Probability Generating Function of Compound Distribution

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Abstract-
The present paper introduced the compound distribution and obtained a new derivation of PGF of compound distribution in general case. Also discussed how to obtain it.
Keywords- *Compound distribution, Mean, Variance, MGF of compound distribution, Poisson distribution.*

Introduction

Compound Distributions have many Applications. Insurance is one of the application of Compound Distribution. In an individual insurance setting, we wish to model the aggregate claims during a fixed policy period for an insurance policy. In this setting, more than one claim is possible. Auto insurance and property and casualty insurance are examples. In a group insurance setting, we wish to model the aggregate claims during a fixed policy period for a group of insureds that are independent. In other words, we discuss distributions that can either model the total claims for an individual insured or a group of independent risks over a fixed period such that the claim frequency is uncertain (no claim, one claim or multiple claims).we discussed a specific type of compound distribution with the simplifying assumption of having at most one claim. We now discuss models for aggregate claims where the claim frequency includes the possibility of having multiple claims. We first define the notion of compound distributions.

In our insurance context there are number of claims which depends on the policy period. We first define the notion of compound distributions. We then discuss probability generating function.

The random variable Y is said to have a compound distribution if Y is of the form $Y = X_1 + X_2 + \cdots + X_N$

Where,

The number of terms N is uncertain the random variable X_i are independent and identically distributed (All X_i common distributed) and each X_i is Independent of N.

The sum Y as defined random sum. If $N = 0$ is realized, then we have $Y = 0$.

In case of insurance the variable N represent the number of claims generated by an individual policy or a group of independent insureds over a policy period.

The variable X_i represents the i^{th} claim. Then Y represents the aggregate claims over the fixed policy period. The random sum Y is a mixture.

Properties

We have known following properties of compound distribution. Our interest is to find probability generating function (PGF) of compound distribution we know that

1) Mean - compound distribution

The mean of Y is

$E(Y) = E[N].E[X]$

2) Variance - Compound Distribution

The Variance of the aggregate claims is

$Var(Y) = E[N]Var[X] + Var[N]E[X]^2$

3) Moment Generating function Compound Distribution

The moment generating function

$$M_y(t) = M_N[l_n M_x(t)]$$

4) Probability generating function of compound Distribution

The probability generating function $\mathbb{P}_y(t)$ is

$$\mathbb{P}_y(s) = \mathbb{P}_N(\mathbb{P}_x(s))$$

Where $\mathbb{P}_x(s)$ is probability generating function of X (claims)

Derivation

The Following is the derivation

$$\mathbb{P}_y(s) = E(s^y) = E_N[E(s^y/N)]$$
$$= E_N[E(S^{(x_1+x_2+\cdots+x_N)}/N)]$$
$$= E_N[E(s^{x1}).E(s^{x2}).....E(s^{x_N})/N]$$

Where, X_i independent identically distributed

$$= E_N\left[(\mathbb{P}_x(s))^N\right]$$
$$\mathbb{P}_y(s) = \mathbb{P}_N(\mathbb{P}_x(s))$$

This is the P.G.F. (Probability generating function) of Compound distribution

Example:

Let Y be the a random variable counting total number of casualty in traffic accident.

N -A number of accident per year.

X_i - The number of casualties in ith accident.

Y=$X_1 + X_2 + \cdots + X_N$

Here, X and N both follows Poisson distribution.

Suppose $X_i \sim P(\mu)$ and $N \sim P(\lambda)$

$\mathbb{P}_X(s) = e^{-\mu(1-s)}$ and $\mathbb{P}_N(t) = e^{-\lambda(1-t)}$

$$\mathbb{P}_y(s) = \mathbb{P}_N(\mathbb{P}_x(s)) = e^{-\lambda(1-\mathbb{P}_X(s))}$$

$$= e^{-\lambda(1-e^{-\mu(1-s)})}$$

Conclusion

In this paper we have to obtained a new derivation of PGF of compound distribution. At the end we have obtained formula is the general case of compound distribution. $\mathbb{P}_X(s)$ is the PGF of X (claims) and $\mathbb{P}_y(s)$ PGF of compound distribution.

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