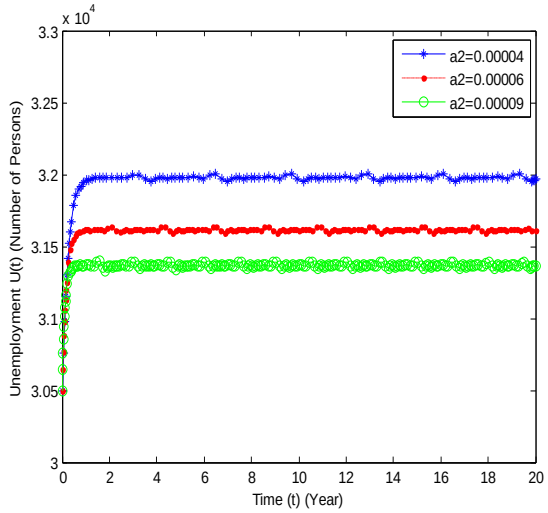


Figure-2:

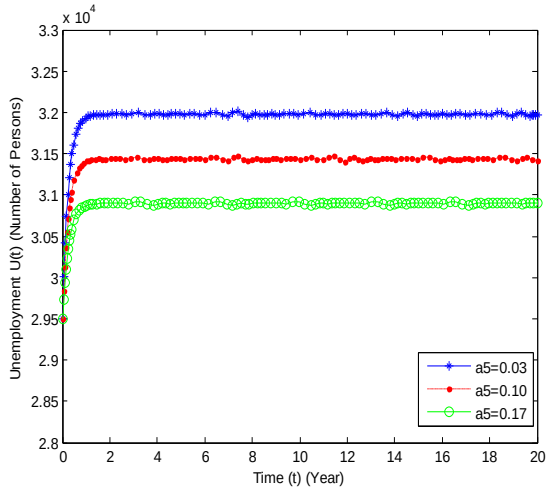
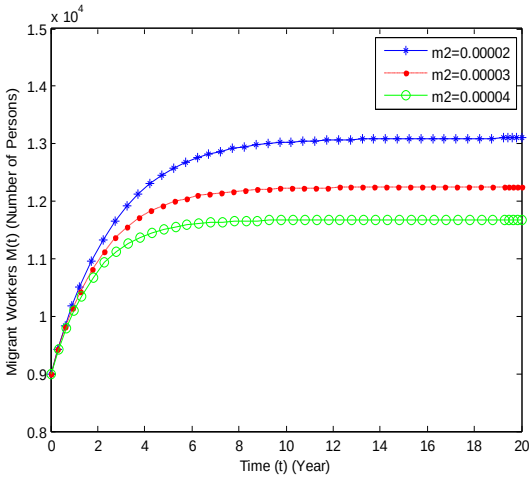


Figure-3:



VI. CONCLUSION

The paper presented a nonlinear mathematical model for unemployment using four dynamic variables: Number of unemployed persons, number of migrant workers, number of employed persons and number of newly created vacancies by government and private sector. It shows the theoretical calculation and compare it with numerical simulation using MATLAB 7.6.0.

Fig.1 shows that unemployment is lower for higher rate of persons who joined employed class. Similarly From Fig.2 it can be observe that unemployment is lower for higher self-employment rate. That is to control unemploymentmore and more people have to join employed class which is possible with efforts of government and private sector by creating new vacancies and also efforts of unemployed by create chances for self-employment. From Fig.3 we can observe that rate of unemployment of migrant workers goes lower as they joined employed class. That is government and private sector should create new vacancies for both native unemployed and migrant workers to control unemployment.

We observed that if territory allow new migrant workers then it should be create new vacancies proportional to native unemployed as well as migrant workers. We get the equilibrium point without any condition in absence of delay. In presence of delay equilibrium point is stable if it satisfies the eq. 26. Equilibrium point is unstable if it cross the critical value of delay (τ) given by eq. 27.

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