Gray markov using on population forecast

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Abstract: *The population problem is one of the important indices for a regional sustainable development, the scale of population has a profound impact which is reasonable for social development of a region There are many population prediction methods, combining the grey model and markov process to predict can largely reduce the prediction error. According to Beijing at the end of the calendar year population data, caculating more accurately population of 2015, verify the validity of the method, drawing the conclusion that Beijing's population would continue to grow in the future.*

Keywords: *GM (1, 1); Markov; population forecast*

I. Introduction

The population problem has always been the important problems affecting China's economic development, holding the change of population trends play an important role in the construction of harmonious society and realizing rapid economic development . In recent years, China's population showed a trend of increasing year by year,on the one hand, the constant growth of the population provides motive effect on the economic and social development; on the other hand, increases the pressure on resource, ecology and environment. So accurately predicting Beijing population can help government department make the right decisions, and take corresponding measures, which has very important significance on solve the employment problem, developing population planning, accounting national economy and improving the social welfare.

There are a lot of research on population, haing a lot of population prediction method, such as time series model, Logistic model with differential equation method, using mathematical statistic method to predict the linear regression model, using matrix method to predict Leslie model of concrete including population growth, the method of exponential function, power function method, gray system GM (1, 1) model. Using logistic model, modified index model, grey model to structure population prediction model, upon inspection, predicted results of the combination model proves to be much better than the other three models(Wang Peng, 2009) . The combination of grey prediction model and markov chain prediction can be applied to the population forecast. According to the calculation of nanjing historical population data, we can more accurately predict the population size of 2008. Grey Markov chain can be regarded as a feasible and effective method for population prediction(Bian Huanqing, 2012). Population is an important index to measure a region development, grasping a region's population scale is good for the government to make relevant policy(Shan Chuanpeng, 2015). GM (1, 1) model can be easily affected by random disturbance of he data, and the model has a poor stability, so we use the Grey Markov model to predict the population. It shows that the prediction accuracy of the grey Markov forecasting model is better than traditional GM (1,1) prediction model and the optimization of GM (1, 1) prediction model, which has a stronger applicability and better stability(Li keshao, 2016).

II. The basic theory of gray markov

A. grey forecasting model

The grey model is given by using the differential equation to give the long-term and continuous change process of the system generation sequence. GM (1, 1) is the most commonly used model in grey forecasting. In order to weaken the randomness of original time series, firstly we need deal with the data and accumulate the cumulative generation sequence.

Setting the original time series is $X^0 = \{x_t^0, t = 1,2,\cdots,n\}$, and the accumulation generation sequence is: $X^1 = \{x_t^1, t = 1,2,\cdots n\}$, $x_t^1 = \sum_{i=1}^{t} x_i^0$. X_0 is the actual population of Beijing from 2000 to 2015, and X_1 is generated by the actual number of 2000-2015. Then using the generated sequence to establish differential equations as follows:

$$\frac{dX^1}{dt} + \alpha X^1 = \beta$$

parameter estimation of the differential equation can be show in the matrix form :

$$\left(\hat{\alpha}, \hat{\beta}\right) = \left(B^T B\right)^{-1} B^T Y$$

$$B = B_{(n-1)\times 2} = \begin{pmatrix} -\frac{1}{2}(x_1^1 + x_2^1) & 1 \\ -\frac{1}{2}(x_2^1 + x_3^1) & 1 \\ \cdots & \cdots \\ -\frac{1}{2}(x_{n-1}^1 + x_n^1) & 1 \end{pmatrix}, Y = Y_{(n-1)\times 1} = \begin{pmatrix} x_2^0 \\ x_3^0 \\ \cdots \\ x_n^0 \end{pmatrix}.$$

Y is the population size from 2001-2015. Solving the differential equations we can get:

$$\hat{x}_{t+1}^1 = \left(x_1^0 - \frac{\hat{\beta}}{\hat{\alpha}}\right) \exp(-\hat{\alpha} t) + \frac{\hat{\beta}}{\hat{\alpha}}$$

Returning to the fitting sequence:

$$\hat{x}^0_{t+1} = \hat{x}^1_{t+1} - \hat{x}^1_t = (x^0_1 - \frac{\hat{\beta}}{\hat{\alpha}})\exp(-\hat{\alpha} t)(1-\exp(\hat{\alpha}))$$

The sequence of $\left\{\hat{x}^0_t\right\}$ is to predict the number of people.

B. model evaluation

the residual test $\begin{cases} \Delta^0(t) = \left|X^{(0)}(t) - \hat{X}^{(0)}(t)\right| \\ e(t) = \Delta^{(0)}(t) / X^{(0)}(t) \end{cases}$

(2) the model accuracy test

Average residual relative value:

$$\bar{e} = \frac{1}{n-1}\sum_{t=2}^{n}|e(t)| \times 100\%$$

The average precision of the model:

$$\rho^0 = (1 - e\ (t)\ ) \times 100\%$$

C. Markov prediction model

Mathematical description of the Markov chain: if the random process to satisfy:

S is the state space of the R that can be finitely set;

(2) $n \geq 1, t_1 < t_2 < \cdots < t_n$,

There are

$$P\{X(t_n) < i_n | X(t_1) = i_1, \cdots, X(t_{n-1}) = i_{n-1}\} = P\{X(t_n) < i_n | X(t_{n-1}) = i_{n-1}\}$$

;

Then, $\{X(t), t \in T\}$ is called a markov chain.

According to conditional probability:

$$P_{ij}(m,n) = p\{X_{m+n} = j | X_m = i\}, i, j \in I, m \geq 0, n \geq 0$$

$\{X_n, n \geq 0\}$ is the n step transition probability. When transfer probability has nothing to do with the starting time m, which can be called as the homogeneous markov chain. $\hat{P}^n - p_{ij}(n)$ record as n step transfer matrix, and the elements of matrix are non negative, sum of the row element is 1.

$Y(k) = x^{(0)}(k)$, a nonstationary random sequence with markov property can be divided into state $E_i, i = 1, 2, \cdots, s$. Any state is expressed as: $E_i = [a_{1i}, a_{2i}]$.

Thus we get the transfer probability matrix :

$$P = (p_{ij})_{s \times s} = \left(\frac{n_{ij}}{n_i}\right)_{s \times s}$$

In the practical application , we may use formula: $P_n = P^n$

III. The population data of Beijing

A. data description

This article chooses the data from 2000 to 2015 of Beijing's population, the data is shown in the table below:

The year	2001	2002	2003	2004	2005
The population	1385	1423	1456	1493	1538
The year	2006	2007	2008	2009	2010
The population	1601	1676	1771	1860	1962
The year	2011	2012	2013	2014	2015
The population	2019	2069	2115	2142	2170

IV. Grey Markov Forecasting

A. grey GM (1, 1) forecast

According to the formula (1) and (2) to calculate,we can get $\hat{u} = \left(\hat{\alpha}, \hat{\beta}\right)^T = (-0.0374, 1281.103)^T$

Obtained the corresponding functions: $\hat{X}^{(1)}(k+1) = 35577.45e^{0.0374k} - 34213.45$

Then decreasing can get analog sequence of the original sequence

$$\hat{x}^0_{k+1} = 35577.45e^{0.0374k} \times (1 - e^{-0.0374})$$

Original data sequence, simulation, detailed numerical can be seen in the table below

project		GM(1，1)		
year	actual values	simulation value	residuals	relative error (%)
2000	1364	1364	0	0
2001	1385	1357.4	27.57	2.03
2002	1423	1409.2	13.78	0.98
2003	1456	1463.0	-6.99	-0.48
2004	1493	1518.8	-25.81	-1.70
2005	1538	1576.8	-38.76	-2.46
2006	1601	1636.9	-35.92	-2.19
2007	1676	1699.4	-23.38	-1.38
2008	1771	1764.2	6,78	0.38
2009	1860	1831.5	28.47	1.55
2010	1962	1901.4	60.59	3.19
2011	2019	1974	45.05	2.28
2012	2069	2049.3	19.73	0.96
2013	2115	2127.5	-12.46	-0.59
2014	2152	2208.6	-56.63	-2.56

Calculated by the above, $\bar{\varepsilon} = 1.87\%$ and $\rho^0 = (1 - \bar{\varepsilon}) \times 100\% = 98.13\%$, $\rho^0 > 95\%$,to a certain extent, this model is reasonable. And the relative errorbetween the actual value and predicted values is within 5%. Prediction results is reliable. Predicted with GM (1, 1) we get Beijing population quantity of

2015 for $\hat{X}^{(0)}_{2015} = 2292.9$; 2016 population size is $\hat{X}^{(0)}_{2016} = 2380.38$.

B. gray markov forecast

1) tate space partitioning

Markov prediction is based on every year's change of the actual values to determine the state transition probability matrix. According to related results of the above table, the relative prediction error is used to divide the state space.

This article selects four state, E1: [-2.56, -0.48]. E2: [-0.48, 0.98]; E3: [0.98, 2.28]; E4: [2.28, 3.19]. The state of the original data shows in the following table:

	0	1	2	3	4
state	E2	E3	E3	E2	E1
	5	6	7	8	9
state	E1	E1	E1	E2	E3
	10	11	12	13	14
state	E4	E4	E2	E1	E1

2) Step transtion-probablity matrix

According to the c - k equation, the 4 step transition probability matrix can be calculated as follow:

$$P^{(1)} = \begin{bmatrix} 4/5 & 1/5 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix} \quad P^{(2)} = \begin{bmatrix} 5/6 & 1/6 & 0 & 0 \\ 4/5 & 1/5 & 0 & 0 \\ 1/2 & 1/4 & 0 & 1/4 \\ 1/2 & 1/4 & 0 & 1/4 \end{bmatrix}$$

$$P^{(3)} = \begin{bmatrix} 5/6 & 1/6 & 0 & 0 \\ 5/6 & 1/6 & 0 & 0 \\ 2/3 & 2/9 & 0 & 1/8 \\ 2/3 & 2/9 & 0 & 1/8 \end{bmatrix} \quad P^{(4)} = \begin{bmatrix} 5/6 & 1/6 & 0 & 0 \\ 5/6 & 1/6 & 0 & 0 \\ 3/4 & 1/5 & 0 & 1/9 \\ 3/4 & 1/5 & 0 & 1/9 \end{bmatrix}$$

3) population prediction of Beijing

(1)Using population data of 2009, 2010, 2011 and 2012 to predict the number of 2013, results shown in the table:

The initial year	initial state	step length	E1	E2	E3	E4
2012	2	1	1	0	0	0
2011	4	2	1/2	1/4	0	1/4
2010	4	3	2/3	2/9	0	1/8
2009	3	4	3/4	1/5	0	1/9
total			2.90	0.67	0	0.44

If the predicted sequence is currently in the state E_i, then if the p_{ij} has the maximum value of the i row, then the next moment it may be the most likely to turn to the state E_j. 2013's data is in E_1, the same as the state of original division, so the prediction result is correct. The predicted value is

$$\hat{Y}^{(0)}_{2013} = \hat{X}^{(0)}_{2013} * [1 + 0.005 * (a_1 + a_2)] = 2095.12$$.

The actual value is 2115, and the relative error is 0.94%. It has a higher prediction accuracy than the gray model.

(2)Using population data of 2010, 2011 2012 and 2013 to predict the number of 2014, results shown in the table:

The initial year	initial state	step length	E1	E2	E3	E4
2013	1	1	4/5	1/5	0	0
2012	2	2	4/5	1/5	0	0
2011	4	3	2/3	2/9	0	1/8
2010	4	4	3/4	1/5	0	1/9
total			3.00	0.82	0	0.19

Seen from the result in the table above: in 2014, the population is in E_1, the same as the state of original division. The forecast is effective. Prediction value is 2175.06, and the actual value is 2152, so the relative error of 1.07%, which is higher than the gray model.

(3)Using population data of 2011 2012 2013 and 2014 to predict the number of 2015, results shown in the table:

The initial year	initial state	step length	E1	E2	E3	E4
2014	1	1	4/5	1/5	0	0
2013	1	2	5/6	1/6	0	0
2012	2	3	5/6	1/6	0	0
2011	4	4	3/4	1/5	0	1/9
total			3.23	0.71	0	0.06

Seen from the result in the table above: in 2015, the population is in E_1. Predictive value is correct. In 2015, the population of grey GM (1, 1) prediction is 2292.9, the grey markov prediction is 2258.05, the sample investigation database of 1% Sample Survey of Population shows that Beijing's population is 2170.5, prediction results more close to actual value, which has a higher prediction accuracy.

(4)Using population data of 2012, 2013, 2014 and 2015 to predict the number of 2016, results shown in the table:

The initial year	initial state	step length	E1	E2	E3	E4
2015	1	1	4/5	1/5	0	0
2014	1	2	5/6	1/6	0	0
2013	1	3	5/6	1/6	0	0
2012	2	4	5/6	1/6	0	0
total			3.3	0.70	0	0

Seen from the result in the table above: in 2015, the population is in E_1. Predictive value is $\hat{Y}^{(0)}_{2016} = 2344.20$.

V.Conclusion

Through the above analysis,we can conclde that the gray markov prediction model has a lower prediction error than the predicted results of GM (1, 1) model. It is feasible in practical application, especially in the population forecast.

According to the model, the result shows that the population of Beijing's future will maintain the trend of rising, the relevant departments can appropriately adjust the population policy.Due to the effectiveness of the grey markov model, it can be used to predict the population, which can have a positive effect on the country's population development goals and the planning of the national economy.

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