Secure Complementary Tree Domination Number of a Graph

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Abstract:
Let G be a nontrivial connected graph, a secure dominating set D of V is said to be a secure complementary tree dominating set if the induced subgraph < V – D > is a tree. A secure complementary tree dominating sets of the graph G, having minimum cardinality is called the secure complementary tree domination number denoted by γ_{sctd} of G. We have determined the exact values of secure complementary tree domination number for some standard graphs and obtained bounds for this new parameter. NORDHAUS – GADDUM type results are attained .The relationship of this parameter with other graph theoretical parameters are also discussed.

Keywords: *Domination number, Secure domination number, Complementary tree dominating set, Secure Complementary tree dominating set, Secure Complementary tree domination number.*

1. Introduction:

By a **graph** we mean a finite, simple, connected and undirected graph G = (V,E), where V is the vertex set and E is the edge set of G. Unless otherwise stated, the graph G has p vertices and q edges. For the general concepts and notations, we refer the reader to [1, 2, 13, 14].

A subset D of V is called a **dominating set** of G if every vertex in V– D is adjacent to some vertex in D. The **domination number** $\gamma(G)$ of G is the minimum cardinality taken over all the dominating sets of G. A dominating set D of a connected graph G is said to be a **connected dominating set** if the induced subgraph < D > is connected. The **connected domination number** γ_c is the minimum cardinality of a connected dominating set of G [2,3,4].

A dominating set D of V in G is a **secure dominating set** if for every $u \in V - D$, there exist a vertex *v* in D such that $v \in N(u) \cap D$ and (D – {*v*}) ∪{*u*} is a dominating set of G. The minimum cardinality of a secure dominating set is the secure domination number $\gamma_s(G)$ of G. A secure dominating set with cardinality $\gamma_s(G)$ is the $\gamma_s(G)$-set of G. Let D be a connected dominating set in G, a vertex $v \in D$ is said to **D- defend u,** where $u \in V - D$, if $uv \in E(G)$ and $(D - \{v\} \cup \{u\}$ is a connected dominating set of G. D is a **secure connected dominating set** in G if for every *u*∈ V–D, there exists $v \in D$ such that *v* is D- defends *u*. The secure connected domination number $\gamma_{sc}(G)$ of G is the minimum cardinality of a secure connected dominating set of G [3,4,5,6].

R.Kulli and B.Janakiram[10], introduced the concept of non-split domination in graphs. A dominating set D of a connected graph G is a **non-split dominating *set***, if the induced subgraph < V(G) – D > is connected. The non-split domination number $\gamma_{ns}(G)$ of G is the minimum cardinality of a non-split dominating set. In [9], S.Muthammal et. al., introduced complementary tree domination number of a graph and found many results on them.

Let D be a dominating set of a non-trivial connected graph G, if the induced subgraph < V(G) – D > is a tree then D is a **complementary tree dominating set of G.** The minimum Cardinality of the complementary tree dominating set is called the complementary tree domination number of G , denoted by $\gamma_{ctd}(G)$.

For a real number *x*, $\lfloor x \rfloor$ denotes the largest integer less than or equal to *x*.

A **Nordhaus- Gaddum-type** results is a (tight) lower or upper bound on the sum and product of parameter of a graph and its complement.

2. Secure Complementary Tree Domination Number of a Graph

Definition: 2.1

A non-empty subset D of V of a non-trivial connected graph G is called a secure complementary tree dominating set (γ_{sctd} - set), if D is a secure dominating set of G and the induced subgraph < V–D > is a tree. The minimum cardinality of a secure complementary tree dominating set is the secure complementary tree domination number $\gamma_{sctd}(G)$ of G. A set with $\gamma_{sctd}(G)$ vertices is called γ_{sctd}-set of G.

Example: 2.2. For the graph G_2 in figure 2.1

$\gamma(G_2) = 1$, $\gamma_{ctd}(G_2) = 2$, $\gamma_{sctd}(G_2) = 4$.

G2:

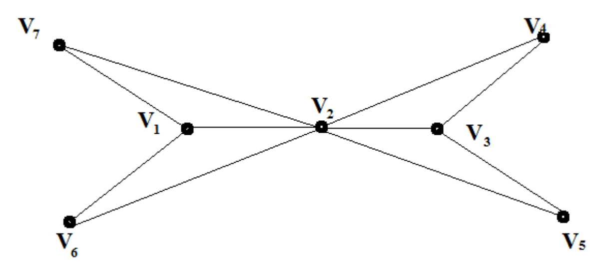

Figure.2.1: Graph with $\gamma_{ctd}(G_2) < \gamma_{sctd}(G_2)$

Example: 2.3. For the graph G_3, in figure 2.2

$$\gamma_s(G_3) = 2, \quad \gamma_{ctd}(G_3) = 2, \quad \gamma_{sctd}(G_3) = 3$$

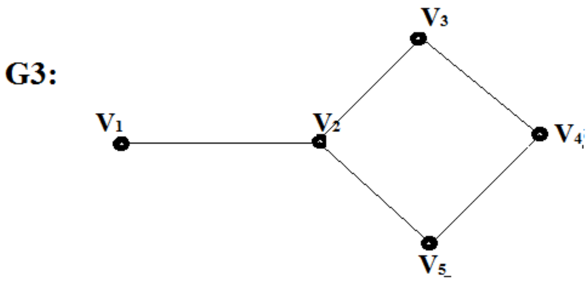

Figure.2.2: Graph with $\gamma_s(G_3) = \gamma_{ctd}(G_3) < \gamma_{sctd}(G_3)$

Example: 2.4 For the graph G_4, in figure 2.3

$\gamma(G_4) = \gamma_s(G_4) = 1, \quad \gamma_{ns}(G_4) = 1, \quad \gamma_{ctd}(G_4) = 2, \quad \gamma_{sctd}(G_4) = 2$

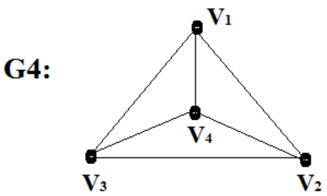

Figure: 2.3. Graph with $\gamma_{ns}(G_4) < \gamma_{sctd}(G_4)$

3. Characterization of Secure Complementary Tree Dominating Sets:

Observation: 3.1

For any connected graph G,

$$\gamma(G) \le \gamma_s(G) \le \gamma_{sctd}(G).$$

Observation: 3.2

Let G be a connected graph, then

$$\gamma_{ctd}(G) \le \gamma_{sctd}(G)$$

Proposition: 3.3

Every secure complementary tree dominating set of G, contains all the pendent vertices of G.

Proof:

Let u be a vertex of G such that deg(u) =1. Let D be a secure complementary tree dominating set of G, suppose, u is in V – D, then a vertex adjacent to u must be in D. Then the induced graph < V – D > is disconnected. Thus u must lie in D.

Theorem 3.4[10]

A secure complementary tree dominating Set D of G is minimal if and only if for every vertex *v* in D, one of the following conditions holds,

(i) *v* is an isolated vertex of < D >.

(ii). there exists a vertex u in V – D for which $N(u) \cap D = \{v\}$.

(iii). N (*v*) ∩ (V – D) = O

(iv).The induced sub graph $< (V - D) \cup \{v\} >$ of V – D $\cup \{v\}$ is either disconnected or contains a cycle.

Proof:

Let D be minimal. Assume the contrary, if there exists a vertex *v* of D such that *v* does not satisfy any of the given conditions, then by (i) and (ii), the set D' = D – {*v*} is a dominating set. By (iii) < V – D'> is connected and by (iv) < V –D'> is a tree. Hence D' is a secure complementary tree dominating set of G, a contradiction.

Conversely, let D be a secure complementary tree dominating set of G and for each vertex *v* in D, one of the four stated conditions holds. We prove that D is a minimal secure complementary tree dominating set of G. If D is not a minimal secure complementary tree dominating set, then there must exists a vertex *v* in D , such that D – {*v* } is the secure complementary tree dominating set of G. Thus *v* is adjacent to at least one vertex in D – { *v*}. Thus condition (i) does not hold. Also if D – { *v*}is a dominating set , then every vertex in V – D is adjacent to at least one vertex in D – { *v* }, condition (ii) does not hold. Since, D –{ *v* } is a secure complementary tree dominating set < V – (D – { *v* }) > is a tree , a contradiction to condition (iii) and (iv) . Hence there exists no vertex *v* in D such that *v* not satisfying any of the four conditions.

Observation: 3.5.

Let H be a spanning sub graph of a connected graph G. If H has a secure complementary tree dominating set, then $\gamma_{sctd}(G) \le \gamma_{sctd}(H)$.

4. Exact values for some standard Graphs:

(i).For a path $P_p, p \ge 4$, $\gamma_{sctd}(P_p) = p - 2$.

(ii).For a cycle $C_p, p \ge 3$, $\gamma_{sctd}(C_p) = p - 2$.

(iii) For a complete graph $K_p, p \ge 3$, $\gamma_{sctd}(K_p) = p - 2$.

(iv). For a complete bipartite $K_{m,n}$, where p = $m + n$

$$\gamma_{sctd}(K_{m,n}) = \begin{cases} \max(m,n), & form \neq n \\ \max(m,n) + 1, & form = n \end{cases}$$

(v). For a star $K_{1,p-1}, p \ge 2$,

$$\gamma_{sctd}\left(K_{1,p-1}\right) = p-1.$$

(vi).For a Bistar $B_{r,s}$with p= $r+s+2$

$$\gamma_{sctd}\left(B_{r,s}\right) = p-2 \quad for\ p \geq 6$$

(vii). For a wheel graph,W_p

$$\gamma_{sctd}\ (W_p) = \begin{cases} 2 & ; 4 \leq p \leq 7 \\ p-5 & ; \quad p \geq 8 \end{cases}$$

(viii). For Fan graph F_p

$$\gamma_{sctd}\ (F_p) = \begin{cases} 1 & ; p=3,4 \\ 2 & ; p=5,6 \\ 3 & ; p=7,8 \\ p-5 & ; p>8 \end{cases}$$

(ix). For $C_p \circ K_1$,

$$\gamma_{sctd}\left(C_p \circ K_1\right) = p \qquad p > 2$$

(x). For $K_{1,p-1} \circ K_1$,

$$\gamma_{sctd}\left(K_{1,p-1} \circ K_1\right) = p, \quad p > 2$$

5. Bounds:

Observation: 5.1

Let G be a connected graph with p ≥ 2, then

$\gamma_{sctd}(G) \leq p-1$.

Theorem: 5.2[9]

For any connected graph G(p,q) with $\delta \geq 2$, $\gamma_{sctd}(G) \geq 3p-2q-2$. The bound is sharp for the cycle $C_n, n \geq 3$.

Proof:

Let D be a γ_{sctd}-set of G. Let s be the number of edges of G having one end in D and the other in V – D. Then the number of vertices and edges in <V – D > is $p-\gamma_{sctd}(G)$ and $p-\gamma_{sctd}(G)-1$, respectively. Then, by Euler theorem

$$2[q-(p-\gamma_{sctd}(G)-1)] = \sum_{v_i \in D} d(v_i) + s$$

Since, $|V(G)-D| = p-\gamma_{sctd}(G)$ there exists at least $p-\gamma_{sctd}(G)$ edges from (V – D) to D. As deg($v_i) \geq \delta(G)$, we have,

$[q-(p-\gamma_{sctd}(G)-1)] \geq \delta(G) .\gamma_{sctd}(G) + p-\gamma_{sctd}(G)$. For $\delta(G) \geq 2$,the equation becomes, $2[q-(p-\gamma_{sctd}(G)-1)] \geq 2.\gamma_{sctd}(G) + p-\gamma_{sctd}(G)$.

(i.e) $\gamma_{sctd}(G) \geq 3p-2q-2$.

When G≅ $C_n, n \geq 3$, the bound is sharp.

Theorem: 5.3[9]

Let G be a connected graph and $\delta(G) = 1$, then$\gamma_{sctd}(G) \geq 3p-2q-m-2$, where m is the number of pendent vertices.

Proof:

Let the $\gamma_{sctd}(G)$-set of G be D, then $|D| = \gamma_{sctd}(G)$. Let s be the number of edges of G having one end in D and the other in V – D. By theorem 5.2

$$2[q-(p-\gamma_{sctd}(G)-1) = \sum_{v_i \in D} d(v_i) + s.$$

$$\geq m + 2(\gamma_{sctd}(G) - m) + p - \gamma_{sctd}(G).$$

Thus, $\gamma_{sctd}(G) \geq 3p-2q-m-2$.

When $G \cong P_p, p \geq 3$, the bounds is sharp

Theorem: 5.4

Let G be a connected graph with $p \geq 2$, then $\gamma_{sctd}(G) = p-1$ if and only if $G \cong K_{1,p-1}$.

Proof:

If $G \cong K_{1,p-1}$, then the set of end vertices of $K_{1,p-1}$ form a minimal secure complementary tree dominating set of G. Thus , $\gamma_{sctd}(G) = p-1$. Conversely, Let $\gamma_{sctd}(G) = p-1$. That is , there exists a secure complementary tree dominating set D containing p – 1 vertices, then$V-D = \{v\}$.Since D is a dominating set of G, *v* must be adjacent to atleast one vertex of D, say *u*. If u is adjacent to any one of the vertex of D, then the vertex u must be in V – D. Since D is minimal, u is adjacent to none of the vertices in D. Thus, $G \cong K_{1,p-1}$.

Theorem.5.5

Let G be a connected graph containing a cycle, then $\gamma_{sctd}(G) = p-2$ if and only if $G \cong C_p$ or K_p or to a graph G, obtained from a cycle or complete graph by attaching pendent vertices to at least one of the vertex of a complete graph or a cycle.

Proof:

It is obviously seen that for all graphs mentioned in the theorem, $\gamma_{sctd}(G) = p-2$.Conversely, let G be a connected graph containing a cycle for which $\gamma_{sctd}(G) = p-2$ (i.e) the secure complementary tree dominating set of G is $|D| = p-2$. Then, $V-D = 2 = \{u,v\}$ and$< V-D >$ is isomorphic to K_2.

Case (i):

Since we know, every pendent vertex is a member of D, any vertex of degree 1 in D is adjacent to at most one vertex in V– D and $< V-D >$ is isomorphic to K_2.

Let $D' = D - \{pendent\ vertices\}$. Then $D' \cup$ {u,v } is either a complete graph or a cycle. Otherwise there exists a vertex w in D' such that w is not adjacent to any of the vertices of $D' - \{w\}$, which is not possible.

Case (ii): $\delta(G) = 2$.

Since $(V-D) = \{u,v\}$. Let *w* be a vertex of degree ≥ 3 in G and let $w \in V-D$. This is possible only if w =u or v ,consider $w = \{u\}$. Let each vertex of D be adjacent to both u and v. If $< D >$ is complete then G is complete. Assume $< D >$ is not complete. Then there exists atleast one pair of non-adjacent vertices in D, say a and$b \in D$ and $V - \{a,b,u\}$ is a secure complementary tree dominating set of G containing p– 3 vertices, a contradiction. Therefore, there exists a vertex in D which is adjacent to exactly one of u and v and again the secure complementary tree dominating set of G has p – 3 vertices, hence $w \in D$.Since deg(w) ≥ 3, there exists atleast one vertex say $b \in D$, which is adjacent to w. Then either $V - \{b, w, u\}$ or $V - \{b, w, v\}$ will be a secure complementary tree dominating set of G. Thus there exists no vertex with degree greater than 3 in G. Hence degree of each vertex is 2. Thus $G \cong C_p$.

Case (iii), $\delta(G) \geq 3$

Let a and b be non-adjacent vertices in $< D >$, then either $V - \{a, b, u\}$ or $V - \{a, b, v\}$ will be a secure complementary tree dominating set, a contradiction. Therefore, $< D \cup \{u, v\} >$ is a complete graph .Hence $G \cong K_p$.

Theorem:5.6

For a connected graph G, with $p \geq 2$, $\gamma_{sctd}(G) \leq 2q - p + 1$.

The bound is sharp if$G \cong K_{1,p-1}$

Proof:

We have, $\gamma_{sctd}(G) \leq p - 1, p \geq 2$,. This implies

$\gamma_{sctd}(G) \leq p - 1 = 2(p - 1) - p + 1 = 2q - p + 1$.

Thus, $\gamma_{sctd}(G) \leq 2q - p + 1$.

For $G \cong K_{1,p-1}$, $\gamma_{sctd}(G) = p - 1$

Proposition: 5.7[9]

$\gamma_{sctd}(G) \geq m$,where m is the number of pendent vertices.

Proof:

Since every pendent vertex is the member of secure complementary tree dominating set, the proposition is obvious.

Theorem: 5.8[10]

For connected graph G, $\gamma_{sctd}(G) \leq p - \omega(G) + 1$, where ω(G) is the clique number.

Proof:

Let D be a set of vertices of G such that < D > is complete and let,$|D| = \omega$(G). Then for any vertex u of D , V $- D \cup \{u\}$ is a secure complementary tree dominating set of G.

Theorem: 5.9

For a tree T, $\gamma_{sctd}(T) = m$ if and only if every vertex of degree atleast 2, is a support, where m is the number of pendent vertices in T.

Proof:

Assume every vertex of degree atleast 2, is a support. If D is the set of pendent vertices of T, then D is a dominating set in T and also $< V - D >$ is a tree. Hence D is a secure complementary tree dominating set of T. Therefore $\gamma_{sct}(T) \leq m$. By prop 5.7, $\gamma_{sctd}(T) \geq m$. Thus, $\gamma_{sctd}(T) = m$. Conversely, let u be a vertex in T, such that deg (u) ≥ 2 and let D be a secure complementary tree dominating set of T. If u is not a support, then u is not adjacent to any of the vertices in D, a contradiction.

Theorem: 5.9

If T is a tree which is not a star then , $\gamma_{sctd}(T) \leq p - 2$.

Proof:

Since the tree T, is not a star, then there exists two adjacent cut vertices u and v with deg(u) and deg(v) ≥ 2. Then $V - \{u, v\}$ is a secure complementary tree dominating set of T. Hence $\gamma_{sct}(T) \leq p - 2$.

Theorem: 5.10

Let T be a tree but not a star, with p vertices, then $\gamma_{sctd}(T) = p - 2$if and only if$T \cong P_p$ or $B_{r,s}$.

Proof:

For a tree which is not a star we can easily verify that,$\gamma_{sctd}(T) = p - 2$,when T is isomorphic to either P_p or $B_{r,s}$. Conversely, let $\gamma_{sctd}(T) = p - 2$. That is, D is a secure complementary tree dominating set of T containing p – 2 vertices. Then, $V(T) - D = \{u, v\}$ and $< V(T) - D > \cong K_2$. Since T is a tree each vertex in D is adjacent to at most one vertices in$V(T) - D$ and also each vertex in $V(T) - D$ is adjacent to at least one vertex in D, as D is a dominating set. Thus

(i).if $< D >$ is an independent set, then $T \cong B_{r,s}$.

(ii). if < D > is not independent, then there exists a vertex $u \in < D >$,such that, deg(u) ≥ 1in$< D >$.

Also either $|N_j(u)| = 1$, $1 \leq j \leq diam(T) - 3$ or if $|N_j(u)| \geq 2$ for some j,$j \leq diam(T)$- 4, then, $< N_j(u) >$ in D is independent, since otherwise D – {u} is a complementary tree dominating set of T. Thus T is path for case (i) and it is a graph obtained from a path by attaching pendent vertices to atleast one of its end vertices.

6.Nordhaus – Gaddum Type results:

Theorem 6.1

Let G be a graph such that G and its complement$\bar{G}$are connected graphs with no isolates, then$\gamma_{sctd}(G) + \gamma_{sctd}(\bar{G}) \leq 2(p - 2)$ and $\gamma_{sctd}(G) . \gamma_{sctd}(\bar{G}) \leq (p - 2)^2$.

The bound is sharp if and only if $G \cong P_4$.

Proof:

From Theorem 5.1, it follow that $\gamma_{sctd}(G) \leq p - 1$and the bound is sharp if only if $G \cong K_{1,p-1}$. But for a star the complement is disconnected. Thus we have, $\gamma_{sctd}(G) \leq p - 2$. Hence the bounds directly follow.

When, $G \cong P_4$, the bounds are sharp.

7. Relation with other Graph Theoretical Parameters:

Theorem 7.1

$\gamma_{sctd}(G) +$ κ(G) $= p$ if and only if $G \cong K_{1,p-1}$ *or* C_p, where κ(G) is the vertex connectivity of G.

Proof:

Assume that G is isomorphic to $K_{1,p-1}$ or C_p, then the connectivity κ(G) of C_p is 2 and for $K_{1,p-1}$, it is 1.Thus the result follows. Conversely let $\gamma_{sctd}(G) +$ κ(G)= p.This is possible only

(i). if $\gamma_{sctd}(G) = p-1$ *and* κ(G) = 1, that is when G ≅ $K_{1,p-1}$.

(ii). if $\gamma_{sctd}(G) = p-2$ and κ(G) = 2 and so $G \cong C_p$.

Theorem 7.2

$\gamma_{sctd}(G) +$ κ$(G) \leq 2p-3$, if and only if $G \cong K_p$.

Proof:

For K_p, $\gamma_{sctd}(G) = p-2$ and κ(G) = p– 1.Thus, $\gamma_{sctd}(G) +$ κ(G) = 2p – 3.Conversely, let $\gamma_{sctd}(G) +$κ(G) = 2p – 3. This is possible only if $\gamma_{sctd}(G) = p-2$ and κ(G) = p – 1.But κ(G) = p – 1, implies $G \cong K_p$.

Theorem 7.3

For a connected graph G,$\gamma_{sctd}(G) + \Delta(G) \leq 2p-2$. The bound is sharp if and only if $G \cong K_{1,p-1}$.

Proof:

For any graph G with p vertices, $\Delta(G) \leq p-1$. By observation5.1, $\gamma_{sctd}(G) \leq p-1$,thus the proof of the theorem follows.

When $G \cong K_{1,p-1}$, $\gamma_{sctd}(G) + \Delta(G) = 2p-2$. conversely, let $\gamma_{sctd}(G) + \Delta(G) = 2p-2$. This is possible only if $\gamma_{sctd}(G) = p-1$ and $\Delta(G) = p-1$,which is possible only if $G \cong K_{1,p-1}$.

Theorem 7.4

For a connected graph G, $\gamma_{sctd}(G) + \Delta(G) = 2p-3$ if and only if $G \cong C_3, P_3, K_p$ or G is the graph obtained from a complete graph by attaching pendent vertices to exactly one of the vertex of the complete graph.

Proof:

For the graphs given in the theorem, it is clear that $\gamma_{sctd}(G) + \Delta$(G)= $2p-3$. Conversely, let $\gamma_{sctd}(G) + \Delta$(G)= $2p-3$. This is possible only,

(i).if $\gamma_{sctd} = p-1$ and Δ(G)= $p-2$

(ii).if $\gamma_{sctd} = p-2$ and Δ (G) $= p-1$

But ,$\gamma_{sctd} = p-1$, implies that, the graph G has to be a star on p vertices and for a star Δ(G)= $p-1$.Thus case (i) is not possible.

$\gamma_{sctd} = p-2$, only if G is isomorphic to C_p,K_p or to a graph G which is obtained from a complete graph by attaching pendent vertices to atleast one of the vertex of the complete graph.

Let $G \cong C_p$, then $\gamma_{sctd} = p-2$ and Δ (G) = $p-2$. Since $\gamma_{sctd}(G) + \Delta$ (G) = $2p-3$, implies $p = 3$. Thus, $G \cong C_3$.

Assume G be a graph obtained from a complete graph by attaching pendent vertices to at least one of the vertex. Let s be the number of vertices in the complete graph and t is the number of pendent vertices attached.

Then Δ (G) = $s-1+t$. Thus $\gamma_{sctd}(G) + \Delta$ (G) = $2p-3$ implies $p = s+t$. Hence G is the graph obtained from a complete graph by attaching pendent vertices at exactly one of the vertex of G.

When, $G \cong P_p$, then $\gamma_{sctds}(G) + \Delta$ (G)= $2p-3$,implies $p = 3$. Hence $G \cong P_3$.

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